Abstract
We examine the effects of two different types of commodity taxation, specific and ad valorem, on wages and profits. We analyze two models of wage determination, one with efficiency wage setting and one with union-firm bargaining. In the former, a (locally) revenue-neutral shift from specific to ad valorem taxation leads to an increase in both employment and wages, and a reduction in profitability. In the latter, the effect on wages and profits may be reversed: predominantly ad valorem taxation raises employment but lowers wages, and under certain circumstances, the net effect is an increase in profits.

JEL Classification: H2, H32, J41, J5

Keywords: Commodity taxation; specific tax; ad valorem tax; efficiency wage; bargaining.

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# European Bank for Reconstruction and Development
COMMODITY TAXES, WAGE DETERMINATION AND PROFITS

1. Introduction

Most recent developments in the theory of commodity tax incidence look at how imperfect competition in product markets and the choice of tax (specific or ad valorem) affect the impact of taxes on prices and profits. For example, Delipalla and Keen (1992) compare the effects of ad valorem and specific taxation in an oligopolistic product market and show that a shift from specific to ad valorem taxation is associated with a relatively lower consumer price and lower profits. However, the interaction between the product and labor markets has been ignored in much of this literature. In this paper, we bring together two strands in the theoretical literature: incidence of (different forms of) commodity taxation, a fundamental issue in Public Economics, and models of wage and employment determination, an issue of great interest in Labor Economics. As we show below, extending the Delipalla and Keen model to incorporate different wage-setting theories leads to new insights on the relative effects of the two forms of commodity taxation.

The importance of the employer’s performance in the product market for wage and employment outcomes has long been recognised in the theoretical literature, dating back to post-war labor economists, and in particular in empirical studies (see, for example, Carruth and Oswald 1989; Nickell 1999). But there seems to be little systematic investigation of how the interaction between imperfectly competitive product and labor markets affects the impact of taxes on the economy (Lockwood 1990). Moreover, to the best of our knowledge, no study looks at the relative effects of different commodity taxes on the labor market. Johnson and Layard (1986), Pisauro (1991), and Petrucci (1994), using different versions of efficiency
wage models, examine the relative effects of ad valorem and specific labor taxes on wages and employment, but commodity taxation is ignored.

The main contribution of this paper is the comparison of commodity taxes in the presence of interactions between product and labor markets. For analytical simplicity, and to focus on the purpose in hand, we assume away labor taxation. The purposes of this paper are: first, to look at the (relative) effects of specific and ad valorem commodity taxation on wages and employment, and second, to examine how the interaction between wages and prices affects the impact of taxes on a firm’s profits. As we show below, the tax effect on wages is model-dependent. One key result of the paper is that a change in the mixture of taxes in favour of ad valorem may actually increase the firm’s profits, contrary to one of the results in Delipalla and Keen (1992) where the labor market is not considered. This is interesting given that tobacco multinationals often lobby for specific taxation.

We employ two popular models, one with efficiency wages and one with a bargaining structure, to analyze the effects of commodity taxation on wage and employment. There is a whole class of efficiency wage models; in all of them the underlying idea is that employers are willing to pay higher than the market-clearing wage and, thus, there is involuntary unemployment in equilibrium. Various microeconomic foundations for the efficiency wage model have been suggested in the literature: higher wage payments than necessary to hire current workers reduce shirking, due to a higher cost of job loss (see, for example, Shapiro and Stiglitz 1984); lower labor turnover costs (Stiglitz 1974; Salop 1979); improvements in the average quality of job applicants (Weiss 1980).\footnote{Even in recessions, firms are slow to cut wages to the market-clearing level because of the effect this would have on workers’ morale} Akerlof (1982) provides a sociological explanation for the efficiency wage hypothesis.
For our purposes, we employ a model that describes some of the general implications of the efficiency wage hypothesis in a simple form.

As for union bargaining, the simplest model is the monopoly union model, where the union sets the wage unilaterally and the firm chooses the level of employment (Oswald 1982). The more general case of bargaining over wages while the firm has unilateral control over employment is the right-to-manage model (Nickell and Andrews 1983). The wage is typically determined in these models as the outcome of a Nash bargain between the union and the firm, and bargains are constrained to take place on the firm’s labor demand curve. Both of these models are subject to the criticism that the wage-employment outcome is not efficient. In contrast, when the union and the firm bargain over both the wage and employment, we have the efficient bargain models. The union is constrained by the firm’s profit level and not directly by the labor demand curve, and outcomes are on the contract curve (McDonald and Solow 1981). For our purpose, we use a framework that combines elements of all three classes of model.

In the efficiency wage model, predominantly ad valorem taxation implies relatively higher industry output and employment. With a relative decrease in the industry unemployment rate, wages must rise, as otherwise workers would exert less effort and shirk on the job. Not surprisingly, the result of lower prices and higher wages is that profits fall. In the bargaining model however, the outcome is determined by factors such as the degree of union power, and the extent of collusion among firms. Interestingly, in such a model predominantly ad valorem taxation may increase a firm’s profits. The intuition is that, in bargaining models, wages tend to move in line with profit-per-employee and in some circumstances the tax shift may lead to
a rise in employment, lower profit-per-employee and hence lower wages, but higher total profit.

The structure of the paper is as follows. The efficiency wage model is presented and comparative statics of wage and employment outcomes with respect to specific and ad valorem taxes are analysed in section 2. The relative effects of the two types of taxes on wages, employment and profits are systematically examined. Section 3 presents and analyses the wage-bargaining model. Section 4 discusses the results derived in the two models employed, and concludes.

2. An Efficiency Wage Model

The framework is based on Delipalla and Keen (1992), amended for the purpose in hand. There are $n$ identical firms in the industry and each firm sells its product at price $P$. The representative firm’s production function is $x = f(e(w,u)\ell)$, where $e$ is the effort put in by its typical employee and $\ell$ is the firm’s employment; that is, we make the conventional assumption that effort and labor are multiplicative (see, for example, Solow 1979). Following much of the efficiency wage literature (see Wadhwani and Wall 1991 for supporting empirical evidence), effort is assumed to be a positive function of both the wage ($w$) and the unemployment rate ($u$), with $e_{wu} < 0$ and $e_{ww} < 0$. Workers elsewhere receive wage $\bar{w} \leq w$.

For the representative firm, profits are:

$$\pi = [(1-v)P(X) - s]f(e\ell) - w\ell, \tag{2.1}$$

2 The sign of $e_{wu}$ is unclear a priori; for simplicity, we assume it is equal to zero.
where $v$ and $s$ are the ad valorem and specific tax rates respectively; $f' > 0$, $f'' < 0$. We assume each firm forms a conjecture about how the industry output ($X$) responds to a change in its own output. That is, $dX/dx = \lambda$, where $\lambda \in (0, n]$ is assumed constant throughout.

Analogously, each firm forms a conjecture about how the industry’s unemployment rate, $u = 1 - (L/L^*)$, where $L^*$ and $L$ are the industry labor force and industry employment respectively, responds to a change in its own employment. That is, $du/d\ell = \beta$, where $\beta \in [-n/L^*, 0)$ is also assumed fixed.

Then the first-order conditions for profit maximisation are:

$$\pi_w = [(1-v)(P + fP_X\lambda) - s]f'(e_x + \ell\beta) - w = 0$$

(2.2)

$$\pi_\ell = [(1-v)(P + fP_X\lambda) - s]f'(e + \ell\beta) - w = 0$$

(2.3)

where subscripts denote partial derivatives. Dividing (2.2) by (2.3), and rearranging, yields the modified Solow condition

$$\pi_w = [(1-v)(P + fP_X\lambda) - s]f'(e + \ell\beta) - w = 0.$$  

(2.4)

Totally differentiating (2.2) and (2.3) we get

$$\begin{bmatrix} \pi_{ww} & \pi_{we} \\ \pi_{ew} & \pi_{ee} \end{bmatrix} \begin{bmatrix} dw \\ d\ell \end{bmatrix} = \begin{bmatrix} -\pi_{vv} & -\pi_{we} \\ -\pi_{ev} & -\pi_{ee} \end{bmatrix} \begin{bmatrix} dv \\ ds \end{bmatrix}$$

(2.5)

with the Hessian matrix on the left-hand side of (2.5) being negative-definite, i.e.

$$\pi_{ww} < 0, \quad \pi_{ee} < 0$$

and

$$\pi_{ww}\pi_{ee} > \left(\pi_{we}\right)^2$$

(2.6)

---

Note that equation (2.4) implies a wage at which the elasticity of effort with respect to the wage is less than one, in contrast to Solow’s result whereby a profit-maximising firm sets the wage such that the elasticity exactly equals one.
from the second-order conditions, and (2.6) implying that the determinant $\Delta$ of the matrix on the left-hand side of (2.5) is positive.

The impact on wages and employment of changes in commodity taxation is examined using comparative static analysis based on equation (2.5). Commodity taxes have a negative effect on both employment and wages. The intuition is that they increase the price of the good and hence reduce demand and output, and so employment falls. Since unemployment rises, effort also increases and firms can compensate at the margin by reducing the wage. But what is of interest here is the relative responsiveness of wages and employment to changes in specific and ad valorem taxes. This leads us to:

**Proposition 1**: In the efficiency wage model, both wages and employment are more elastic with respect to specific than ad valorem taxation.

**Proof**: See Appendix A.

Thus, commodity taxation reduces both wages and employment but the magnitude of the effect depends on the type of the tax. Both wages and employment are less elastic with respect to ad valorem taxation. This suggests that a shift away from specific towards ad valorem taxation may lead to an increase in both wages and employment. This is the issue we now address explicitly.

Following Delipalla and Keen (1992), we consider a $P$-shift, that is, a tax change of the form

$$Pdv = -ds > 0$$

(2.7)

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4 See Appendix A for the derivation of terms in equation (2.5) and comparative statics.
which tilts the balance towards ad valorem taxation whilst leaving total tax revenue unchanged at the initial equilibrium price. Before we look at the effect of such a $P$-shift on wage and employment, we show that the Delipalla and Keen result, that a $P$-shift from specific to ad valorem taxation reduces both price and profits, still holds in our model. This is summarised as

**Proposition 2:** In the efficiency wage model, a $P$-shift from specific to ad valorem taxation, in the sense of (2.7), leads to a reduction in (a) consumer price, and (b) profits, except in the special case of joint collusion ($\lambda / n = 1$) in which case they are unaffected.

**Proof:** See Appendix A. \[ \square \]

Turning now to the effect of a $P$-shift on wages and employment, the next proposition confirms the intuition underlying Proposition 1, and helps us see why a $P$-shift reduces profits.

**Proposition 3:** In the efficiency wage model, a $P$-shift from specific to ad valorem taxation leads to an increase in both wages and employment.

**Proof:** The effect on wage of an arbitrary tax reform is given by

$$dw = \left( \frac{\partial w}{\partial v} \right) dv + \left( \frac{\partial w}{\partial s} \right) ds. \tag{2.8}$$

Substituting into (2.8) for the particular reform given by (2.7) and using (A.9) gives:

$$dw = (f_P x \lambda) \left( \frac{\partial w}{\partial s} \right) dv > 0, \tag{2.9}$$

since $\partial w / \partial s < 0$ and $dv > 0$. Then, a similar argument to the above gives
\begin{equation}
\frac{d\ell}{fP_s}\left(\frac{\partial\ell}{\partial s}\right)ds > 0
\end{equation}

where use has been made of (A.10) and (A.11).

Note here that since the number of firms is fixed, total industry employment increases. The intuition underlying Proposition 3 is analogous to the one for a rise in output discussed in Delipalla and Keen (1992): under ad valorem taxation, but not under specific, output expansion becomes profitable since part of the implied reduction in sales revenue on intra-marginal units is borne by the Exchequer rather than by the firm. So at the margin, a $P$-shift gives firms an incentive to raise output and hence employment. However, with a fall in unemployment, firms have an incentive to raise the wage in order to mitigate the consequent reduction in effort.

3. **A Wage Bargaining Model**

We turn now to an alternative non-competitive model of wage and employment, namely a bargaining model, where it is assumed that each firm in the industry has to bargain with its workers, who have formed a union. As discussed above, the literature on union-firm bargaining is dominated by three models: monopoly union, right-to-manage and efficient bargain.\footnote{Manning (1987) integrates these three models into one analytical framework.} Our model combines elements of all three, as will become clear below.

In order to derive clear-cut analytical results, it is necessary to specify relatively simple functional forms for both the firm production function and the union utility function. The production function of a representative firm is simply $x = f(\ell)$. For analytical convenience,
we assume that the union is risk-neutral and cares only about the wage, hence, \( U(.) = w \). The assumption that the union is indifferent to the level of employment can be justified when there is majority voting in the union and layoffs are determined by seniority (Oswald 1993). We assume that the actual outcome is determined by a Nash-bargain, where the union’s fall-back utility is \( b \) (representing strike pay or casual work wages), assumed constant throughout, and the firm’s fall-back profit is 0. Note that because the union cares about wages only, the efficient bargain leads to outcomes that are on the labor demand curve. The extension to the case where the union cares about employment also is discussed in the next section.

The Nash solution is given by the values of \( w \) and \( \ell \) which maximise the weighted product of the union and firm's payoffs, \( G \), where:

\[
G = \mu \log(w - b) + (1 - \mu) \log \pi.
\]  

(3.1)

Here \( \mu \) represents the union’s bargaining power, \( 0 \leq \mu \leq 1 \), and

\[
\pi = \{(1 - \nu)P(X) - s\} f(\ell) - w\ell.
\]  

(3.2)

The first-order conditions are:

\[
G_w = \frac{\mu}{w - b} - \frac{(1 - \mu)}{\pi} = 0,
\]  

(3.3)

\[
G_\ell = (1 - \mu) \frac{1}{\pi} \left\{ (1 - \nu)P - s \right\} f' + (1 - \nu)P_x \lambda_{\text{ff}}' - w = 0.
\]

(3.4)

Using (3.2) and (3.4), equation (3.3) can be rewritten as:

\[
\left\{ (1 - \nu)P - s \right\} \left( f' - \frac{H}{\ell} \right) + (1 - \nu)P_x \lambda_{\text{ff}}' - (1 - \mu)b = 0.
\]  

(3.5)

Comparative statics (see Appendix B) show that commodity taxes have a negative effect on both employment and wages. The magnitude of the effect, once again, depends on the form of taxation. Thus,

\[6\] We assume the corresponding second-order conditions are satisfied.
**Proposition 4:** In the wage-bargaining model, employment is more elastic with respect to specific than ad valorem taxation.

**Proof:** See Appendix B.

Turning now to the effect of a $P$-shift, as defined in (2.7), on profits we get:

**Proposition 5:** A $P$-shift from specific to ad valorem taxation has an ambiguous effect on profits. But

(a) if the union has no bargaining power, it reduces profits, except in the special case of joint collusion ($\lambda/n = 1$) in which case profits are unaffected, and

(b) when both the union and the firm have bargaining power and $\lambda/n = 1$, it increases profits.

**Proof:** See Appendix B.

This result, the most important one of the paper, highlights the contrast with the efficiency wage model. A shift towards ad valorem taxation may (but not necessarily will) increase profits. For this result to happen it is essential that the union has some bargaining power, otherwise wages are always forced down to the competitive level and hence we are back in the model of Delipalla and Keen (1992). Note also that there is an implicit zero profit constraint in the model, and in the monopoly union case where the firm has no bargaining power, profits are always forced to zero. In the intermediate case where both parties have bargaining power, profits increase when there is joint collusion among firms. The intuition of this result is enhanced by examining the following:
Proposition 6: In the wage-bargaining model specified, a P-shift from specific to ad valorem taxation leads to an increase in employment (and hence a reduction in wages).

Proof: A P-shift implies

\[ d\ell = \frac{\partial \ell}{\partial v} dv + \frac{\partial \ell}{\partial s} ds = \left[ \frac{\partial \ell}{\partial v} - P \frac{\partial \ell}{\partial s} \right] dv = \frac{P \lambda f}{E} dv > 0. \] (3.6)

Since there is a negative relationship between employment and the wage (unless \( \mu = 0 \) – see equation (B.4)), a P-shift leads to a reduction in the wage.

As the number of firms in the industry is assumed fixed, total employment and output rise, wages fall and, in the case of joint profit maximisation, profits increase, as the relative wage reduction outweighs the price reduction.

4. Conclusions and Extensions

The main purpose of this paper is to examine the interaction between product and labor markets in the presence of different types of commodity taxation and different models of wage and employment determination. Previous work by Delipalla and Keen (1992) showed shifting the balance of taxation from specific to ad valorem in the presence of competitive labor markets unambiguously reduces profits. Our analysis shows that this strong result may not hold if wage-determination is non-competitive. In a standard efficiency wage model, the intuition behind the Delipalla and Keen result is actually strengthened: (specific and ad valorem) commodity taxes reduce both employment and wages, and a locally revenue-neutral shift in the balance towards ad valorem taxation increases both. In the bargaining model, the
tax effects on employment and wages go in opposite directions. It is this difference that gives rise to the interesting result that predominantly ad valorem taxation can be advantageous to the firm. Shifting the balance towards ad valorem taxation will increase employment and hence reduce wages. In the example above, when firms are engaged in tacit collusion, the relative reduction in the wage outweighs the price reduction and profits increase.

The result is interesting because the prevailing view, in the theoretical literature and the business world, is that it is specific taxation that favours profits (see, for example, Keen 1998, for an excellent survey on specific versus ad valorem taxation). Incorporating the labor market into the analysis, we show that this is not necessarily true. Depending on the labor and product market characteristics, predominantly ad valorem taxation can be favourable to profits. However, higher profits in our bargaining model come at the expense of lower wages for workers, and hence ad valorem taxes may encounter opposition from unions rather than firms.

The results in this paper have been derived from simple models with very specific functional forms to obtain explicit solutions. In particular, it is worth stressing that we have not shown that a shift from specific to ad valorem taxation will necessarily result in higher profits if there is joint collusion among firms in the industry. In general, these types of comparative statics cannot be signed if the union has a more general utility function that incorporates concern for employment or other variables, and perhaps risk aversion also. Furthermore, the models in this paper have all been partial equilibrium and it is unclear the extent to which they might apply in a general equilibrium framework. Nickell (1999), for example, points out that some partial equilibrium results in this general literature carry over to general equilibrium (e.g. a rise in union power typically lowers employment and raises equilibrium unemployment) but
others do not (e.g. a general rise in union power does not necessarily make all workers in the economy better off). This is a topic that merits further examination but is beyond the scope of this paper.

Our main point in this paper is the effects of a shift from one form of commodity taxation to another are far from clear-cut, and that the interaction between the product and labor markets must be considered before policy decisions are made. In future work we intend to analyse several extensions, including the introduction of labor income taxation and the optimal mix of direct and indirect taxes in different models of wage determination; allowing free entry of firms, and possible collusion among different groups of workers; and extending to an open economy.

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7 Chang (1995) looks at optimal rates of ad valorem taxes on commodities and labour in an efficiency-wage model.
APPENDIX A

• On Equation (2.5):

Differentiating (2.2) w.r.t. \( w \), and (2.3) w.r.t. \( \ell \), gives

\[
\pi_{ww} = \ell \left\{ \frac{f'w + f^2w}{f'} (1 - v) (f')^2 \ell \lambda (2P_x + \lambda f_{xx}) \right\} < 0 \quad (A.1)
\]

and

\[
\pi_{\ell \ell} = \frac{f''w^2}{f'} + \frac{f'w}{f'} + (1 - v) (f')^2 \ell \lambda (2P_x + \lambda f_{xx}) < 0 \quad (A.2)
\]

respectively, where use has been made of (2.2) and (2.4) in derivation of (A.2).

Differentiating (2.2) w.r.t. \( \ell \) and making use of (2.4), we get

\[
\pi_{\ell w} = \frac{f''w^2}{f'} + (1 - v) (f')^2 w \ell \lambda (2P_x + \lambda f_{xx}) . \quad (A.3)
\]

Accordingly

\[
\pi_{\ell s} = -f'w \ell < 0 , \quad (A.4)
\]

\[
\pi_{ss} = (P + \lambda f_{x}) \pi_{ss} < 0 , \quad (A.5)
\]

\[
\pi_{s\ell} = -f'w \ell = w \ell \pi_{ss} < 0 , \quad (A.6)
\]

and

\[
\pi_{\ell v} = (P + \lambda f_{x}) \pi_{ss} \pi_{sv} = \frac{w}{\ell} \pi_{ss} < 0 . \quad (A.7)
\]

• Comparative Statics:

From (2.5):

\[
\frac{\partial w}{\partial s} = \frac{1}{\Delta} \left\{ -\pi_{\ell \ell} \pi_{ss} + \pi_{s\ell} \pi_{ss} \right\} = \frac{1}{\Delta} \left[ -\pi_{\ell \ell} + \frac{w}{\ell} \pi_{ss} \right] \pi_{ss} \quad (A.8)
\]

using (A.4). Substituting for \( \pi_{\ell \ell} , \pi_{s\ell} \) and \( \pi_{ss} \),

\[
\frac{\partial w}{\partial s} = \frac{1}{\Delta} \left\{ f''w^2 (\ell - 1) + f'w (2e + \ell e_0 \beta) \right\} < 0 \quad (A.8)'\]
\[
\frac{\partial w}{\partial v} = \frac{1}{\Delta} \left[ -\pi_{vw} + \pi_{wv} \pi_{\ell v} \right] = \frac{1}{\Delta} \left[ -\pi_{w\ell} + \frac{w}{\ell} \pi_{wv} \right] \pi_{wv} \tag{A.9}
\]
\[
= (P + \lambda f P_X) \frac{\partial w}{\partial s} < 0
\]

(using A.5). Similarly,
\[
\frac{\partial \ell}{\partial s} = \frac{1}{\Delta} \left[ \pi_{w\ell} \pi_{ws} - \pi_{wv} \pi_{\ell s} \right] = \frac{1}{\Delta} \left[ \pi_{w\ell} - \frac{w}{\ell} \pi_{wv} \right] \pi_{ws} \tag{A.10}
\]
\[
= \frac{1}{\Delta} \ell f ' e_{ws} w < 0
\]
and
\[
\frac{\partial \ell}{\partial v} = \frac{1}{\Delta} \left[ \pi_{w\ell} \pi_{wv} - \pi_{wv} \pi_{\ell v} \right] = \frac{1}{\Delta} \left[ \pi_{w\ell} - \frac{w}{\ell} \pi_{wv} \right] \pi_{wv} \tag{A.11}
\]
\[
= (P + \lambda f P_X) \frac{\partial \ell}{\partial s} < 0.
\]

- **Proof of Proposition 1:**

Equations (A.8) and (A.9) imply
\[
\epsilon_{wv} = (P + \lambda f P_X) \frac{\epsilon_{wv} v}{s} < 0 \tag{A.12}
\]
and equations (A.10) and (A.11) imply
\[
\epsilon_{\ell v} = (P + \lambda f P_X) \frac{\epsilon_{\ell v} v}{s} < 0 \tag{A.13}
\]

Starting from a situation where we have equivalent specific and ad valorem taxes, in the sense of \(s = v P\), that is, both types of tax yield the same revenue at initial price, (A.12) and (A.13) imply
\[
\epsilon_{wv} = \frac{P + \lambda f P_X}{P} \epsilon_{wv} \tag{A.14}
\]
and
\[
\epsilon_{\ell v} = \frac{P + \lambda f P_X}{P} \epsilon_{\ell v} \tag{A.15}
\]
respectively.
Proof of Proposition 2:

(a) Noting that \( X = nf[e(w,u(\ell))\ell] \), we derive

\[
\frac{\partial X}{\partial s} = nf' \left\{ (e + \ell e_w) \frac{\partial \ell}{\partial s} + \ell e_w \frac{\partial w}{\partial s} \right\} < 0
\]  \hspace{1cm} (A.16)

and

\[
\frac{\partial X}{\partial v} = nf' \left\{ (e + \ell e_w) \frac{\partial \ell}{\partial v} + \ell e_w \frac{\partial w}{\partial v} \right\} = (P + \lambda f P_x) \frac{\partial X}{\partial s} < 0
\]  \hspace{1cm} (A.17)

using (A.11) and (A.9). Using (A.19) and that \( \frac{\partial P}{\partial s} = P_x \frac{dX}{ds} \) and \( \frac{\partial P}{\partial v} = P_x \frac{dX}{dv} \), a \( P \)-shift as defined by (2.7) leads to

\[
dP = \frac{\partial P}{\partial v} dv + \frac{\partial P}{\partial s} ds
\]  \hspace{1cm} (A.18)

\[
= \lambda f P_x \frac{\partial P}{\partial s} dv < 0.
\]

(b) From

\[
\Pi = n\pi = [(1-v)P-s]nf[e(w,u(\ell))\ell] - nw\ell
\]  \hspace{1cm} (A.19)

we derive

\[
\frac{\partial \Pi}{\partial s} = [(1-v)P-s + (1-v)P_x X] \frac{\partial X}{\partial s} - X - n \left( w \frac{\partial \ell}{\partial s} + \ell \frac{\partial w}{\partial s} \right)
\]  \hspace{1cm} (A.20)

where use has been made of (2.2), (A.16) and (2.4); and

\[
\frac{\partial \Pi}{\partial v} = P \frac{\partial \Pi}{\partial s} + n\lambda f P_x \left( w \frac{\partial \ell}{\partial s} + \ell \frac{\partial w}{\partial s} \right) (1-v)(1-\gamma)XP_x f 'e_w
\]  \hspace{1cm} (A.21)
where use has been made of (2.2), (A.17), (A.20) and (2.4). Then, using (A.20) and (A.21), a
$P$-shift implies

$$d\Pi = \frac{\partial \Pi}{\partial v} dv + \frac{\partial \Pi}{\partial s} ds$$

$$= n \lambda f P_x \left( w \frac{\partial \ell}{\partial s} + \ell \frac{\partial w}{\partial s} \right) (1 - \nu)(1 - \gamma) Xf e_{w} < 0.$$  \hspace{1cm} (A.22)
APPENDIX B

• Comparative Statics:

Totally differentiating equation (3.5),

\[
\begin{align*}
\{[(1-v)P-s+(1-v)P_x\lambda f^\prime]\n\left[f^\prime\prime+2(1-v)P_x\lambda (f^\prime)^2+(1-v)P_x\lambda^2 f (f^\prime)^2\right]-[(1-v)P-s]\mu\left(\frac{f^\prime}{\ell^2}-\frac{f}{\ell}\right)-(1-v)P_x\lambda f^\prime \mu \frac{f}{\ell}\right]d\ell \\
\{f^\prime-\frac{\mu f}{\ell}\} P+P_x\lambda f^\prime d\ell -\{f^\prime-\frac{\mu f}{\ell}\} ds = 0.
\end{align*}
\]

(B.1)

Denote the term in front of \( d\ell \) by \( E \). Noting that \( E = \pi_{\ell} - G_{\omega,\ell} \), and using the second-order conditions, we can show that \( E < 0 \). So, the effect of taxes on employment is given by:

\[
\frac{\partial \ell}{\partial s} = \frac{1}{E} \left( f^\prime - \frac{\mu f}{\ell} \right) < 0
\]

(B.2)

and

\[
\frac{\partial \ell}{\partial v} = P \frac{1}{E} \left( f^\prime - \frac{\mu f}{\ell} \right) + \frac{P_x\lambda f^\prime}{E} - \frac{P \frac{\partial \ell}{\partial s} + \frac{P_x\lambda f^\prime}{E} < 0. \)

(B.3)

The relationship between wage and employment is given by

\[
\frac{\partial w}{\partial \ell} = \frac{\mu}{\ell} \left\{ [(1-v)P-s] \left( f^\prime - \frac{f}{\ell} \right) + (1-v)P_x\lambda f^\prime \right\} < 0,
\]

(B.4)

and, thus,

\[
\frac{\partial w}{\partial s} = \frac{\partial w}{\partial \ell} \frac{\partial \ell}{\partial s} > 0,
\]

(B.5)

\[
\frac{\partial w}{\partial v} = \frac{\partial w}{\partial \ell} \frac{\partial \ell}{\partial v} > 0.
\]

(B.6)

• Proof of Proposition 4:

From equations (B.2) and (B.3), it follows that

\[
\eta_{\ell v} = P\eta_{\lambda v} + \frac{P_x\lambda f^\prime}{E} \frac{\nu v}{\ell}.
\]

(B.7)
For $s = vP$, 

$$\eta_v = \eta_0 + \frac{P_X \lambda f f^' v}{E \ell}.$$  \hspace{1cm} (B.8)

- **Proof of Proposition 5:**

On noting that $X = nf(\ell)$,

$$\frac{\partial X}{\partial s} = nf \cdot \frac{\partial \ell}{\partial s}$$ \hspace{1cm} (B.9)

and

$$\frac{\partial X}{\partial v} = P \frac{\partial X}{\partial s} + nf \cdot \frac{P_X \lambda f f^'}{E}$$ \hspace{1cm} (B.10)

using (B.3). Then, the effect of a specific tax on profits is given by

$$\frac{\partial \Pi}{\partial s} = \left[ (1-v) P - s + (1-v) P_X X \right] \frac{\partial X}{\partial s} - X - n \left( w \frac{\partial \ell}{\partial s} + \ell \frac{\partial w}{\partial s} \right)$$ \hspace{1cm} (B.11)

and the effect of the ad valorem tax is given by

$$\frac{\partial \Pi}{\partial v} = \left[ (1-v) P - s + (1-v) P_X X \right] \frac{\partial X}{\partial v} - PX - n \left( w \frac{\partial \ell}{\partial v} + \ell \frac{\partial w}{\partial v} \right)$$

$$= P \frac{\partial \Pi}{\partial s} + n \frac{P_X \lambda f f^'}{E} \left[ (1-v) P_x f f^'(n-\lambda) + (1-\mu)(w-b) \right]$$ \hspace{1cm} (B.12)

where use has been made of (B.10), (B.3), (B.6) and (3.5) and (3.4). Then a $P$-shift implies

$$d\Pi = \frac{\partial \Pi}{\partial v} dv + \frac{\partial \Pi}{\partial s} ds = \left[ \frac{\partial \Pi}{\partial v} - P \frac{\partial \Pi}{\partial s} \right] dv$$

$$= n \frac{P_X \lambda f f^'}{E} \left[ (1-v) P_x f f^'(n-\lambda) + (1-\mu)(w-b) \right] dv.$$ \hspace{1cm} (B.13)

If the union has no bargaining power, then $w = b$ and equation (B.13) $< 0$. If $0 < \mu < 1$ and $n = \lambda$, equation (B.13) $> 0$. 

REFERENCES


