ISSN 1466-0814

THE INTERTEMPORAL SUBSTITUTION MODEL OF LABOR SUPPLY IN AN OPEN ECONOMY

João Ricardo Faria* and Miguel León-Ledesma#

October 2000

Abstract

The intertemporal substitution model of labor supply has been based on closed economy models. This paper studies the intertemporal substitution hypothesis in an open economy. It derives the long run labor supply as a function of the real wage, real interest rate and real exchange rate from a standard open economy optimizing representative agent model. The paper tests the steady state solution of the model for the US and, in order to avoid the Lucas critique, it tests for the superexogeneity of the interest rate and exchange rate. In accordance with the theory, the empirical evidence is supportive of the intertemporal substitution hypothesis, the significant impact of the real exchange rate, and is robust to the Lucas critique.

JEL Classification: D90, J22, E32

Keywords: Intertemporal substitution, Labor supply, Interest rate, Exchange rate

Acknowledgements: We would like to thank Joaquim Andrade, Jorge S. Arbache, Alan Carruth, Andy Dickerson, Emmanuel Kohlscheen, Kevin Nell, Adolfo Sachsida and José Sanchez-Fung for useful comments. All errors remain our own.

Address for Correspondence: Miguel León-Ledesma, Department of Economics, Keynes College, University of Kent, Canterbury, Kent CT2 7NP, UK. Tel: +44 1227 823026; Fax: +44 1227 827850; email: mal@ukc.ac.uk

* School of Finance and Economics, University of Technology, Sydney, Australia
# Department of Economics, University of Kent, Canterbury, United Kingdom

THE INTERTEMPORAL SUBSTITUTION MODEL OF LABOR SUPPLY IN AN OPEN ECONOMY

1. Introduction

This paper presents an empirical analysis of the intertemporal substitution model of labor supply in an open economy. The intertemporal substitution hypothesis lies at the heart of labor market fluctuations, and cyclical fluctuations in employment are one of the leading forces behind the business cycle[1]. The hypothesis gained strength in the literature after the seminal paper of Lucas and Rapping (1969). The main idea is that workers, as rational maximizing agents, compare actual and expected future real wages and adjust their labor supply accordingly. If, due to an increase in the real interest rate, workers expect the future real wage to decrease relatively to the present real wage, they increase their labor supply and vice versa.

The intertemporal substitution model has been tested exhaustively. For example, the findings of Hall (1980), Abowd and Card (1987), Alogoskoufis (1987a, 1987b), Dutkowsky and Foote (1992) and Dutkowsky and Dunsky (1996) support the hypothesis. In contrast, Card (1991) in a survey of the microeconometric literature, Altonji (1982) and Mankiw *et al* (1985) present evidence against the intertemporal substitution model.

However, the theoretical and empirical work done in this area has been based on closed economy models. This paper analyses the intertemporal substitution hypothesis in an open economy. It presents a standard open economy model in which the derived labor supply is a function not only of the real wage and the real interest rate, but also of the real exchange rate. By empirically testing the long run predictions of the model, the paper is able to assess if the intertemporal substitution effect is affected by the openness of the economy. Moreover, it

---

[1] See, for instance, Barro and King (1984).

allows for examining the direct impact of the real exchange rate on labor supply. As we shall see, the open economy model provides a better specification of the labor supply function.

The paper is structured as follows. The next section presents the representative agent model of an open economy, where the labor supply curve is derived as a function of the real wage, real interest rate and real exchange rate. Section 3 presents the econometric estimation of this function for the U.S. The concluding remarks appear in section 4.

2. The Model

The model is standard in the literature (e.g., Turnovsky, 1995). The representative agent derives utility from the consumption of the domestic good (c) and the imported good $(c*)$ and disutility from labor (L). The representative agent produces a single commodity using the stock of capital (K) and labor through a well-behaved neoclassical production function $F(K,L)$. He faces increasing installation costs of investment, represented by a convex function $C(I)$ $(C(0)=0,\ C'(I)>0,\ C'(0)=1)$. Finally, the representative agent allocates his savings in foreign bonds (b) that pay an exogenously given world interest rate $(i*)$. For simplicity there is no government[2], no labor migration, and no other assets in this economy, such as real money balances.

The optimization problem in a command optimum framework is[3]:

$$\underset{c,c^*LI}{Max}\int_0^\infty U(c,c^*,L)\,e^{-\theta t}dt$$

[2] From an empirical point of view, by not introducing the government in the model we are implicitly assuming that the variables that determine labor supply are superexogenous. This point will be addressed in section 3.
[3] There is equivalence between the centralized and decentralized equilibrium as shown in Blanchard and Fischer (1989).

$$\dot{b} = \frac{1}{\sigma}[F(K,L) - c - \sigma\, c^* + \sigma\, i^*\, b - C(I)]$$

$$\dot{K} = I$$

where σ is the relative price of the foreign good in terms of the domestic good (i.e. the real exchange rate), and θ is the rate of time preference.

The use of continuous time in this model contrasts with the most commonly used stochastic discrete time models that derive the Euler equations for consumption and labor supply (see, for example, Dutkowsky and Foote, 1992, and Dutkowsky and Dunsky, 1996). However, our interest focuses on the long run steady state labor supply and not on the short run intertemporal substitution effect, which would capture transitional dynamics.

Let us assume a separable utility function between consumption and labor supply: $U(c,c^*,L) = u(c,c^*) + V(L)$. The solution of this model is straightforward. The steady state equilibrium is given by the following equations:

$$I = 0 \quad (1)$$

$$\lambda(\theta - i^*) = 0 \quad (2)$$

$$q = \frac{\lambda}{\sigma} \quad (3)$$

$$u_c(c,c^*) = \frac{\lambda}{\sigma} \quad (4)$$

$$u_{c^*}(c,c^*) = \lambda \quad (5)$$

$$F_K(K,L) = \theta \quad (6)$$

$$V'(L) = \frac{-\lambda}{\sigma} F_L(K,L) \quad (7)$$

$$F(K,L) + \sigma\, i^*\, b = c + \sigma\, c^* \quad (8)$$

where λ and q are the shadow prices of net foreign bonds and capital. The system of equations (1) to (8) is clearly block recursive. Equation (1) determines investment in the steady state. Equation (2) gives the equilibrium value of λ, which can be assumed constant, equal to 1, without loss of generality. In addition, equation (2) shows that the rate of time preference is equal to the international interest rate $(\theta = i^*)$. Given $\lambda = 1$, the rest of the system is determined. Equation (3) gives the equilibrium value of capital's shadow price (q) as the inverse of the real exchange rate. Equations (4) and (5) define the equilibrium values of consumption of the domestic good (c), and the imported good (c^*). In the same vein, equations (6) and (7) determine simultaneously the equilibrium values of *L* and *K*. Finally, equation (8) gives the optimal value of net foreign bonds.

The focus of this article is on the labor supply curve derived from this model. The labor supply curve emerges from equations (6) and (7). It is easy to see that the labor supply is a function of the real wage w (which is given by the marginal productivity of labor, $F_L(K,L)$), the real interest rate r (given by the marginal productivity of capital, $F_K(K,L)$, which in equilibrium is equal to the rate of time preference, θ, and the international real interest rate i^*), and of the real exchange rate σ:

$$L = L(w, r, \sigma) \tag{9}$$

The expected behavior of this function is that an increase in the real wage leads to an increase in labor supply. In the same fashion, an increase in the real interest rate is associated with an increase in the labor supply, which captures the intertemporal substitution effect in the steady state. The impact of the real exchange rate can be negative or positive[4]. An appreciation of the exchange rate is an improvement of the terms of trade. Thus, imported consumption goods become cheaper *vis a vis* domestic goods. Therefore, workers can keep the same level of utility by working fewer hours or they may choose to increase their labor supply to take advantage of the higher value of their income in order to increase future consumption.

3. Empirical Evidence

As already noted above, most of the empirical attempts to capture the intertemporal substitution effect in both consumption and labor supply concentrate on short run effects. Our interest rests in the long run equilibrium impact of the interest rate (intertemporal effect) and the real exchange rate (openness effect). By doing so, we can identify not only the equilibrium properties of the model, but also the transition properties as captured by an equilibrium correction mechanism. Hence, we estimate a more general model in which the dynamics are data-determined, with a theory-determined steady state solution.

In order to do so, we estimated the general solution (9) for labor supply. The model was estimated for the U.S. using quarterly data ranging from 1972:1 to 1996:4[5]. All the data used for estimation were obtained from the OECD Statistical Compendium 1997:2 and have been seasonally adjusted. The linear form of equation (9) for estimation purposes is:

$$LS_t = \alpha_0 + \alpha_1 ER_t + \alpha_2 W_t + \alpha_3 R_t + u_t \qquad (10)$$

where LS_t is labor supply, measured as the index of labor force over working age population; ER_t is the real exchange rate, measured as an index of the effective real exchange rate calculated using trade weights by the OECD (that is, $1/\sigma$); W_t is the real wage rate, which is measured as the index of after-tax total wages and salaries, adjusted by the CPI, divided by total hours of work of the employed population; and R_t is the real interest rate, and it is measured as one plus the real yield of long term US Government bonds. Other measures of the real interest rate, such as the real yield of the composite index of the NYSE, were also

---

[4] Lahiri (1996) presents a model that associates a sustained real appreciation of the domestic currency with an increase in labor supply over time.

[5] The starting date was determined by data availability. The empirical literature on exchange rates has usually focused on the analysis of the post-Bretton Woods float (1973). However, the data show enough variability of the real exchange rate before that date.

used. However, the results using the government bonds yield gave the most robust estimates, while other measures gave, in some cases, implausible results.

Pre-tests for the order of integration of the variables using the ADF and Phillips-Perron tests showed that, in all cases, the variables involved have a unit root. For this reason, and in order to allow for dynamics without imposing a priori exogeneity properties, we estimated equation (10) as a long run equilibrium correction mechanism (ECM) using Johansen's (1991, 1992) VAR method. Note also that, since we are testing the steady state solution of the model, our interest is on the long run cointegration vector between the variables involved.

The number of lags was chosen using several criteria. We started with a maximum number of 8 lags and analyzed the sensitivity of the cointegration tests and parameter values to a reduction in the number of lags. In addition, the Schwarz Bayesian Criteria (SBC) and Akaike Information Criteria (AIC) were used as additional information. Three lag lengths were finally chosen for estimation, i.e. 5, 4 and 3. The results using these lags were remarkably stable and we will report the ones using 4 lags to avoid under and over-parametrization of the model[6]. Regarding the choice of deterministic trends we chose a model without deterministic trends in the cointegrating vector and a separate drift in the equilibrium correction model. In no case deterministic trends were found to be significant in the cointegration vector. Also, this specification allows for different growth patterns of the variables involved[7].

Table 1 presents the maximum eigenvalue LR test for the null hypothesis of r cointegration vectors versus $(r+1)$. The results show that there is only one cointegration

---

[6] Insufficient lag length can lead to size distortions and overparametrization can lead to a loss of power.

[7] This is also in accordance with the results obtained in the unit roots tests where the variables were found to be random walks with a drift. See Maddala and Kim (1998) for an analysis of these issues.

vector between the variables involved. The normalized cointegration vector resulting from the maximum likelihood estimation is as follows (absolute values of *t*-ratios in parentheses):

Open Economy Model

$$LS_t = 0.2414 - 0.0766 ER_t + 0.1320 W_t + 0.6798 R_t$$
$$(3.9468) \quad (10.074) \quad (6.3435)$$

Determinant Residual Covariance: 0.0000

Log Likelihood: 1391.990; AIC: -27.705; SBC: -25.662

The results show that all the variables involved are significant at the usual confidence levels. As expected, the real wage has a significant positive impact on labor supply. Two of the results are of especial relevance. First, the real exchange rate enters the labor supply function with a negative and significant effect. An appreciation of the real exchange rate leads to a decrease in the labor supplied with an elasticity of around 0.08. Second, the effect of the interest rate is highly significant. In addition, the positive sign is evidence in favor of the intertemporal substitution hypothesis.

Comparing our results with previous studies, we can see that our estimate of the elasticity of labor supply with respect to real wages is substantially lower than obtained by Alogoskoufis (1987b) Dutkowsky and Dunsky (1996) and Mankiw *et al* (1985). In all these studies, the elasticity varies between 0.26 and 1[8]. Regarding the intertemporal effect, the semi-elasticity of labor supply with respect to the interest rate is very similar to that obtained in those three studies. This comparison, however, should be taken with care, since our

---

[8] The elasticity of LS_t with respect to W_t evaluated at the mean is 0.1664 whereas the semi-elasticity of LS_t with respect to R_t at the mean is 0.6939.

estimates provide long run elasticities, while most of the previous results refer to short-run effects.

In order to show the relevance of the inclusion of the real exchange rate in the determination of labor supply, we also estimated the cointegration vector for the solution of the model without the real exchange rate, which corresponds to the closed economy model used in previous empirical studies. The model was estimated also using 4 lags in the VAR and the results of the LR test showed the existence of only one cointegration vector[9]. The normalized vector obtained is as follows:

Closed Economy Model

$$LS_t = 0.5010 + 0.1768W_t + 0.3146R_t$$

(35.574) (7.2562)

Determinant Residual Covariance: 0.0000

Log Likelihood: 1164.935; AIC: -23.578; SBC: -22.368

The results show that, by excluding ER_t the model performs worse than the previous one. This can be seen through the smaller absolute values of the SBC and AIC (in spite of the smaller number of explanatory variables) as well as the Log Likelihood statistic. Hence, we can argue that the best specification for the labor supply function is the one including the real exchange rate. In an open economy, thus, the real exchange rate is a significant determinant of the decision to supply labor and, by assuming closed economy models, the labor supply functions could be mispecified. Note that this seems to hold true for a relatively closed

[9] The LR values for the existence of zero and at most one vector were 35.701 and 8.993 respectively for the model without a trend in the cointegration vector and a trend in the VAR. The steps taken in this case are the same as in the previous equation.

economy such as the USA, where the amount of foreign goods affected by changes in the real exchange rate is lower than in any small open economy[10].

Table 2 reports the results of the equilibrium correction model for the change in the labor supply (ΔLS_t) derived from the cointegrating vector obtained from the open economy model. As it can easily be seen, many of the lags used are not significant and it would be desirable to obtain a more parsimonious specification. We report these results, though, for consistency with the VAR assumptions. The parameter of the ECM_{t-1} is significant and shows a velocity of convergence of around 16% per quarter. Despite the number of insignificant variables, the ECM model passes all the basic specification tests although it presents slight problems of normality of the errors.

An important feature to address in this model is that of the Lucas (1976) critique. If the government wishes to minimize the business cycle by affecting the labor supply in the economy, it may choose to do it by influencing the interest rate or the exchange rate[11]. However, if agents are forward looking, government intervention can affect the way the policy variables enter the model generating variable-dependent parameters. This would make the estimated model useless for policy issues. In a time series context, Davidson *et al* (1978) argued that, if *superexogeneity* of the regressors holds, the Lucas critique does not apply. Superexogeneity holds if both the regressors in the conditional model (10) are weakly

---

[10] We test the model for a relatively closed economy such as the USA because most previous time-series tests of the intertemporal substitution hypothesis have been carried out for this country. In doing so, we are able to show that omitting the effect of the exchange rate may lead to mispecification problems in previous work.

[11] In a small open economy, the interest rate would not be considered as a policy variable. This is not the case for a large economy as the US. In addition, the real wage can hardly be considered a policy variable and, thus, we concentrate on the other two variables. Of course if the nominal wage is indexed to the interest rate, we would have a quite different picture (see, for instance, VanHoose and Waller, 1989).

exogenous, and if the model presents structural invariance[12]. Thus, we carried out an analysis of superexogeneity of ER_t and R_t in the model presented.

In order to test for weak exogeneity, we estimated marginal models for both ER_t and R_t. The instrumental variables used in the marginal model for ER_t were: a short run real interest rate such as the Federal Funds interest rate and the current account deficit as a percentage of GDP. In the marginal model for R_t we used the investment-output ratio and an indicator of the cycle such as the firms' stock over GDP. In both cases cointegration was found and we used the same procedure as for the conditional *ECM*.[13] Following Hendry and Ericsson (1991), if the lagged value of the *ECM* enters significantly in the conditional equilibrium correction equation but not in the marginal ones, then the regressors considered are weakly exogenous. Table 3 reports the F-statistics for the significance of ECM_{t-1} in each model. The results show that both ER_t and R_t can be considered weakly exogenous with respect to labor supply[14].

The test for structural stability follows Engle and Hendry (1993). Their test consists of including the squared residuals of the marginal equations and its lags in the conditional equation. If an F-test of joint significance cannot reject the null of zero coefficients, then we would accept that the effect of both variables is structurally stable and this would point to

[12] The results here presented should be taken with caution. Lindé (2000), for example, suggests that the superexogeneity tests are not capable of detecting the relevance of the Lucas critique in practice in small samples.

[13] For sake of space the results are not reported here but are available from the authors upon request.

[14] These results are reinforced by the fact that the lagged conditional *ECM* does not enter significantly the equilibrium correction equations for R_t and ER_t in the original open economy VAR.

superexogeneity. Table 4 shows that we cannot reject the null hypothesis and, thus, we can accept the superexogeneity of both ER_t and R_t[15].

Overall, thus, the empirical results seem to be supportive of the open economy model of intertemporal labor supply. We have also showed that the model estimated seems to be robust to the Lucas critique.

4. Concluding Remarks

Most of the theoretical and empirical literature on the intertemporal substitution hypothesis has been based in closed economy models. This paper studies the intertemporal substitution hypothesis in an open economy. It presents a standard representative agent open economy model in which the long run labor supply is derived as a function of the real wage, real interest rate and real exchange rate. The expected effect of these variables is in line with the neoclassical growth model. Increases in the real wage and real interest rate lead to increases in labor supply. The impact of the real exchange rate is ambiguous. The paper tests the steady state solution of the model for US data and, in order to avoid the Lucas critique, tests for the superexogeneity of the interest rate and exchange rate. Overall, the empirical evidence is supportive of the intertemporal substitution hypothesis and robust to the Lucas critique.

---

[15] For consistency with the previous results we included 4 lags of the squared errors of the marginal equations.

References

Abowd, J. M. and D. Card (1987) Intertemporal Labor Supply and Long-term Employment Contracts, *American Economic Review* 77, 50-68.

Alogoskoufis, G.S. (1987a) Aggregate Employment and Intertemporal Substitution in the UK, *Economic Journal* 97, 403-415.

Alogoskoufis, G.S. (1987b) On Intertemporal Substitution and Aggregate Labor Supply, *Journal of Political Economy*, 95, 938-960.

Altonji, J.G. (1982) The intertemporal substitution model of labour market fluctuations: An empirical analysis, *Review of Economic Studies* 49, 783-824.

Barro, R.J. and R.G. King (1984) Time-separable preferences and intertemporal substitution models of business cycles, *Quarterly Journal of Economics* 99, 819-839.

Blanchard, O.J. and S. Fischer (1989) *Lectures on Macroeconomics*, MIT Press, Cambridge.

Card, D. (1991) Intertemporal labor supply: An assessment, NBER Research Working Paper No.3602.

Davidson, J.E.H, D.F. Hendry; F. Srba and S. Yeo (1978) Econometric modelling of the aggregate time-series relationship between consumer's expenditure and income in the United Kingdom, *Economic Journal* 88, 661-692.

Dutkowsky, D.H. and R.M. Dunsky (1996) Intertemporal substitution, money, and aggregate labor supply, *Journal of Money, Credit, and Banking* 28, 216-232.

Dutkowsky, D.H. and W.G. Foote (1992) Intertemporal substitution in macroeconomics: Consumption, labor supply, and money demand, *Review of Economics and Statistics* 74, 333-338.

Engle, R.F. and D.F. Hendry (1993) Testing super exogeneity and invariance in regression models, *Journal of Econometrics* 56, 119-139.

Hall, R.E. (1980) Labor supply and aggregate fluctuations, Carnegie-Rochester Conference Series on Public Policy 12, 7-33.

Hendry, D.F. and N.R. Ericsson (1991) An econometric analysis of U.K. money demand in Monetary trends in the United States and the United Kingdom by Milton Friedman and Anna J. Schwartz, *American Economic Review* 81, 8-39.

Johansen, S. (1991) Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models, *Econometrica* 59, 1551-1580.

Johansen, S. (1992) Determination of cointegration rank in the presence of a linear trend, *Oxford Bulletin of Economics and Statistics* 54, 383-397.

Lahiri, A. (1996) Macroeconomic Effects of Devaluation Rate Changes: Dynamic Implications Under Alternative Regimes of Capital Mobility, University of California, Los Angeles, Department of Economics Working Paper: 760.

Lindé, J. (2000) Testing for the Lucas critique: a quantitative investigation. Research Department, Sveriges Riksbank, mimeo.

Lucas, R.E., Jr. (1976) Econometric policy evaluation: A critique, *Carnegie-Rochester Conference Series on Public Policy* 1, 19-46.

Lucas, R.E., Jr. and L.A. Rapping (1969) Real wages, employment, and inflation, *Journal of Political Economy* 77, 721-754.

Maddala, G.S. and I.M. Kim (1998) *Unit roots, cointegration, and structural change*, Cambridge University Press, Cambridge.

Mankiw, N.G., J.J. Rotemberg and L.H. Summers (1985) Intertemporal substitution in macroeconomics, *Quarterly Journal of Economics* 100, 225-251.

Turnovsky, S.J. (1995) *Methods of Macroeconomics Dynamics*, MIT Press, Cambridge.

VanHoose, D.D. and C.J. Waller (1989) Optimal Monetary Policy and Alternative Wage Indexation Schemes in a Model with Interest-Sensitive Labor Supply, *Journal of Macroeconomics* 11, 163-80.

Table 1: LR test for the number of cointegration vectors

Eigenvalue	Likelihood Ratio	5 percent critical value	1 percent critical value	No. of CV(s)
0.3377	61.020	47.21	54.46	None
0.1290	21.872	29.68	35.65	At most 1
0.0818	8.749	15.41	20.04	At most 2
0.0068	0.645	3.76	6.65	At most 3

Table 2: Equilibrium correction equation for labor supply

Variable	Coefficient	Std. Error	*t*-statistic	Prob.
ECM_{t-1}	-0.1604	0.0707	-2.2697	0.0260
ΔLS(-1)	-0.1537	0.1125	-1.3660	0.1759
ΔLS(-2)	-0.2131	0.1067	-1.9972	0.0493
ΔLS(-3)	-0.1628	0.0979	-1.6624	0.1005
ΔLS(-4)	0.6200	0.0919	6.7447	0.0000
ΔER(-1)	-0.0053	0.0126	-0.4221	0.6741
ΔER(-2)	-0.0067	0.0131	-0.5119	0.6102
ΔER(-3)	-0.0040	0.0130	-0.3089	0.7582
ΔER(-4)	0.0158	0.0122	1.2965	0.1987
ΔW(-1)	-0.0453	0.0722	-0.6275	0.5322
ΔW(-2)	0.0526	0.0799	0.6583	0.5123
ΔW(-3)	0.1624	0.0824	1.9702	0.0524
ΔW(-4)	-0.1781	0.0745	-2.3896	0.0193
ΔR(-1)	-0.0327	0.0663	-0.4936	0.6230
ΔR(-2)	-0.0217	0.0706	-0.3076	0.7592
ΔR(-3)	-0.0912	0.0699	-1.3043	0.1960
ΔR(-4)	-0.0303	0.0717	-0.4225	0.6737
C	0.0015	0.0007	2.2949	0.0245

R^2	0.9104	Mean dependent var	0.0016
Adjusted R^2	0.8906	S.D. dependent var	0.0097
S.E. of regression	0.0032	Akaike info criterion	-8.4820
Sum squared resid	0.0008	Schwarz criterion	-7.9982
Log likelihood	420.90	F-statistic	46.022
Durbin-Watson	1.9390	Prob(F-statistic)	0.0000
$LM_{SC}(1)$	0.1491	White heteroskticity	0.9828
$LM_{SC}(4)$	1.0269	Jarque-Bera Normal	2.1881
ARCH(1)	0.2026		

Table 3: Significance of ECM_{t-1} in marginal equations (p-values)

In conditional model	F-statistic: 5.1514 (0.026)
In marginal model for ER_t	F-statistic: 1.0571 (0.307)
In marginal model for R_t	F-statistic: 2.1688 (0.145)

Table 4: Significance of squared errors of marginal model in conditional model

In marginal model for ER_t	F-statistic: 1.2323 (0.303)
In marginal model for R_t	F-statistic: 0.1949 (0.964)