ISSN 1466-0814

SECURITY AND PRICE ARBITRAGE

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March 2000

Abstract

We examine the effect of international price arbitrage on the effectiveness of unilateral export controls. The restriction on the quality of exports of security sensitive products limits the outside option of domestic customers: if the product available on the international market is of low quality the firm can charge a high price to domestic customers for its latest technology. This effect leads the government to set looser export controls on security sensitive products.

JEL Classification: F10, L13, D74

Keywords: Arms Control, Price Arbitrage.

Acknowledgments: I am grateful to Pierre Regibeau for his valuable comments. This paper has been presented in seminars at the University of Birmingham and the University of York, I would like to thank the people who attended those seminars for their suggestions. However, I am responsible for all remaining mistakes.

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1. Introduction

The development of dual use technologies has made it difficult to give a clear answer to which products are 'security-sensitive', i.e. products which have potential military applications. It is particularly interesting to observe how the evolution of information technologies and computer capacity has provoked a change in the perception of national security among the most developed countries. This change in perception is especially evident in the US. The increasing dependency that this country has on information networks makes it the 'most vulnerable objective in the World'[1] to eventual attacks with 'viruses' or 'logic bombs' from a foreign enemy. The Presidential Commission for the Protection of Essential Infrastructures (DARPA) has been created to design defensive mechanisms in the event of 'Information Warfare'. Besides, the Bureau of Export Administration (BXA) implements several export controls in order to avoid state of art technologies becoming a threat to US national security when exported to other countries. The National Defense Authorization Act on High Performance Computers[2] controls and restricts the exports of powerful computers to Tier 3 countries. This is a group of 50 countries, which include Russia, China and Israel. Some of these countries are also producers of high tech computers although US computers have a higher quality.

[1] This is the opinion of people like John McConnell, former director of the National Security Department, and William Studeman, former subdirector of the CIA.

[2] High Performance Computers (HPC) are those of speeds above 2000 theoretical operations per second (MTOPS). The most recent regulations on the exports of HPC have been implemented in February of 1998 (Federal Register, Vol. 63, No. 22. Tuesday, February 3, 1998. Rules and Regulations).

The purpose of this paper is to examine the effect of quality restrictions on the domestic and foreign producers' profits and on the welfare of the country that sets export controls.

This work is related to a large number of papers about 'strategic' trade policy in which governments use various trade tools (e.g. tariffs, quotas or subsidies) in order to affect the rivalry between foreign and domestic firms[3]. The present paper introduces two main innovations. Firstly, because of the security concern of the governments, we focus on a new policy tool, namely restrictions on the quality of exported goods. Secondly, in order to reflect the 'free-flowing' nature of the international markets, we consider the effect of international price arbitrage on the quality restriction. The security concern of exporters of weapons has been analyzed in the Arms Trade literature[4].

There are very few papers that analyze trade policies in the presence of price arbitrage. Donnenfeld (1988) examines the effect of commercial policy on the composition and quality of imports which are supplied by a multiproduct foreign monopolist that sells goods of variable quality to a population of buyers which differ in their willingness to pay for quality. In his paper the foreign monopolist uses product differentiation as a device to discriminate among domestic consumers whose preferences differ. He examines the implications for trade policy of this type of discrimination. Restricting the quantity of imports is shown to have a positive effect on the welfare of consumers.

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[3] See e.g. Brander and Spencer (1983, 1985). For a review of this literature see Helpman and Krugman (1987, 1989).
[4] E.g. Levine *et al*. (1994), Levine and Smith (1995) and García-Alonso (1999).

Markusen and Venables (1988) compare the effect of trade and industrial policies in segmented with non segmented markets. They use a model in which firms located in two different countries compete in quantities under the existence of transport costs between the two countries. In the segmented market situation, firms decide the domestic and foreign quantities, taking as given the quantity that the foreign competitor sells in the domestic and foreign country individually. In the non segmented market situation, however, firms decide the aggregate quantity, taking as given the aggregate quantity that the competitor sells in both markets. The paper concludes that the effect of policy is greater when markets are segmented than when they are not segmented.

Our paper analyzes the effect of price arbitrage on the effect of exported quality restrictions on domestic welfare. We consider two firms located in different countries which produce a good with an exogenously given quality. One of the firms is restricted by the domestic government to sell a lower quality in the foreign market. There are no transport costs between countries. However, each consumer is located at a different distance from the firms; therefore they each perceive the goods that the firms offer as imperfect substitutes. We compare the optimal restriction on the quality exported in the case in which the non restricted firm can set two different prices in the home and foreign market (i.e. no price arbitrage) with the case in which it must sell its product at the same price in both markets (i.e. perfect price arbitrage).

It is shown that, under perfect price arbitrage, a looser restriction on the quality of the security sensitive good exported has a negative effect on the domestic profits of the restricted firm and a positive effect on consumer welfare. We prove that when the optimal proportion of quality exported in the absence of price arbitrage is sufficiently high, the introduction of perfect price arbitrage implies an increase in the optimal proportion of quality exported. This result does not change with the introduction of elastic individual demands.

The rest of the paper is organized as follows. Section 2 presents the model and compares the results with and without perfect price arbitrage between the two countries. Section 3 extends the model to the case in which consumers have elastic demands. Finally, section 4 presents the main conclusions of the paper.

2. The Model

Consider two firms located in different countries, *A* and *B*. Firms produce a good with quality q_A and q_B respectively. Both firms sell their product in the home and the foreign market.

Consumers buy either one or zero units of the good. Their willingness to pay is increasing in the quality of the product, but they perceive the goods produced by the two firms as imperfect substitutes. This is modeled by assuming that consumers are 'located' at different distances from the firms. This, together with the existence of transportation costs, different for each consumer, allows a firm with a lower quality product to have a positive share in the market. More precisely, consumers in both countries are uniformly distributed along the 0-1 line according to their preference for the two firms' products (firm and consumers in country *A* will be usually referred to as *domestic* firm and consumers). Firms *A* and *B* are located at '0' and '1' on the line respectively. The utility function of a domestic consumer *i* that buys from firm *j* has the following form:

$$U_{ij}[q_j, P_j] = R + q_j - \lambda d_{ij} - P_j \,, \qquad (1)$$

where U_{ij} is the utility that consumer *i* gets from buying good *j*, $j = A, B$; d_{ij} is the distance between consumer *i* and firm *j*; λ is a positive parameter and q_j and P_j are the quality[5] and price that firm *j* offers. *R* is a positive constant which we assume to be high enough for the participation constraint of consumers to be fulfilled. Foreign consumers have the same utility function.

The government in country *A* restricts the quality that the domestic firm, firm *A*, can export; it only allows the firm to export a proportion of the quality available in the domestic market (we denote this proportion γ). Therefore, we will have that the domestic firm offers a quality q_A in its domestic market and a quality q_A^* in its foreign market such that:

$$q_A^* = \gamma q_A . \tag{2}$$

The government in country *B* does not restrict quality, hence, the foreign firm, firm *B*, sells the same quality at home and abroad since the willingness to pay for consumption is increasing in quality; we denote this quality q_B.

The domestic firm sets a price P_A in its domestic market and a price P_A^* in its foreign market and the foreign firm sets prices P_B^* and P_B its home and foreign markets respectively.

In equilibrium, it must be the case that domestic consumers in country *A* are not better off by buying the lower quality good that the home firm sells in the foreign market. In other words,

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[5] As in Economides (1989), quality is normalized so that the increase of one unit in its level pushes up the utility for the product by one unit. One could think of it as the speed of a computer.

we must have that $U_{iA}[q_A, P_A] \geq U_{iA}[q_A^*, P_A^*]$. For this to be the case the following arbitrage condition must hold:

$$q_A(1-\gamma) \geq P_A - P_A^*. \tag{3}$$

There is a distance from firm *A*, denoted *y*, at which a consumer in country *A* is indifferent between buying from the domestic or the foreign firm. Correspondingly, the consumer from country *B* which is at a distance *x* from firm *A* is indifferent between buying from either of the two firms. Therefore, a proportion *y* of consumers in country *A* buy from the domestic firm, firm *A*, and proportion $(1-y)$ of the consumers of country *A* import from firm *B*. Also, we have that a proportion *x* of consumers of country *B* import from firm *A* and a proportion $(1-x)$ of country *B*'s consumers buy from their domestic firm, firm *B* (see Figure 1).

Figure 1: The security sensitive market

0 ______________ *y* __________ 1

FIRM *A* FIRM *B*

0 _____ *x* ___________________ 1

2.1 No Price Arbitrage

We start by assuming that firm *B* can discriminate between the home and foreign consumers and therefore set different domestic and export prices. We will use this as a benchmark for comparison with the case when there is perfect price arbitrage between the two countries.

2.2.1 Firms' problem

In order to solve the maximization problem of the firms, we first have to derive the domestic and foreign demand of each of the two firms. These are given by the locations of the indifferent domestic and foreign consumers: y and x. Using equation (1), we obtain the following firm A's export demand:

$$x = \frac{\gamma q_A - q_B + P_B^* - P_A^*}{2\lambda} + \frac{1}{2}. \tag{4}$$

Similarly, we obtain firm A's domestic demand:

$$y = \frac{q_A - q_B + P_B - P_A}{2\lambda} + \frac{1}{2}. \tag{5}$$

Since firm A sells goods with different qualities in the domestic and foreign market, it has two decision variables in the maximization problem which are domestic and exports price (for simplicity, we assume zero production costs):

$$\underset{\{P_A, P_A^*\}}{Max}\ \alpha\pi_A + \pi_A^* = \alpha y P_A + x P_A^*, \tag{6}$$

where π_A are the profits that firm A obtains in the domestic market, π_A^* are the profits it obtains in the export market and α is the relative size of the market in country A with respect to the market in country B. We can think of it as the density of consumers along the line in country A.

Substituting x and y (equations (4) and (5)) into the profit function of firm A (equation (6)) we can derive the first order conditions for its maximization problem:

$$P_A = \frac{q_A - q_B}{2} + \frac{P_B}{2} + \frac{\lambda}{2}, \tag{7}$$

$$P_A^* = \frac{\gamma q_A - q_B}{2} + \frac{P_B^*}{2} + \frac{\lambda}{2}. \tag{8}$$

These are the reaction functions of the domestic and export price of firm A with respect to the export and domestic price of firm B. They are both upward sloping.

In the absence of price arbitrage, firm *B* can also discriminate between the home and the foreign market even though it sells the good with the same quality in both markets. The maximization problem is:

$$\underset{\{P_B^*, P_B\}}{Max}\ \pi_B^* + \alpha\pi_B = P_B^*(1-x) + P_B\alpha(1-y),$$

where, π_B^* are the profits that firm *B* obtains in the domestic market and π_B are the profits it obtains in the export market.

Substituting *x* and *y* into the profit function of firm *B* and differentiating we get the following first order conditions:

$$P_B = \frac{1}{2}(\lambda - q_A + q_B + P_A), \qquad (9)$$

$$P_B^* = \frac{1}{2}(\lambda - \gamma q_A + q_B + P_A^*). \qquad (10)$$

These are the reaction functions of firm B which are clearly upward sloping.

Introducing equations (7) and (8) in the above expressions we obtain the equilibrium domestic and export price of firm *B* in the absence of price arbitrage:

$$P_{Bna}^* = \lambda - \frac{\gamma q_A - q_B}{3}, \qquad (11)$$

$$P_{Bna} = \lambda - \frac{q_A - q_B}{3}. \qquad (12)$$

Introducing equation (12) into equation (7), we get the domestic price of firm *A*:

$$P_{Ana} = \lambda + \frac{q_A - q_B}{3}, \qquad (13)$$

and introducing equation (11) into equation (8) we get the exports price of firm *A*:

$$P_{Ana}^* = \lambda + \frac{\gamma q_A - q_B}{3}. \qquad (14)$$

Finally we derive the domestic and foreign demand of firm *A*:

$$x_{na} = \frac{\gamma q_A - q_B + P^*_{Bna} - P^*_{Ana}}{2\lambda} + \frac{1}{2} = \frac{\gamma q_A - q_B}{6\lambda} + \frac{1}{2}, \tag{15}$$

$$y_{na} = \frac{q_A - q_B + P_{Bna} - P_{Ana}}{2\lambda} + \frac{1}{2} = \frac{q_A - q_B}{6\lambda} + \frac{1}{2}. \tag{16}$$

We summarize the relevant properties of prices and demand in the following proposition:

Proposition 1: *In the absence of price arbitrage between countries, an increase in the proportion of quality exported,* γ*, has a positive effect on the home firm export price,* P^*_{Ana} *and the exports demand for the domestic firm,* x_{na}*, and a negative effect on the competitor's domestic price,* P^*_{Bna}.

However, a variation in the proportion of quality exported does not affect the domestic price, P_{Ana}, or domestic demand of the restricted firm. Neither does it affect the exports price of the non-restricted firm, P_{Bna}. Let us also note that the relative size of markets does not have any influence on the equilibrium prices and market shares.

2.1.2 Country *A*'s problem

The government of country *A* sets the proportion of quality which it allows the domestic firm to export. The optimal proportion is the one that maximizes welfare in country *A*. Welfare in country *A* is a function of the domestic firm's profits, domestic consumer surplus, *CS*, and security, $S[\gamma]$, which is assumed to be a decreasing function of the proportion of quality exported, $S'[\gamma] < 0$:

$$W = \alpha\pi_A + \pi^*_A + \alpha CS + S[\gamma]. \tag{17}$$

The consumer surplus in country *A* has the following form:

$$CS = \int_0^y U_{iA} \partial d_{iA} + \int_0^{1-y} U_{iB} \partial d_{iB} \ . \tag{18}$$

The first element in the RHS of the above expression is the consumer surplus of consumers who buy from the home firm and the second element is the consumer surplus of the consumers who buy from firm *B*.

The first order condition of the maximization problem of the government is:

$$\frac{\partial W}{\partial \gamma} = \alpha \frac{\partial \pi_A}{\partial \gamma} + \frac{\partial \pi_A^*}{\partial \gamma} + \alpha \frac{\partial CS}{\partial \gamma} + S'[\gamma]. \tag{19}$$

Let us analyze the effect of a variation in γ on the different elements of the welfare function separately. In the absence of price arbitrage, the profits that the domestic firm obtains in the domestic market do not depend on the proportion of quality exported since neither the domestic demand nor the domestic price depend on it. Also, consumer surplus is not affected by a variation in γ because the prices at which domestic consumers buy from the domestic or the foreign firm are not affected by it.

However, an increase in the proportion of quality exported has a positive effect on the export profits of the domestic firm:

$$\frac{\partial \pi_A^*}{\partial \gamma} = \frac{\partial x_{na}}{\partial \gamma} P_{Ana}^* + \frac{\partial P_{Ana}^*}{\partial \gamma} x_{na} = \left(\frac{q_A}{3\lambda} \right) \left(\lambda + \frac{\gamma q_A - q_B}{3} \right). \tag{20}$$

Using equation (19) and the fact that security is the only term in the welfare function which is negatively affected by an increase in the proportion of quality exported, we can state the following proposition:

Proposition 2: *In the absence of price arbitrage the government of the security concerned country will restrict the quality exported by the domestic firm only if the concern about security is sufficiently high.*

2.2 Price Arbitrage

Let us now analyze what happens in our model when we introduce perfect price arbitrage between the two countries. Since firm *B* sells the same quality in the domestic and foreign market, it must charge the same price in both markets. If the export price were lower than the domestic price, consumers in country *B* could always buy the good with the same quality and a lower price in the international market and *vice versa*.

2.2.1 Firms' problem

Using equation (1) and taking into account that the price of firm *B* under perfect price arbitrage coincides in both markets, i.e. $P_B^* = P_B$ (we use P_B in what follows) we obtain the following export demand for firm *A*:

$$x = \frac{\gamma q_A - q_B + P_B - P_A^*}{2\lambda} + \frac{1}{2}. \tag{21}$$

Similarly, we get firm *A*'s domestic demand:

$$y = \frac{q_A - q_B + P_B - P_A}{2\lambda} + \frac{1}{2}. \tag{22}$$

Notice that the non-arbitrage condition (equation (3)) on P_A and P_A^* is fulfilled *iff* $y > x$. We will proceed under the assumption that the arbitrage constraint is not binding for firm *A*. The intuition for this is that since the home firm is not restricted in the domestic market, competition there is more intense; therefore the consumer will always be able to obtain a 'better deal' there. We will then check that the assumption is indeed satisfied in equilibrium.

Substituting *x* and *y* (equations (21) and (22)) in the profit maximization problem of firm *A* (equation (6)) we derive the first order conditions:

$$P_A = \frac{q_A - q_B}{2} + \frac{P_B}{2} + \frac{\lambda}{2}, \tag{23}$$

$$P_A^* = \frac{\gamma q_A - q_B}{2} + \frac{P_B}{2} + \frac{\lambda}{2}. \tag{24}$$

Using equations (23) and (24), it can be seen that the arbitrage condition is fulfilled.

Firm *B* must now set the same price in the export and domestic market due to the existence of perfect price arbitrage between the two countries:

$$\underset{\{P_B\}}{Max}\, \pi^*_B + \alpha\pi_B = P_B(1 - x + \alpha(1-y)). \tag{25}$$

Substituting *x* and *y* into equation (25) we get the first order condition for firm *B*:

$$P_B = \frac{\lambda}{2} + \frac{q_B}{2} - q_A \frac{\gamma + \alpha}{2(1+\alpha)} + \frac{P_A^* + \alpha P_A}{2(1+\alpha)}. \tag{26}$$

Equations (23), (24) and (26) are the reaction functions of firms *A* and *B*. Note that they are upward sloping with respect to the competitors' price.

We now have a three equation system in P_A, P_A^* and P_B. In order to derive the equilibrium prices, first introduce equations (23) and (24) into equation (26) and isolate P_B in order to obtain its equilibrium value:

$$P_B = \lambda + \frac{q_B}{3} - q_A \frac{\gamma + \alpha}{3(1+\alpha)}. \tag{27}$$

It is interesting to compare the no arbitrage prices with the prices we get with perfect price arbitrage between the two countries. The export price that firm *B* sets if there is no arbitrage is smaller than the price it sets when there is arbitrage. However, its domestic price without arbitrage is higher than the price firm *B* sets when there is perfect price arbitrage. The reason is that firm *B* cannot discriminate between foreign and domestic consumers, hence, it must set a unique price for its good which lies in-between the two prices it sets in the absence of perfect price arbitrage.

The properties of firm *B*'s equilibrium price are also affected by the fact that, due to the existence of perfect price arbitrage, firm *B* cannot discriminate between domestic and foreign consumers. The price of firm *B* is decreasing in the proportion of quality exported by the competitor. If firm *B* could set different prices in markets *A* and *B*, a variation in the quality restriction for firm *A* would only affect the domestic price of firm *B*. With price arbitrage, however, a variation in the quality gap with firm *A* in the domestic market affects the price of firm *B* in both markets.

Also, notice that the price of firm *B* is decreasing in the relative size of the market of country *A* (i.e. α) as long as there is a restriction on the quality that firm *A* can export (i.e. if $\gamma < 1$). Since firm *B* must face the highest quality from the competitor in country *A*, if firm *B* could set different prices, it would set a lower price in market *A* than in market *B*. Since it cannot discriminate between the foreign and home markets it sets a price that is in-between the two prices it would set if it could discriminate. However, if the importance of market *A* increases (i.e. if α increases), firm *B* will set its price closer to the optimal price it would set in market *A* if it could discriminate.

Now, substituting equation (27) into equations (24) and (26) we get the equilibrium domestic price of firm *A*:

$$P_A = \lambda - \frac{q_B}{3} + \frac{q_A}{2}\left(1 - \frac{\gamma + \alpha}{3(1+\alpha)}\right). \qquad (28)$$

The price that firm *A* sets in its home market is decreasing in the proportion of quality exported. The intuition is that an increase in the proportion of quality that firm *A* can export makes the price of the competitor decrease. This leads firm *A* to decrease its domestic price as well. Hence, even though the variation in quality exported does not directly influence the domestic price of firm *A* (since it does not affect the quality of its domestic product), there is an induced effect due to the existence of perfect price arbitrage between the two countries. We can see in Figure 2 that, in market *A*, an increase in γ shifts the reaction function of firm *B* down, but it does not affect the reaction function of firm *A*. This leads to a new equilibrium point with a lower firm *A* domestic price and a lower firm *B*'s price.

<FIGURE 2 HERE>

We can also see that when $\gamma < 1$, firm *A*'s domestic price is decreasing in the size of the home market due to the indirect negative effect that this has on firm *B*'s price. As we know when there is a restriction ($\gamma < 1$), the higher the relative size of market *A* the smaller the price of firm *B* and this induces a lower domestic price of the restricted firm, firm*A*.

Finally, we derive the equilibrium export price of firm *A*:

$$P_A^* = \lambda - \frac{q_B}{3} + \frac{q_A}{2}\left(\gamma - \frac{\gamma + \alpha}{3(1+\alpha)}\right). \tag{29}$$

Firm *A*'s export price is increasing in the proportion of quality exported. An increase in this proportion allows the firm to set a higher price for any given P_B. Even though firm *B* reacts to the increase in *A*'s quality by decreasing its price, the net effect on P_A^* is clearly positive. This is illustrated in Figure 3 where firm *A*'s reaction function shifts to the right and firm *B*'s reaction function shifts down.

<FIGURE 3 HERE>

For the same reason as before, the export price is decreasing in the size of the home market.

We can now compare the equilibrium prices of firm *A* with those that prevail in the absence of price arbitrage. Firm *A*'s domestic price is higher when there is arbitrage than when there is not. Meanwhile, firm *A*'s export price is smaller when there is perfect price arbitrage than when there is not. The intuition is based on the fact that the competitor sets an export price which is smaller than the unique price it could set with perfect price arbitrage. Since the competitor's export price is now smaller, firm *A*'s domestic price becomes smaller too. The same argument follows for the export price of firm *A*.

Substituting the equilibrium prices in equation (21) and (22) we derive the domestic and foreign demand of firm *A*:

$$x = \frac{1}{2\lambda}\left(\frac{q_A}{2}\left(\gamma - \frac{\gamma+\alpha}{3(1+\alpha)}\right) - \frac{q_B}{3}\right) + \frac{1}{2}, \tag{30}$$

$$y = \frac{1}{2\lambda}\left(\frac{q_A}{2}\left(1 - \frac{\gamma+\alpha}{3(1+\alpha)}\right) - \frac{q_B}{3}\right) + \frac{1}{2}. \tag{31}$$

Both the domestic and foreign demands of firm *A* are decreasing in the relative size of market *A*. An increase relative size of market *A* has a negative effect on firm *B*'s price and this effect outweighs the decrease in the prices of the domestic firm and therefore reduces its domestic and export demand. An increase in the proportion of quality allowed to be exported has a positive effect on the foreign demand of firm *A* and a negative effect on the domestic demand of firm *A*. Let us recall that firm *B* must sell at the same price in both markets. The increase in the quality exported by firm *A* makes firm *B* reduce the price of its good, this makes firm *B*'s product more attractive for the consumers in country *A*.

Comparing the above expressions with equations (15) and (16) we see that the foreign demand of firm *A* is smaller with perfect price arbitrage between the two countries; however its domestic demand is higher. In Figure 1, *x* and *y* are closer in the absence of price arbitrage.

Let us summarize the main properties in the following proposition:

Proposition 3: *When there is perfect price arbitrage between countries, an increase in the proportion of quality exported has a negative effect on the price of the non-restricted firm and the domestic price of the restricted firm and a positive effect on the export price of the restricted firm. Also, an increase in the proportion of exported quality has a positive effect of the export demand of the restricted firm and a negative effect on its domestic demand. An increase in the importance of the home market of the security concerned country has a negative effect on all prices as well as on the domestic and foreign demand of the restricted firm.*

We now analyze the optimal security policy of the security concerned government. The government in country *A* sets the proportion of quality which the domestic firm can export.

2.2.2 Country *A*'s problem

Using equation (19), we analyze the effect of a variation in the proportion of quality exported on the different elements of the welfare function.

An increase in the proportion of quality allowed to be exported has a positive effect on the profits that firm *A* obtains from selling in the foreign market, i.e. $\partial\pi_A^* / \partial\gamma > 0$ and a negative effect on the profit it obtains in the home market, i.e. $\partial\pi_A / \partial\gamma < 0$.

$$\frac{d\pi_A^*}{d\gamma} = \frac{\partial x}{\partial \gamma} P_A^* + \frac{\partial P_A^*}{\partial \gamma} x = \frac{q_A}{3} \frac{(2+3\alpha)}{(1+\alpha)} x > 0,$$

$$\frac{d\pi_A}{d\gamma} = \frac{\partial y}{\partial \gamma} P_A + \frac{\partial P_A}{\partial \gamma} y = -\frac{q_A}{3(1+\alpha)} y < 0.$$

The negative effect of an increase in the quality allowed to be exported on firm *A*'s domestic profits is due to the fact that firm *B* must sell its good at the same price in both markets. As a consequence of the increase in the proportion of quality exported, firm *A* faces a lower price from firm *B*. This reduces the profits that firm *A* gets in the home market.

An increase in the proportion of quality exported will have a negative effect on the total profits if the proportion of domestic demand of the home firm is sufficiently higher than its foreign demand. It can be proved that an increase in the proportion of quality exported has a negative net effect on the domestic firm's profits, i.e. $\partial(\alpha\pi_A + \pi_A^*)/\partial\gamma < 0$, if $x - y\dfrac{\alpha}{2+3\alpha} < 0$. In this case, the increase in export profits due to the increase in export price and demand does not compensate for the decrease in profits due to the decrease in domestic price and demand since the foreign demand of firm *A* is not sufficiently high.

Let us note that the LHS of this expression is decreasing in the size of the domestic market, α. Therefore, it is more likely that the net effect of an increase in the proportion of quality exported on the domestic firm profits is negative if the relative size of the domestic market is higher. However, in order to make the welfare analysis simpler we will work under the assumption that the global profits of firm *A* are increasing in the proportion of quality they are allowed to export. Otherwise we might face a situation in which the firm would want to further restrict itself beyond the restriction set by the government.

We now analyze the effect of a decrease in the restriction on the consumer surplus in country *A*. The effect of a variation in the proportion of quality exported on consumer surplus can be stated in the following way:

$$\frac{\partial CS}{\partial \gamma} = (R + q_A - P_A - \lambda y)\frac{\partial y}{\partial \gamma} - y\frac{\partial P_A}{\partial \gamma} - (R + q_B - P_B - \lambda(1-y))\frac{\partial y}{\partial \gamma} - (1-y)\frac{\partial P_B}{\partial \gamma}.$$

Substituting $\partial y/\partial \gamma$ above we get that an increase in the proportion of quality exported has a positive effect on consumer surplus:

$$\frac{\partial CS}{\partial \gamma} = -y\frac{\partial P_A}{\partial \gamma} - (1-y)\frac{\partial P_B}{\partial \gamma} = \frac{q_A}{3(1+\alpha)}\left(1 - \frac{y}{2}\right) > 0.$$

This comes from the fact that an increase in γ has a negative effect on the prices that domestic consumers must pay for buying the good either from the domestic or from the foreign firm.

Let us now derive the effect of a variation in the proportion of quality exported on the sum of profits that the domestic firm gets in the domestic market and consumer surplus:

$$\frac{\partial \pi_A}{\partial \gamma} + \frac{\partial CS}{\partial \gamma} = P_A\frac{\partial y}{\partial \gamma} - (1-y)\frac{\partial P_B}{\partial \gamma}.$$

The RHS of this expression is the sum of two effects: the first term represents the decrease in domestic profits due to the decrease in the domestic demand induced by an increase in the proportion of quality exported. The second term is the increase in consumer surplus of the domestic consumers who buy from the foreign firm. This effect is due to the decrease in firm *B*'s price induced by the increase in the proportion of quality exported. Notice that the decrease in the domestic price due to an increase in γ is irrelevant here because it simply implies a transfer from the domestic firm to the consumers. The following proposition states a condition that determines the sign of the final effect:

Proposition 4: $\frac{\partial \pi_A}{\partial \gamma}+\frac{\partial CS}{\partial \gamma}>0$ *if and only if* $\gamma>(3+2\alpha)-(2+2\alpha)\frac{(\lambda+q_B)}{q_A}$

Proof: $$\frac{\partial (yP_A)}{\partial \gamma}+\frac{\partial CS}{\partial \gamma}=-\frac{q_A}{3(1+\alpha)}y+\frac{q_A}{3(1+\alpha)}\left(1-\frac{y}{2}\right)>0$$

$$\Leftrightarrow y<\frac{2}{3}$$

$$\Leftrightarrow \gamma>(3+2\alpha)-(2+2\alpha)\frac{(\lambda+q_B)}{q_A}. \blacksquare$$

Intuitively, a high proportion of quality exported leads to a high proportion of consumers in country *A* that buy the product from firm *B* (i.e. a low *y*). If this is the case, the positive effect that an increase in the proportion of quality exported has on the consumer surplus due to the decrease in import price will outweigh the decrease in the home firm's domestic profits.

We can now compare the optimal proportion of quality exported with perfect price arbitrage and without price arbitrage between the two countries. As was said, in the absence of perfect price arbitrage a variation in the proportion of quality exported does not affect the sum of consumer surplus and the profits that the home firm gets in the domestic market. However, we must take into account two effects that influence the incentives of the government. First, we have the effect identified in Proposition 4. If the condition stated in that proposition holds, the government has an incentive to increase the proportion of quality exported with the introduction of perfect price arbitrage. Second, we must take into account that the effect of an increase in the proportion of quality exported on export profits might be different with and without perfect price arbitrage. We expect that a variation in the proportion of quality exported has a stronger positive effect on the exports profits of the restricted firm in the case of price arbitrage than in the absence of price arbitrage. For a given export price this is clearly the case[6]. A high γ will ensure that the export price of firm *A* with and without price arbitrage are sufficiently close for this effect to be the relevant. The assumption of a high γ is quite intuitive because in reality the proportion of quality which is exported tends to be quite high. Therefore, we can state the following proposition.

Proposition 5: *When the optimal proportion of quality exported in the absence of price arbitrage is bigger than* $(3+2\alpha)-(2+2\alpha)\frac{(\lambda+q_B)}{q_A}$*, the introduction of perfect price arbitrage implies an increase in the optimal proportion of quality exported.*

Proof: follows from the previous proposition.■

In the case in which the negative effect of an increase in the proportion of quality exported on the home firm's domestic profits outweighs the positive effect on consumer surplus, it is not possible to derive the effect of the introduction of price arbitrage analytically. However, we have done simulations of the model that show that even in this case the introduction of perfect

[6] For a given exports price of firm *A* we have that with perfect price arbitrage:

$$\left.\frac{\partial \pi_A^*}{\partial \gamma}\right|_a = \frac{P_A^* q_A}{2\lambda}\left(1-\frac{1}{2(1+\alpha)}\right)$$

while in the absence of price arbitrage we have:

$$\left.\frac{\partial \pi_A^*}{\partial \gamma}\right|_{na} = \frac{P_{Ana}^* q_A}{2\lambda}\left(\frac{1}{2}\right).$$

If the exports price of firm *A* is not given this will still be the case if the difference between the exports price with and with no price arbitrage is not big; a sufficiently high proportion of quality exported will ensure this.

price arbitrage increases the optimal proportion of quality exported (in the appendix we show a table with the parameter values we have used for the simulations).

3. Linear Individual Demands

In the previous section, consumers had unitary demands, that is to say, they consume one unit of the good they decide to buy and zero units of the other good. Therefore, the individual demand functions are completely inelastic. Due to this, variations in the price of the restricted firm on its home market simply imply transfers of rents between home firms and the domestic consumers that buy from them. In this section we want to allow consumers to decide the quantity of the good that they want to buy. When the consumers' demand depends on quantity an increase in the price that a consumer must pay causes a welfare loss. This effect is neglected if we use unitary demands.

Keeping the basic structure of the model we have described in the previous section, we introduce the possibility that consumers buy different quantities of the good produced by each of the firms. As before, consumers are uniformly distributed along the (0-1) line but they now have a utility function which depends on the quantity of good purchased:

$$U_{ij} = (q_j - \lambda d_{ij})Q - Q^2 + R - P_j Q\,, \tag{32}$$

where Q is the quantity of good purchased.

In this framework, consumers' choice is a two stage process. In the first stage the consumer chooses the system for which $\lambda d_{ij} + P_j$ is a minimum. In the second stage the consumer purchases Q units of the preferred good. The location of the indifferent consumer takes the same form as in the previous model.

The first order condition of the consumers' maximization problem gives the corresponding linear demand function,

$$Q = \frac{q_j - \lambda d_{ij} - P_j}{2} . \tag{33}$$

In the case of perfect price arbitrage between the two countries the profit function of the firms *A* and *B* are:

$$\alpha\pi_A + \pi_A^* = \frac{\alpha P_A}{2}\int_0^y (q_A - \lambda d_{iA} - P_A)\partial d_{iA} + \frac{P_A^*}{2}\int_0^x (\gamma q_A - \lambda d_{iA} - P_A^*)\partial d_{iA} ,$$

$$\pi_B^* + \alpha\pi_B = \frac{P_B}{2}\int_0^{1-x} (q_B - \lambda d_{iB} - P_B)\partial d_{iB} + \frac{\alpha P_B}{2}\int_0^{1-y} (q_B - \lambda d_{iB} - P_B)\partial d_{iB} .$$

In the absence of price arbitrage, the profit function of firm *B* is:

$$\pi_B^* + \alpha\pi_B = \frac{P_B^*}{2}\int_0^{1-x} (q_B - \lambda d_{iB} - P_B^*)\partial d_{iB} + \frac{\alpha P_B}{2}\int_0^{1-y} (q_B - \lambda d_{iB} - P_B)\partial d_{iB} .$$

Substituting the demand function (equation (33)) in the utility function (equation (32)) we get the utility as a function of quality and prices:

$$U_{ij}[q_j, P_j] = R + \frac{(q_j - \lambda d_{ij} - P_j)^2}{4} .$$

Therefore, consumer surplus takes the following form:

$$\begin{aligned} CS &= \int_0^y U_{iA}\partial d_{iA} + \int_0^{1-y} U_{iB}\partial d_{iB} \\ &= \int_0^y R + \frac{(q_A - \lambda d_{iA} - P_A)^2}{4}\partial d_{iA} + \int_0^{1-y} R + \frac{(q_B - \lambda d_{iB} - P_B)^2}{4}\partial d_{iB}. \end{aligned}$$

We have done simulations of this model (see the Appendix). The results of those simulations show that the optimal proportion of quality exported will be higher in the case of perfect price arbitrage. With linear individual demands, an increase in the proportion of quality exported has an additional positive effect because with perfect price arbitrage it decreases the prices that domestic consumers must pay to the domestic and foreign firms therefore implying a welfare gain. The simulations show that an increase in the proportion of quality exported has a stronger positive effect on the sum of consumer surplus and the home firm profits in the case of perfect price arbitrage.

4. Concluding Remarks

It is becoming increasingly difficult to define the boundaries between military and civil technology since nowadays the spin-offs between them go in both directions. For this reason, the restrictions on the quality that the firms of security-concerned countries are allowed to export affect a wide range of civil products. The most important examples are information technology and computers.

This paper has analyzed the effect of unilateral export controls on the domestic and foreign producers' profits and the welfare of the country that sets the export controls under the existence of perfect price arbitrage.

It has been proved that when there is perfect price arbitrage between countries, a looser restriction on the quality of the security sensitive product which is allowed to be exported makes the price of the foreign competitor of the restricted firm and the domestic price of restricted firm lower and the export price of the restricted firm higher. In contrast, in the absence of price arbitrage between countries, a looser restriction only affects the market in which the restricted quality is sold. It has a positive effect on the home firm export price and the export demand for the domestic firm and a negative effect on the competitor's domestic price.

The effect of the introduction of perfect price arbitrage on the optimal restriction of the quality exported has also been analyzed. In the absence of perfect price arbitrage a looser restriction has a positive effect on the export profits of the restricted firm. However, with perfect price arbitrage, a looser restriction also has a negative effect on the domestic profits of the restricted firm and a positive effect on consumer welfare. We cannot rule out the possibility of a net negative effect on welfare of a looser restriction. It is more likely that it arises if the importance of the domestic market is higher.

We have proved, however, that when the optimal proportion of quality exported in the absence of price arbitrage is sufficiently high, the introduction of perfect price arbitrage implies an increase in the optimal proportion of quality exported. The introduction of elastic individual demands for security sensitive products confirm this result.

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APPENDIX

Linear Individual Demands

A.1 Price Arbitrage

Let us recall that, with price arbitrage:

$$x = \frac{\gamma q_A - q_B + P_B - P_A^*}{2\lambda} + \frac{1}{2},$$

$$y = \frac{q_A - q_B + P_B - P_A}{2\lambda} + \frac{1}{2}.$$

<u>Firms' problem</u>

Firm *A* maximizes its profit function deciding on both the domestic and the foreign price:

$$\pi_A = \frac{\alpha P_A}{2}\int_0^y (q_A - \lambda d_{iA} - P_A)\partial d_{iA} + \frac{P_A^*}{2}\int_0^x (\gamma q_A - \lambda d_{iA} - P_A^*)\partial d_{iA}$$
$$= \frac{\alpha P_A}{2}\left((q_A - P_A)y - \lambda\frac{(y)^2}{2}\right) + \frac{P_A^*}{2}\left((\gamma q_A - P_A^*)x - \lambda\frac{(x)^2}{2}\right).$$

Maximizing the profit function with respect to P_A and P_A^* yields

$$\frac{\partial \pi_A}{\partial P_A}:\ \frac{\alpha}{2}\left((q_A - P_A)y - \lambda\frac{(y)^2}{2}\right) - \frac{\alpha P_A}{2}y + \frac{\alpha P_A}{2}\left(\frac{y}{2} - \frac{(q_A - P_A)}{2\lambda}\right) = 0,$$

$$\frac{\partial \pi_A}{\partial P_A^*}:\ \frac{\alpha}{2}\left((\gamma q_A - P_A^*)x - \lambda\frac{(x)^2}{2}\right) - \frac{\alpha P_A^*}{2}x + \frac{\alpha P_A^*}{2}\left(\frac{x}{2} - \frac{(\gamma q_A - P_A^*)}{2\lambda}\right) = 0.$$

Rewriting these two equations we get:

$$(2q_A - 3P_A)y - \lambda(y)^2 - P_A\frac{(q_A - P_A)}{\lambda} = 0,$$

$$(2\gamma q_A - 3P_A^*)x - \lambda(x)^2 - P_A^*\frac{(\gamma q_A - P_A^*)}{\lambda} = 0.$$

Firm *B* maximizes the following profit function:

$$\pi_B^* + \alpha\pi_B = \frac{P_B}{2}\int_0^{1-x}(q_B - \lambda d_{iB} - P_B)\partial d_{iB} + \frac{\alpha P_B}{2}\int_0^{1-y}(q_B - \lambda d_{iB} - P_B)\partial d_{iB}$$

$$= \frac{P_B}{2}\left((q_B - P_B)(1-x) - \lambda\frac{(1-x)^2}{2}\right) + \frac{\alpha P_B}{2}\left((q_B - P_B)(1-y) - \lambda\frac{(1-y)^2}{2}\right).$$

The first order condition of the maximization problem is:

$$\frac{1}{2}\left((q_B - P_B)(1-x) - \lambda\frac{(1-x)^2}{2} - P_B(1-x) - P_B\frac{(q_B - P_B)}{2\lambda} + P_B\frac{(1-x)}{2}\right) +$$
$$\frac{\alpha}{2}\left((q_B - P_B)(1-y) - \lambda\frac{(1-y)^2}{2} - P_B(1-y) - P_B\frac{(q_B - P_B)}{2\lambda} + P_B\frac{(1-y)}{2}\right) = 0$$

Rewriting we get:

$$\alpha\left((2q_B - 3P_B)(1-y) - \lambda(1-y)^2 - P_B\frac{(q_B - P_B)}{\lambda}\right) +$$
$$\left((2q_B - 3P_B)(1-x) - \lambda(1-x)^2 - P_B\frac{(q_B - P_B)}{\lambda}\right) = 0$$

Country *A*'s problem

In order to obtain the consumer surplus, we first substitute the demand function (equation (33)) in the utility function (equation (32)) and we get utility as a function of quality and prices:

$$U_{ij}[q_j, P_j] = R + \frac{(q_j - \lambda d_{ij} - P_j)^2}{4}.$$

The consumer surplus takes the following form:

$$CS = \int_0^y U_{iA}\partial d_{iA} + \int_0^{1-y} U_{iB}\partial d_{iB}$$

$$= \int_0^y R + \frac{(q_A - \lambda d_{iA} - P_A)^2}{4}\partial d_{iA} + \int_0^{1-y} R + \frac{(q_B - \lambda d_{iB} - P_B)^2}{4}\partial d_{iB}$$

$$= R + \frac{1}{4}\left[y(q_A - P_A)^2 - \lambda(y)^2(q_A - P_A) + \frac{\lambda^2(y)^3}{3}\right]$$
$$+ \frac{1}{4}\left[(1-y)(q_B - P_B)^2 - \lambda(1-y)^2(q_B - P_B) + \frac{\lambda^2(1-y)^3}{3}\right]$$

A.2 No Price Arbitrage

In the case when there is not price arbitrage:

$$x = \frac{\gamma q_A - q_B + P_B^* - P_A^*}{2\lambda} + \frac{1}{2},$$

$$y = \frac{q_A - q_B + P_B - P_A}{2\lambda} + \frac{1}{2}.$$

Firm B can now set different prices in the domestic and exports market. The profit function is now:

$$\begin{aligned}\pi_B^* + \alpha\pi_B &= \frac{P_B^*}{2}\int_0^{1-x}(q_B - \lambda d_{iB} - P_B^*)\partial d_{iB} + \frac{\alpha P_B}{2}\int_0^{1-y}(q_B - \lambda d_{iB} - P_B)\partial d_{iB} \\ &= \frac{P_B^*}{2}\left((q_B - P_B^*)(1-x) - \lambda\frac{(1-x)^2}{2}\right) + \frac{\alpha P_B}{2}\left((q_B - P_B)(1-y) - \lambda\frac{(1-y)^2}{2}\right).\end{aligned}$$

From here we obtain the first order conditions:

$$(q_B - P_B^*)(1-x) - \lambda\frac{(1-x)^2}{2} - P_B^*(1-x) - P_B^*\frac{(q_B - P_B^*)}{2\lambda} + P_B^*\frac{(1-x)}{2} = 0,$$

$$(q_B - P_B)(1-y) - \lambda\frac{(1-y)^2}{2} - P_B(1-y) - P_B\frac{(q_B - P_B)}{2\lambda} + P_B\frac{(1-y)}{2} = 0.$$

Rewriting, we get the following expressions for the first order conditions:

$$(2q_B - 3P_B^*)(1-x) - \lambda(1-x)^2 - P_B^*\frac{(q_B - P_B^*)}{\lambda} = 0,$$

$$(2q_B - 3P_B)(1-y) - \lambda(1-y)^2 - P_B\frac{(q_B - P_B)}{\lambda} = 0.$$

In the simulations we have used the following values for the underlying parameters:

α	= 1.0
λ	= 1.0
q_A	= 1.5
q_B	= 1.0

Figure 2: Increase in Production of Quality Exported

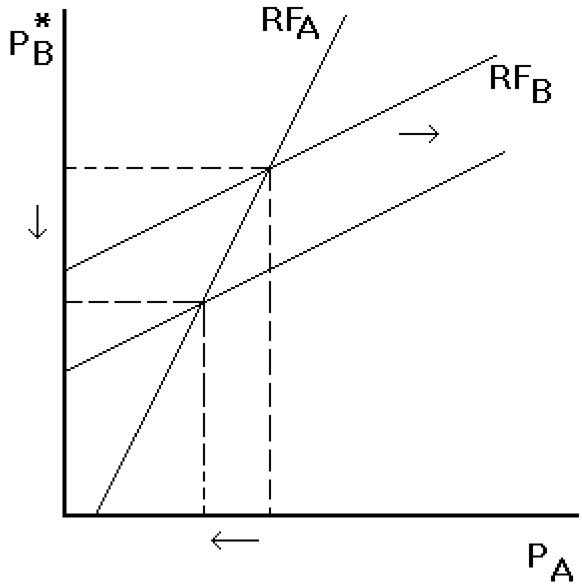

Figure 3: Increase in the Proportion of Quality Exported