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CUMULATIVE GROWTH AND THE CATCHING-UP DEBATE FROM A DIS-EQUILIBRIUM STANDPOINT

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Abstract

This paper presents an overview of the literature on 'cumulative growth'. It is argued that, independently of the 'new' growth theory, these models have achieved the nature of 'endogenous' growth models. Their main differences, however, lie in the assumptions about the equilibrium prevailing in the economy. Cumulative growth models do not assume a general equilibrium setting and, thus, the main driving force of growth is demand. Although the natural rate of growth is endogenous (through the effect of induced productivity growth), it can be shown that these models are compatible with a wide set of outcomes concerning the catching-up and convergence issue. In order to do this, we present a model of cumulative causation that, overcoming some of the weaknesses of previous models, allows for catching-up from followers to leader to occur. We also show how the induced productivity growth effect may lead to a faster catching-up rate, contrary to the popular result that it necessarily leads to divergence.

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Keywords: Cumulative Growth, Catching-up, Endogenous Growth.

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I Introduction

The economic growth debate of the last fifteen years has opened the doors for a wider variety of ideas about why and how countries' and regions' growth rates differ. The main achievement from a theoretical viewpoint is the construction of models that allow growth rates to be positive in the steady state without the help of any exogenous variable. Growth can be positive in the long run and depends on the investment decisions of the agents. This may seem very obvious for the amateur economic growth practitioner. However, this intuitive idea clashes with the complications and constraints that both real data and mathematical models impose for the growth theoretician. How can we explain the continuous growth of output without generating explosive behaviour of per capita income not observed in the data? What factors lie behind this possibility? 'New' growth theory has indeed contributed to specifying growth models in which both questions are addressed. Technical progress, either disembodied or embodied in capital goods, has been put in the core of the analysis. National specific non-exogenous factors may now explain why some countries have succeeded more than others. Even more, a world of diverging per capita incomes has become a most plausible result of the analysis.

However, an important question to be addressed is to what extent these results are 'new'? And even more, do these new theories relax some of the narrowest assumptions of the old Solow growth model? The answer to the first question is 'not too much.' The answer to the second is 'not to the extent that the full employment frictionless economy assumption is kept unchanged'. In this paper we will argue that cumulative causation growth models had already achieved certain features of the 'endogenous growth' nature of these models. We will also argue that the main difference between cumulative and ‘new’ growth models lies in the fact that the former do not assume full employment and do not deal with a general equilibrium economy. Even more, full employment or the ‘natural’ rate of growth is never achieved because growth creates the necessary resources for growth itself. In this sense, the nature of the cumulative growth models allows for a ‘total’ endogeneity of growth to growth itself as a self-reinforcing process. In this context, we will show how a very rich set of dynamics of income distribution across the world is a possible outcome of these models. That is, despite having been ignored by recent debates, cumulative growth models may explain divergence, constant relative differences or catch-up in per capita income levels. The existence of a Verdoorn effect does not necessarily imply divergence or explosive behaviour, although it is the central mechanism that allows for endogenous productivity growth.

The paper will be organised as follows. In the next section we present a brief review of the main cumulative causation models and their strengths and weaknesses. This discussion will aid in understanding the model developed in Section III, in which technical progress, technology diffusion and other relevant variables from an empirical point of view are introduced in the canonical cumulative growth model. The dynamic behaviour of the model will be analysed in Section IV. Section V extracts some relevant conclusions of the arguments put forward.

II Cumulative Growth: An Overview

As a consequence of his *laws of growth*, Kaldor’s (1970) paper on the determinants of regional growth disparities entails a definitive adherence of Kaldor with the cumulative causation theories of development in line with Myrdal (1957) and that can be dated back to Veblen’s (1915) seminal work. Kaldor’s explanation of the differences in growth rates among regions rests on the existence of two interrelated mechanisms: (1) the growth of output is determined by the growth of aggregate demand, concretely by the growth of exports, that is influenced by the degree of competitiveness of the region; (2) productivity growth is a by-product of output growth due to the existence of dynamic increasing returns through the mechanisms underlying Verdoorn's Law. Since prices are set in oligopolistic markets, a mark-up over unit labour costs is the dominant rule of pricing. Growth in productivity stemming from the growth of output would allow for a reduction of unit labour costs and, thus, of prices, increasing the competitiveness of the region (or country). This increased competitiveness allows for further expansions of output through increased exports, and so on. The result is that, given an initial advantage, regions will tend, through the circular and cumulative mechanism described above, to maintain it (or even increase it) over time, resulting in uneven development among regions. In this mechanism, Verdoorn's Law plays the crucial role of transforming the growth of output into the growth of demand and, thus, more growth of output. Dynamic increasing returns are a force running against the existence of converging levels of output per capita among regions.

However, the arguments of Kaldor are far from being clearly formalised. It is not clear whether Kaldor was arguing that *growth rates* tend to diverge or simply the *level of output per capita*. In the first case we would be confronting a rather unstable world with explosive behaviour of some regions and ever declining behaviour of others, which is not observed in the real world. The canonical Kaldorian model of growth was first formalised by Dixon and Thirlwall (1975). It is worth examining this model in order to extract clear consequences and analyse further extensions that have enriched the possible set of dynamics of Kaldor's verbal arguments. For any given region, the discrete time form of the model can be written as:

$$g_t = \gamma x_t, \tag{1}$$

$$x_t = \eta p_{dt} + \delta p_{ft} + \varepsilon z_t, \tag{2}$$

$$p_{dt} = w_t - r_t + \tau_t, \tag{3}$$

$$r_t = r_a + \lambda g_t. \tag{4}$$

Equation (1) states that the growth of output (g_t) is a linear function of the growth of exports (x_t). Equation (2) is a typical export demand function expressed in growth rates, where the growth of exports depends on the growth of domestic prices (p_{dt}), foreign prices (p_{ft}) and the income of the 'rest of the world' (z_t), with η, δ and ε being the respective elasticities. Equation (3) is the expression for the rate of growth of domestic prices, which is derived from a mark-up pricing equation as $P_{dt} = (W_t / R_t)T_t$, where W_t is the level of money wages, R_t is the average product of labour and T_t is one plus the percentage mark-up over unit labour costs. Finally, equation (4) is the expression for the rate of growth of productivity (r_t) derived from the Verdoorn Law relationship, r_a being the autonomous productivity growth. The model is block recursive, and its solution for the equilibrium rate of growth of output is:

$$g_t = \gamma \frac{[\eta(w_t - r_a + \tau_t) + \delta p_{ft} + \varepsilon z_t]}{1 + \gamma\eta\lambda}. \tag{5}$$

This expression is telling us that the growth rate of a region varies positively[1] with the autonomous rate of productivity growth (r_a), the growth of 'world' income (z_t), the income

[1] Bearing in mind that $\eta < 0$.

elasticity of demand for exports (ε), the rate of growth of foreign prices (p_{ft}) and the Verdoorn coefficient (λ), and negatively with the rates of growth of wages and mark-up (w and τ), with the effect of η being ambiguous.

Note that "[...] it is the Verdoorn relation which makes the model circular and cumulative, and which gives rise to the possibility that once a region obtains a growth advantage, it will keep it" (Dixon and Thirlwall, 1975, p. 205). However, the existence of a Verdoorn relation is not a sufficient condition for the existence of growth rate differences among regions unless the value of λ is different between regions or initially the rest of the parameters of the model differ, so Verdoorn's coefficient serves to exaggerate these differences. Summarising, growth rate differences are determined by the structural characteristics of regions that determine their degree of competitiveness, the rate of induced productivity growth and the extent to which all these characteristics influence aggregate demand growth. Boyer and Petit (1991) distinguish between productivity and demand *regimes* in order to clarify the double link between productivity growth and aggregate demand growth in this model. The former would reflect the extent to which aggregate demand influences productivity growth, while the latter would determine the effect of productivity growth on aggregate demand growth. For the model to show a stable pattern, the sensitivity of aggregate demand growth to productivity growth must be higher than that of productivity growth to output growth. Otherwise, we would be in the presence of ever increasing growth rates differences and, as Gordon (1991) states, this is 'too much cumulation'. The stability of the model can be studied by introducing a lag structure in any of the equations (we will use equation (2)) and analysing the asymptotic behaviour of the solution. Introducing a lag in equation (2) yields a first order difference equation, the general solution to which is

$$g_t = g_0(-\gamma\eta\lambda)^t + \frac{\gamma[\eta(w_{t-1} - r_a + \tau_{t-1}) + \varepsilon z_{t-1} + \delta p_{ft-1}]}{1+\gamma\eta\lambda}. \quad (6)$$

For the model to be stable $(-\gamma\eta\lambda)$ must be less than 1[2], and growth would converge towards its equilibrium level. If $(-\gamma\eta\lambda)$ is greater than 1 the rate of growth of the region would be an increasing function of time, and there would not be a stable solution.[3] If this result holds, divergence *in growth rates* would occur, and the outcome would be that the divergence in *per capita incomes* exhibits an explosive behaviour, which is very unlikely to occur given reasonable values for the parameters of the model.[4] The most plausible result is one of sustained equilibrium differences in growth rates between regions.[5]

Setterfield (1997a, 1997b) favours a dis-equilibrium interpretation of this model that helps to realise the role played by irreversible historical time in determining the growth path. If we assume that the general solution (6) has a unit root, that is $(-\gamma\eta\lambda) = 1$, then it is clear that the equilibrium growth will be dependent on the initial conditions. In fact, this is the cumulative nature of the model, in which the starting point of the process determines the rest of the sequences of occurrence. This would also be the case if, even when $(-\gamma\eta\lambda) < 1$, the velocity of convergence towards the determinate equilibrium is too slow in comparison with the changes experienced in the exogenous data determining the equilibrium. In such a case, equation (5) would be rendered irrelevant for explaining the long-run growth rate. Thus, the initial condition and the time position of the system would determine the growth rate that, in this case, is said to be *path-dependent* in the sense that it is not independent of the historical

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[2] Since $\eta < 0$, then the term $(-\gamma\eta\lambda)$ will be greater than 0.

[3] See Guccione and Gillen (1977) and the reply of Dixon and Thirlwall (1978) for a discussion of the stability conditions of the model.

[4] See Dixon and Thirlwall (1975) pp. 212-213.

[5] See Swales (1983) for the introduction of some non-linearities that enrich the set of possible cases of equilibrium.

shocks. However, Setterfield points out that a more satisfactory explanation of growth would be one where it depends not only on the initial conditions, but also on all the steps taken towards the actual position. This case is possible if the model shows *hysteresis* with its parameters changing over time depending on the resulting rate of growth. This possibility will be dealt with when we refer to the possible existence of *lock-in* processes, in line with the arguments of Arthur (1989).

The Kaldorian model analysed is intrinsically putting forward the idea of an endogenous determination of the *natural rate of growth* as discussed and tested in León-Ledesma and Thirlwall (1998). Productivity growth is an endogenous result of the actual rate of growth, and it is assumed that labour adapts automatically to increases in labour demand stemming from increases in aggregate demand. Wages do not play a role in clearing the labour market and, despite the fact that in the canonical model they are exogenously determined, it is possible to introduce a wage and profit bargaining function that determines the distribution of income (see Palley, 1996, 1997 and Boyer and Petit, 1991). Thus, the *natural rate of growth* is an endogenous result of the actual rate of growth that, in turn, is determined by the autonomous components of aggregate demand. The endogeneity of growth proposed in this post-Keynesian approach differs substantially from that proposed by the 'new' neo-classical growth theorists represented, among others, by Romer (1986, 1990), Lucas (1988) and Grossman and Helpman (1991).[6] For this school, the income distribution is determined by the marginal productivity of the factors of production. However, they differ from the 'old' Solovian theories in the assumptions about the returns to capital. New growth theories allow for constant returns to capital[7] and increasing returns to scale, and solve the distribution problem arising from this

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[6] See Skott and Auerbach (1994) for a critique of the underlying assumptions of new growth theories from a perspective embracing both institutionalist and cumulative causation ideas.
[7] Or to the sum of all the reproducible factors as capital, human capital and innovation.

assumption by treating the excess returns over unity as an externality. In any case, since prices adjust to clear the factor market, the economy is always in equilibrium at full employment. That is, the economy is always on the production possibility frontier and, in this case, and assuming that Say's Law applies, aggregate demand plays no role in determining the growth of output. Although 'new' growth theorists correctly point out technical progress as a source of increasing returns and non-convergent growth, their view is pre-Keynesian in the sense that they only concentrate on the supply side of the economy, and use a general equilibrium approach to model the dynamics of growth. In contrast, cumulative growth models assume that the economy is not at all times in equilibrium at the production possibility frontier. The growth of output moves that frontier and, thus, full employment is not attained and the distribution of income cannot be determined by the slope of the frontier. Growth is endogenous because supply and demand interact in a way determined by the structural parameters of the model. This endogeneity, however, does not depend on the assumptions made about the optimising behaviour of microeconomic agents, and it is thus less restrictive.

The model considered so far does not take account for the possibility that the rate of growth of income generates a rate of growth of imports in excess of that of exports. If this was the case, the country or region would be incurring continuous and sustained balance of payments deficits.[8] If one of the policy objectives is the balance of payments equilibrium or, simply, monetary constraints related to the balance of payments position arise, the growth rate can find a constraint in the balance of payments. Thirlwall and Dixon (1979) first introduced this constraint in the model by adding an import growth demand function as:

$$m_t = \psi\left(p_{ft} - p_{dt}\right) + \pi g_t \tag{7}$$

[8] Regions do not have a balance of payments in the normal accounting sense but excess imports still need financing. Thus, regions would also be subject to balance of payments problems due to monetary constraints. See Thirlwall (1980) for a discussion of this topic.

ψ being the price elasticity of imports and π the income elasticity of imports.[9] Starting from an equilibrium balance of payments, the condition for dynamic external equilibrium is $m_t + p_{ft} = p_{dt} + x_t$. Then, the equilibrium rate of growth obtained by satisfying this condition is:

$$g_t = \frac{(1+\eta+\psi)[w_t - r_a + \tau_t - p_{ft}] + \varepsilon z_t}{\pi + \lambda(1+\eta+\psi)}\,. \qquad (5')$$

The interpretation of the equilibrium growth rate is similar to that represented in equation (5). The difference rests in the appearance of the income elasticity of demand for imports in the denominator. The higher π, the higher the sensitivity of import growth to the growth of income and the sooner growth will generate balance of payments deficits. Since deficits must be corrected, at least in the long run, the higher π the lower will be the equilibrium rate of growth due to the existence of a balance of payments constraint. This does not preclude the economy from being export-led, for an increase in the rate of growth of exports will raise the constraint. This shows the two ways in which exports are important: (1) by increasing aggregate demand in an autonomous way - which is the dynamic version of Harrod's foreign trade multiplier; and (2) by relaxing the balance of payments and allowing for further increases in the other components of aggregate demand without incurring external deficits, in other words, the Hicks super-multiplier (McCombie, 1985). This argument helps to explain why macroeconomic performance matters for growth, as pointed out by Fischer (1993), rendering the production function approach to economic growth ill-equipped.

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[9] We are assuming that the price elasticity of imports and the cross-price elasticity of imports with respect to the foreign countries are equal. For simplicity we will make the same assumption in equation (2) for the export demand function, where η is the relative price elasticity of demand for exports.

Making the assumption that relative prices do not vary in the long run, the model with a balance of payments constraint would collapse to

$$g_t = \frac{\varepsilon}{\pi} z_t \qquad (5'')$$

which is known as *Thirlwall's Law* of growth (Thirlwall, 1979). That is, the rate of growth of a region or country is determined by the growth of the rest of the world and the ratio between export and import income elasticities. This ratio reflects the degree of non-price competitiveness of the national (regional) products in the international markets and the non-price competitiveness in the domestic market, both determined by the structural specialisation of the region and the degree of product differentiation. *Thirlwall's Law* has generated a vast literature both on empirical and theoretical grounds that, in general, has led to confirm its relevance as an explanation of why growth rates differ.[10]

The equilibrium solution of the canonical model and its extensions, if stable, presents a world in which growth rate differences are steadily maintained over time. If we assume that the initial per capita income of region A is higher than in region B, and the equilibrium growth rates are $g_t^A > g_t^B$, then the result would be divergence in per capita incomes. If contrawise $g_t^A < g_t^B$, then the result would be initially a catch-up from region B to region A and then region B forging ahead from A. However, this result is not satisfactory when explaining the process of convergence and divergence in the real world. The model rules out the possibility that high growth regions in the past find themselves involved in slow growth processes, and slow growth regions transforming into fast growth. The possible sets of dynamics are limited because they do not make any reference to the influence of the level of income on the rate of

[10] For recent general overviews on Thirlwall's Law see McCombie and Thirlwall (1994), McCombie and Thirlwall (1999) and the mini-symposium in *Journal of Post Keynesian Economics*, 20, 1997.

growth of income. These models cannot explain why some groups of countries tend to converge to a similar level of per capita income and others tend to diverge. Another problem is that it does not explicitly model the importance of non-price competitiveness and technological progress as another possible source of cumulative tendencies. However, the introduction of a richer set of dynamics has recently been approached in several models (Amable, 1992, 1993, Boyer and Petit, 1991, De Benedictis, 1998, Gordon, 1991, Palley, 1997 and Setterfield 1997a, 1997b). These models point out that, from a Kaldorian model of cumulative causation, the final outcome can be convergence (catch-up), divergence or sustained differences. In all of them, technological factors linked with processes of learning by doing, innovation, embodied technical progress and diffusion of technology play a crucial role in the determination of non-price competitiveness of exports (and imports) in the same vein that in the canonical model price competitiveness linked increased productivity with increased aggregate demand. Whether the technological forces favouring convergence (catch-up) are stronger than those favouring divergence (cumulative knowledge) is an empirical question. Two sources of convergence can be pointed out:

(a) *Technological Catch-up.* The original idea of Veblen (1915), that backward countries would tend to grow faster than leading countries, was re-visited by Gerschenkron (1962), Abramovitz (1986) and Gomulka (1990) among others. The idea is based on the possibility that the technological gap between nations opens up the opportunities for backward countries to access the leader’s technology. Higher growth would be attained through the accumulation of new capital embodying more advanced technical characteristics. However, for the catch-up process to take place it is necessary that leaders and followers exhibit some *technological congruence* and that the followers have enough *social capability* to absorb and reward the new technology (Abramovitz, 1986). Catch-up will occur if the technological gap is not ‘too big’, with groups of countries converging towards similar levels of per capita income and others locked out of the process of development (Baumol, 1986).

Amable (1993) has introduced catch-up in a Kaldorian cumulative growth model where the technological gap affects the growth of per capita income, the rate of innovation and the rate of school enrolment. Note that the catch-up hypothesis is perfectly compatible with a Kaldorian world in which technology is not freely accessible and countries are not permanently on the production possibility frontier (Pugno, 1991). By introducing a catch-up term in the canonical model the set of possible convergence-divergence outcomes is enriched. It is possible that two countries converge even if there is an underlying cumulative process leading to divergence. Empirical work also suggests that catch-up is an important factor explaining growth (Amable, 1993 and Targetti and Foti, 1997).

(b) *Lock-in*. When analysing the dis-equilibrium interpretation of the Dixon-Thirlwall Kaldorian model, it was pointed out that if the growth process has a unit root, it will permanently depend on the initial growth conditions. However, Setterfield (1997a, 1997b) points to the possibility that the parameters of the model react endogenously to the rate of growth itself, leading to *path-dependent* processes of growth where the final outcome depends on the initial conditions and all the steps taken towards its equilibrium path position. The cumulative growth characterised by increasing returns leads to increased inter-relatedness among components of the production process that, in turn, are inherited from the past (Arthur, 1989).[11] This inter-relatedness increases the cost of changing from one specialisation to another making the growth process more *inflexible*. Thus, certain region or country can find

[11] *Inter-relatedness* and *roundaboutness* are already mentioned in Young's (1928) verbal exposition of a cumulative growth model.

itself locked into a certain technique of production, or into a certain specialisation. If this is the case, the higher the level of development the lower will be the possibilities of realising dynamic increasing returns based on changes of specialisation[12] and, thus, the Verdoorn coefficient λ will fall. Alternatively, since the income elasticities of demand for imports and exports reflect non-price competitiveness, they reflect the ability of a region to adapt to the changing patterns of consumption due to the growth of disposable income. The *lock-in* effect leads to a higher (lower) value of π (ε) because of the higher cost of changes in specialisation due to inter-relatedness. Thus, combining increasing returns with *path-dependence* can lead to *lock-in* processes that may cause regions with high levels of income to suffer lower rates of growth. Although empirically *catch-up* and *lock-in* would mean that the rate of growth depends inversely on the level of development, both are different forces leading to converging levels of income.

The existence of these forces does not preclude the possibility that the rate of growth leads to divergence in per capita incomes. In the Dixon-Thirlwall model with a catch up term in the productivity growth equation, as in Targetti and Foti (1997), it is still possible that the cumulative forces leading to divergence overwhelm the converging forces of catching up and the final outcome is divergence. However, it is implausible that price competitiveness is the only link between productivity and export growth causing a cumulative process. If we want to address the importance of cumulative growth some link has to be explicitly considered between technological progress and non-price competitiveness. The fact that the countries gaining more market share in international markets are not those experiencing a lower growth of relative unit labour costs (RULC) has come to be known as *Kaldor's Paradox* (Kaldor,

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[12] That is to say, the lower the possibilities of achieving a higher growth of productivity arising from the Smith-Young process of division of labour.

1978). This phenomenon reflects the fact that competition in international markets rests more on technological factors improving the quality and variety of products.

Amable (1992) confronts this problem by introducing a non-price factor in the export equation in a balance of payments constrained growth model *à la* Thirlwall (1979). This factor, reflecting the impact of technical progress in export performance, is cumulative past production reflecting learning-by-doing. In the same vein, De Benedictis (1998) introduces the effects of learning by doing, national innovation and diffusion of technology as determinants of the national degree of technological innovation that, in turn, affects export performance. Both models show that the equilibrium rate of growth may be stable given reasonable values of the parameters, and that the possibilities of a laggard economy catching-up with the leader would mainly depend on its ability to generate and adopt innovations faster than the leader. Otherwise, divergence would be the pattern. Palley (1997) uses a model in which productivity growth depends on capital accumulation through capital deepening and embodied technical progress. Though Palley's model is one of a closed economy, it shows the possibility of multiple equilibria in growth rates and does not rule out the unstable case. Thus, cumulative causation models, which were initially associated with a description of the world in which differences among regions tended to be inevitably increasing, are also able to explain a wide set of dynamics without demanding restrictive a priori assumptions as is the case for the old and new neo-classical models.

The richness of these models is also matched with good empirical performance (see for instance Atesoglu, 1994, Amable 1993, Boyer and Petit, 1991 and Targetti and Foti, 1997, Fingleton, 2000 and León-Ledesma, 1999).[13] Empirical testing, though, is not yet a

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[13] Notably, Fingleton (2000) introduces spatial effects due to the existence of regional spillovers and externalities that also enriches the dynamics of the model.

generalised practice. Lack of regional data, lack of long series for variables related with technological factors and the difficulties associated with the interpretation of cross-sectional growth empirics are some of the problems of these models when confronting empirical data. This is, nonetheless, another field of possible extension and understanding of these models, which could easily benefit from what we have learnt from neo-classical growth empirics.[14]

III Extending the Cumulative Growth Model

From the discussion in the previous section we conclude that the basic cumulative model can be improved in two ways. First, it is a model of growth in which growth rate differences are constant and, hence, it does not allow for the existence of declining (or increasing) growth rates over time. In other words, no reference is made to the relationship between the rate of growth and the level of per capita income (or productivity) and, thus, the analysis of whether there is convergence or divergence in income or productivity levels is not possible. Secondly, there is no explicit reference to the important role of non-price factors that determine competitiveness. In this regard, the role of innovation and the diffusion of technologies seems of crucial importance (see Fagerberg, 1988). Recent developments in growth theory have emphasised the possible beneficial effects of innovation activities and the role of catching-up as major determinants of the growth performance of countries and regions.

The model we present here is an extended version of Dixon and Thirlwall (1975) that introduces technology variables along the lines of Amable (1993) and De Benedictis (1998). These variables are similar to those emphasised in the "new growth theory" analysis, but a different interpretation will be given to them. As will be shown below, there are several

[14] See Temple (1999).

cumulative forces that may lead to divergent growth that interact with the effect that the catching-up – due to the adoption of foreign technologies – has on leading to convergence. Five continuous time equations can describe the relations at work:

$$y = \theta x, \qquad \theta > 0 \tag{8}$$

$$x = \eta(p - pf) + \varepsilon z + \zeta K + \delta(I/O), \qquad \eta < 0,\ \varepsilon > 0,\ \zeta > 0,\ \delta > 0 \tag{9}$$

$$p = w - r \tag{10}$$

$$r = \phi y + \lambda(I/O) + \alpha K + \sigma GAP, \qquad \phi > 0,\ \lambda > 0,\ \alpha > 0,\ \sigma > 0 \tag{11}$$

$$K = \gamma y + \beta q + \omega(edu) + \psi GAP, \qquad \gamma > 0,\ \beta > 0,\ \omega > 0,\ \psi < 0 \tag{12}$$

The first equation (8) states that the growth of output (y) depends on the growth of exports (x), which is equal to equation (1) in the Dixon-Thirlwall model. The growth of exports, in turn, depends negatively on the growth of relative prices ($p - pf$), and positively on world income growth (z), the investment-output ratio (I/O) and a technology variable to account for non-price factors (K) reflecting the flow of innovations that affect export performance. The first two variables on the right hand side of the export equation (9) correspond to the usual specification of an export function expressed in rates of growth. The introduction of the investment-output ratio as a proxy for capital accumulation is due to the fact that the capacity of an economy to deliver in international markets depends on the growth of physical equipment and infrastructures (Fagerberg, 1988). This variable may also capture the effect of embodied technical progress on export performance. Innovation is a key factor affecting the non-price competitiveness of economies. Product differentiation and quality competition characterise modern international trade. These factors determine the national-specific competitiveness and are different from those depending on the product composition of exports

(Amable, 1992). The former will be reflected in the innovation variable, while the latter is captured by the income-elasticity of demand for exports (ε). A country's ability to differentiate and compete in quality will crucially depend on the degree of innovation of its productive structure, which is reflected in the innovation variable introduced in the export equation (K).

The third equation of the model – equation (10) – is equivalent to equation (3) assuming that the mark-up over unit labour costs is constant over time. The fourth relation of this model determines the rate of growth of labour productivity (r). One major determinant of productivity growth is the induced effect of output growth, that is, the Verdoorn-Kaldor mechanism. As mentioned earlier, this mechanism is responsible for the circular nature of the growth process in the canonical model. The Verdoorn-Kaldor mechanism reflects the existence of dynamic economies of scale due to increased specialisation (Young, 1928) and embodied technical progress (Kaldor, 1957) and also the existence of static increasing returns.[15] Embodied technical progress is explicitly captured in this model by the introduction of the investment-output ratio (I/O) as a second determinant of productivity growth. The third determinant of productivity growth is innovative activity (K). Innovation not only leads to a higher degree of product differentiation and quality but also to process innovation leading to increased productivity. The final determinant of productivity growth is the productivity gap (GAP). The existence of productivity differences between the frontier economy and the followers opens up the opportunity for imitation and diffusion of more advanced technologies generated by the leader. In a simplified version, it implies a positive effect of the productivity

[15] Recent empirical developments in the literature on Verdoorn's Law have stressed its importance at the regional level. See, for instance, Fingleton and McCombie (1998), Harris and Lau (1998) and León-Ledesma (2000). In an international context, however, it may well be the case that a single equation estimation of the Law suffers from a high degree of simultaneity bias.

gap on the productivity growth of the follower economies, leading to a potential catch-up in productivity levels.[16] The GAP variable, however, may also be thought of as representing the effect of the *lock-in* specialisation discussed in the previous section.

The final set of relationships defining our model – equation (12) – is the one determining innovative activity or the flow of new national innovations. This will depend on four factors. First, on the rate of growth of output (y), reflecting the demand-led innovation hypothesis of Schmookler (1966). Secondly, on the rate of growth of the cumulative sum of real output (q) as in Amable (1992) and de Benedictis (1998). This variable is a proxy for the effect of learning-by-doing originally formulated by Arrow (1962). Both the new products developed and the new production processes depend crucially on the effect of learning acquired through the accumulated experience of the workers. Thus, the higher the growth of accumulated experience – proxied by cumulative output – the more innovations will be incorporated in the production activities.[17] The third major determinant of the success of an economy to generate innovations is the level of education of its working population (edu).[18] The level of education affects the capacity to innovate not only directly but also indirectly because it raises the ability of the economic system to assimilate and understand the new techniques of production. Finally, the productivity gap negatively affects the innovation activity of an economy. With a low level of development few resources are directed to research and development and patenting activities. In other words, the ability to innovate

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[16] For further qualifications of the concept of catch-up in cumulative growth models see Amable (1993) and Targetti and Foti (1997).

[17] Note that, in this context, innovations do not necessarily mean the creation of new products or production techniques but also the marginal improvements in the existing ones.

[18] The level of education (and not its rate of growth) has been introduced in the model due to the fact that the role played by education is wider than a simple human capital variable in a production function. This implies that a constant level of education ensures a constant flow of innovations due to the technical competence and skills of the working population.

depends on the technological level of the country. Countries with a lower technological level are more likely to rely on the benefits of knowledge created in the leader economies.

We will close the model with the formal definition of both the cumulative output growth (q) and the gap (GAP) variables. Given that $Y(t)$ is the level of output in time *t* we have:

$$q = \frac{d \log \int_{t=0}^{T} Y(t)dt}{dt} . \tag{13}$$

The productivity gap is one minus the ratio of productivity between the follower (R) and the leader economy ($R*$). The gap will be zero if there is no difference in productivity level and approach unity if productivity in the follower country is very low.

$$GAP = 1 - \frac{R}{R*} = 1 - G . \tag{14}$$

Thus, we can identify several forces in the model, some leading to divergence and others to convergence in productivity. On the one hand, the Verdoorn-Kaldor effect is a cumulative force that reinforces initial growth advantages (and disadvantages). This is also the same for the effect of demand-led innovation that affects both non-price and price competitiveness, and has a similar effect to that of the Verdoorn-Kaldor mechanism. Learning-by-doing is another force that may make growth cumulative due to the positive effect of cumulative experience on non-price competitiveness and output growth. The final force acting towards a diverging growth pattern is the negative effect of the productivity gap on innovation that tends to perpetuate low levels of technological innovation. On the other hand, the catching-up effect, arising from the flow of technologies from the leader to the follower economies, is the main convergent force of the model. The final outcome will depend on the combination of these multiple effects and their relative power. It will be shown in the next section that possible outcomes of the model include total catch-up, partial catch-up and divergence from the leader.[19] But, given the cumulative forces at work, can the model generate a stable solution for the rate of growth of output? The relevance of this question lies in the empirical fact that explosive behaviour of the growth of output is not observed in the real world.

IV The Dynamics and Stability Analysis

As commented in the introduction to this paper, cumulative growth models are capable of generating a rich set of dynamics relating to convergence issues from the dis-equilibrium perspective of endogenous growth. In order to describe these dynamics, we solve the model represented by (8)-(12) for the growth of output (y) and the growth of productivity (r). We obtain the following two equations:

$$y = D - EG + Fq - Hr \,, \tag{15}$$

$$r = J - LG + Mq + Ny \,, \tag{16}$$

where,

$$D = \frac{\theta\zeta\psi}{1-\theta\zeta\gamma} + \frac{\theta\eta}{1-\theta\zeta\gamma}(w - pf) + \frac{\theta\varepsilon}{1-\theta\zeta\gamma} z + \frac{\theta\zeta\omega}{1-\theta\zeta\gamma} edu + \frac{\theta\delta}{1-\theta\zeta\gamma}(I/O) \,,$$

$$E = \frac{\theta\zeta\psi}{1-\theta\zeta\gamma} < 0 \,, \qquad F = \frac{\theta\zeta\beta}{1-\theta\zeta\gamma} > 0 \,, \qquad H = \frac{\theta\eta}{1-\theta\zeta\gamma} < 0 \,,$$

$$J = \sigma + \alpha\psi + \lambda(I/O) + \alpha\omega(edu) \,,$$

$$L = \sigma + \alpha\psi >< 0, \qquad M = \alpha\beta > 0, \qquad N = \phi + \alpha\gamma > 0$$

[19] The model even allows for the case of the follower economies forging ahead from the “old” leader once the catch-up has taken place.

Differentiating (13) and re-arranging we obtain the following expression for the rate of growth of output:[20]

$$y = q + \frac{\dot{q}}{q}. \tag{17}$$

From the definition of the gap variable (14), we know that the rate of growth of the productivity ratio between the follower and the leader (*G*) is

$$\frac{\dot{G}}{G} = r - r^* \tag{18}$$

with r^* being the rate of growth of labour productivity in the leader economy.[21] Substituting (17) and (18) into (15) and (16), we have the following system of first order non-linear differential equations:

$$\dot{q} = q[D - EG + (F-1)q - Hr], \tag{19}$$

$$\dot{G} = G[P - LG + M(q - q^*) + N(y - y^*)], \tag{20}$$

where,

$$P = \sigma + \alpha\psi + \lambda[(I/O) - (I/O)^*] + \alpha\omega[edu - edu^*].$$

The stability of the system (19)-(20) can be analysed through the stability of the system in brackets, ruling out the possibility that q and G are equal to zero (D-stability). In the steady state, $\dot{q} = \dot{G} = 0$, and given (17) and (18) then $r = r^*$, $y = q$ and $y^* = q^*$. With the

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[20] All the variables denoted by a dot represent the time derivative of the variable, i.e. $\dot{x} = dx/dt$

[21] All the variables with the superscript * represent the original variable for the leader economy.

steady state solutions for y^* and r^* given in the Appendix we can obtain the following system representing the equilibrium paths of both G and q:

$$-LG + (M+N)q = -T \ \Big|_{\dot{G}=0}, \tag{21}$$

$$-EG + (F-1)q = -S \ \Big|_{\dot{q}=0}, \tag{22}$$

where T and S depend on a set of exogenous variables and the parameters of the model (Θ):

$$T = f(I/O, edu, pf, z, I/O^*, edu^*, w^*, \Theta),$$

$$S = g(I/O, edu, pf, w, z, I/O^*, edu^*, w^*, \Theta)$$

Since all the elements in the off-diagonal of the Jacobian of the system (21)-(22) are positive, the stability conditions of this model require:

(a) $-L < 0$, thus, $L > 0$ or $\sigma + \alpha\psi > 0$;

(b) $|Jac| > 0$, or $\frac{L}{M+N} < \frac{E}{F-1}$, which implies that the slope of the phase path line for $\dot{G} = 0$ has to be smaller than the one for $\dot{q} = 0$;

(c) For condition (b) to hold, it is required that $(F-1) < 0$.

The steady state equilibrium point is one where both productivity level differences remain the same and output growth is stable and equal in the leader and the follower economy. If catch-up is strong enough, during the transition output grows faster in the follower than in the leader. Once we approach the equilibrium, both rates of growth equal one another and productivity level differences remain stable. The parameters of the model determine where the follower stops catching-up and, thus, whether this process is absolute or just partial. The existence of per capita income convergence and a tendency for the rates of growth to be equal in the long run for the advanced countries is one of the growth facts reported in Evans (1996) and Temple (1999). Two possible stable cases of equilibrium can arise. These are depicted in figures 1 and 2, where the combinations of G and q that make $\dot{q}=0$ and $\dot{G}=0$ are represented. The first one is a stable focus (Figure 1), where the path taken towards the equilibrium gap and rate of growth of the cumulative output (and output growth) generates cyclical behaviour.[22] The economy oscillates around the equilibrium point until it is reached. The second case is a stable node (Figure 2).[23] In this case, regardless of the initial point, the economy will follow a direct path towards its equilibrium solution, this adjustment being faster than in the former case. In Figure 1 we have depicted an equilibrium point where the laggard country catches-up with the leader and even forges ahead of it ($G>1$). Contrawise, Figure 2 shows a situation where the equilibrium only allows for a partial catch-up and, thus, differences in levels of productivity would be maintained through time. Of course, it would be possible to find cases of total falling behind ($G=0$), if both lines do not cross before the q axis. However, this is an implausible case especially for developed and emerging economies. It is also important to note that a positive value of the parameter of the GAP variable - or negative value of the parameter of G - does not necessarily imply convergence in levels of productivity. The convergence in productivity levels will also depend on the endogenous cumulative mechanism linking the growth of output, learning-by-doing and innovation with productivity growth and price and non-price competitiveness. Despite the fact that the parameter σ is positive, convergence may not be the outcome if the cumulative

[22] This would be the case if the trace of the Jacobian of the system and its determinant are such that $(tr\ Jac)^2 > 4|Jac|$.

[23] If $(tr\ Jac)^2 > 4|Jac|$.

forces in the leader economy are stronger than in the follower. Note also that from the system of equations (8)-(12) we can obtain a reduced form for productivity growth as follows:

$$r = a_1 + a_2 G + a_3 (edu) + a_4 (I/O) + a_5 q + a_6 (p - pf) + a_7 z \tag{23}$$

This is similar to those convergence equations used when attempting to test the neo-classical hypothesis of convergence, controlling for the variables that determine the steady state level of productivity (see Barro and Sala-i-Martín, 1995). From the perspective of the model here presented, a convergence equation that does not include the rate of growth of the determinants of exports would be mispecified. Thirlwall and Sanna (1996) have shown that one of the most robust variables influencing per-capita income growth in a convergence equation is the rate of growth of exports. This model gives a plausible theoretical explanation of these results and, on the other hand, shows that the Barro-type convergence equations are not necessarily a test of the neo-classical growth model. A test of the structural model underlying the reduced form convergence equation is necessary in order to compare the relevance of competing explanations of the growth and convergence phenomena.[24]

Turning finally to the conditions under which the model presented is stable, it is possible to extract a set of economically meaningful conclusions. The condition (a) that $L > 0$ means that the net effect of the productivity gap on productivity growth must be positive, i.e. that the positive effect of technological catch-up is not offset by the negative impact of the gap on innovation and, in turn, productivity. Condition (b) is stating that the effect of the productivity gap relative to that of the cumulative mechanisms has to be smaller for productivity growth than for output growth. Keeping the effect of the productivity gap constant, the greater the impact of cumulative forces on output growth (F), the less stable the

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[24] See León-Ledesma (1999) for an estimation of the model represented by equations (8)-(12) using simultaneous equations techniques.

model will tend to be. This conclusion, together with condition (c), calls for a limited impact of the cumulative forces on output growth. These conditions are a generalisation, in a continuous time extended system, of the stability conditions stated by Boyer and Petit (1991) in a Kaldorian cumulative growth model, i.e. that the sensitivity of output growth to productivity growth must be smaller than that of productivity growth to output growth. Otherwise, the model would be unstable, giving as a result a world in which differences in per capita income would reach infinite values in a finite time span.

Regarding the Verdoorn-Kaldor coefficient (ϕ), note that the higher the value of it the more stable will the model tend to be. This is because a high impact of output growth on productivity growth will increase the velocity of convergence towards the steady state dynamic equilibrium. In other words, it will make the denominator of the left-hand side of condition (b) higher. Thus, contrary to the intuitive idea of a high Verdoorn coefficient leading to unstable growth, it has the effect of accelerating the convergence towards the long-run equilibrium. This convergence towards the steady state does not ensure, however, the existence of catching-up with the leader economy since it only refers to the process of mean reversion. If, nevertheless, the effect of the catching-up variable is strong enough to lead to an absolute closure of the gap, the Verdoorn coefficient will certainly have the effect of accelerating the process of catching-up. This may explain the results obtained by Fingleton and McCombie (1998) that show how in the European regions it is possible to observe a strong and robust Verdoorn effect together with a high velocity of beta (β) convergence.

V Conclusions

Throughout this paper we have presented and discussed some of the most relevant contributions to the cumulative or dis-equilibrium growth literature. We have argued that cumulative growth models achieved their endogeneity nature before and independently of the 'new' growth theories. Furthermore, these models may present dynamic behaviour compatible with the variety of experiences regarding convergence in developed and developing countries. The standpoint of these models departs from the general equilibrium nature of the recent contributions to endogenous growth theory. In this context it is demand that leads growth and the natural rate of growth becomes endogenous to the actual.

In order to show this and to avoid some of the shortcomings of the traditional cumulative growth models we have presented an extended version of the canonical Kaldorian cumulative growth model. The model allows for the introduction, among other things, of technology variables such as innovation and technology gaps that have been stressed as important factors determining the growth performance of modern economies. It allows for the analysis of productivity convergence generating a richer set of dynamics than the traditional cumulative growth models. It has been shown that the model, under some non-restrictive conditions, can generate a stable pattern of growth. Contrary to the popular idea of cumulative growth generating ever increasing differences in per capita output and productivity levels, a growth process generated by this kind of dynamics is compatible with the existence of catch-up from the followers to the leader economy.

A growth equation similar to those used in recent growth empirical exercises can be derived from the structural form of the model. This fact recommends the estimation of the structural form, in order to avoid second order identification problems that impede discriminating among competing theories of growth. On the other hand, we have shown how the Verdoorn coefficient, counterintuitively, can act as a force that increases the velocity of convergence towards the steady state.

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FIGURES

Figure 1. Stable focus

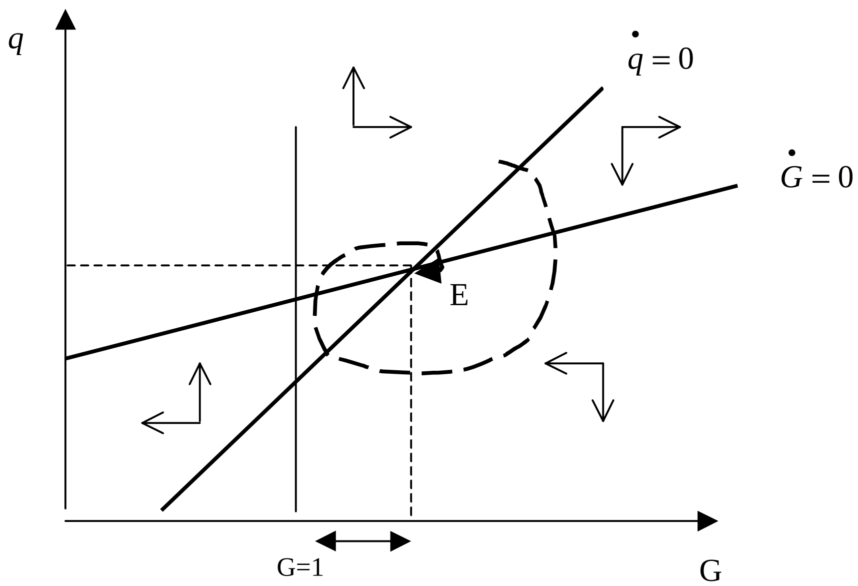
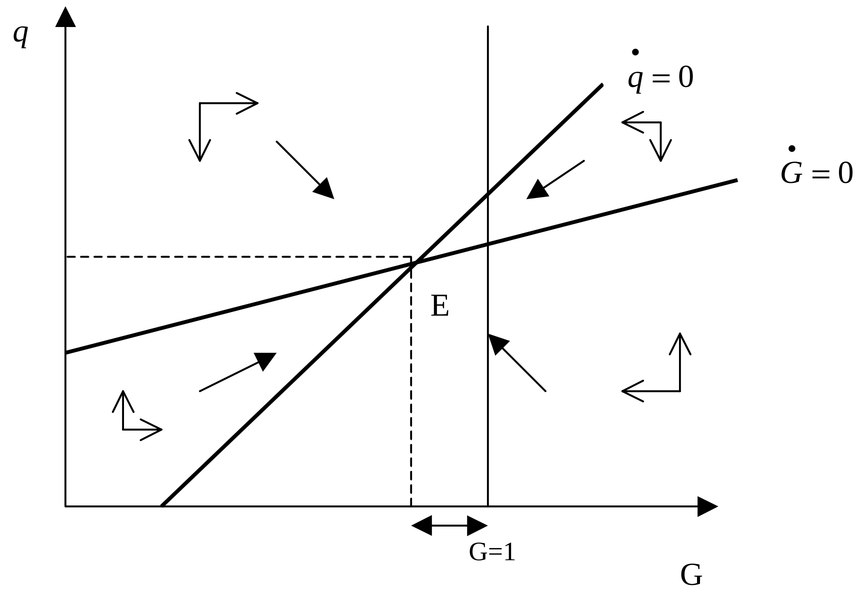


Figure 2. Stable node



APPENDIX

The Dynamics of Output Growth in the Leader Economy

For the leader economy $GAP = 0$, for $G = 1$. Thus, re-writing equations (4) and (5), we obtain,

$$r^* = \phi y + \lambda (I/O)^* + \alpha K^* \tag{4'}$$

$$K^* = \gamma y^* + \beta q^* + \omega (edu)^* \tag{5'}$$

Solving the new system for the rate of growth of output we obtain:

$$y^* = B + Cq^*, \tag{A.1}$$

where,

$$B = \frac{\theta\eta}{1-\theta[\zeta\gamma - \eta(\phi + \alpha\gamma)]}(w^* - pf) + \frac{\theta\varepsilon}{1-\theta[\zeta\gamma - \eta(\phi + \alpha\gamma)]}z - \frac{\theta\eta\lambda}{1-\theta[\zeta\gamma - \eta(\phi + \alpha\gamma)]}(I/O)^*$$
$$+ \frac{\theta\omega(\zeta - \eta\alpha)}{1-\theta[\zeta\gamma - \eta(\phi + \alpha\gamma)]}(edu)^*$$

$$C = \frac{\theta\beta(\zeta - \eta)}{1-\theta[\zeta\gamma - \eta(\phi + \alpha\gamma)]}.$$

From expression (10) we have (Amable, 1993 and De Benedictis, 1998),

$$\dot{q}^* = Bq^* + (C-1)q^{*2} \tag{A.2}$$

Only in the case that $B > 0$ and $(C-1) < 0$, we have a stable and positive solution for the rate of growth of cumulative output (see Amable, 1993). Equation (A.2) is a first order non-linear differential equation of the Bernoulli form, whose solution path is given by:

$$q(t)^* = \frac{1}{\left[\frac{1}{q_0^*} + \frac{C-1}{B}\right]e^{-Bt} - \frac{C-1}{B}} \tag{A.3}$$

where q_0* is the initial rate of growth of cumulative output. Given the relationship (17) between y and q, we obtain the dynamic solution for the rate of growth of output:

$$y(t)^* = B + \frac{C}{\left[\frac{1}{q_0*} + \frac{C-1}{B}\right] e^{-Bt} - \frac{C-1}{B}} \tag{A.4}$$

When $t \to \infty$, $q^* = y^* = -B/(C-1)$, and $\dot{q} = 0$. Since the value of $(C-1) < 0$, we have a positive and stable solution for the rate of growth of output, despite the fact that cumulative forces are at work. This solution will depend positively on z, pf, $(I/O)*$ and $edu*$, and negatively on $w*$.