Firms and Economic Performance:
A View from Trade*

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This draft: November 2018

Abstract

We use transaction-level US import data to compare firms from virtually all countries in
the world competing in a single destination market. Guided by a simple theoretical framework,
we decompose countries' market shares into the contribution of the number of firm-products,
their average attributes (quality and efficiency) and heterogeneity around the mean. To further
explore the role of exceptional firms, we also develop a novel decomposition that separates the
contribution of heterogeneity from that of granularity. Our results show that the number of firm-
products explains half of the variation in sales, while the remaining part is equally accounted
for by average attributes and their dispersion. Quality is the main driver of firm heterogeneity.
While individual firms matter, we find that heterogeneity is more important than granularity for
explaining sales. We then study how the distribution of firm-level characteristics varies across
countries, and we explore some of its determinants. Countries with a larger market size tend to
be characterized by a more dispersed distribution of firms’ sales, especially due to heterogeneity
in quality. These countries also tend to be more likely to host superstar firms, although this is
not the only source of higher heterogeneity.

JEL Classification: F12, F14.

Keywords: US Imports, Firm Heterogeneity, International Trade, Prices, Quality, Variety,
Granularity.

*We thank Andy Bernard, Ed Glaeser, Kiminori Matsuyama, Peter Neary, Steve Redding, Veronica Rappoport and
seminar participants at the Barcelona GSE Summer Forum (2018), CAGE International Research Day (University of
Warwick), DICE Workshop “Firms in a Global Economy”, ECB, ESSEC Business School (Cergy-Pontoise), ESSIM
(Oslo), OECD (Paris), the University of Ancona and the University of Oxford for valuable comments. We acknowledge
financial support from the Spanish Ministry of Economy and Competitiveness through the National Plan (ECO2014-
59805-P) and the Severo Ochoa Programme for Centres of Excellence in R&D (SEV-2015-0563).

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Understanding differences in economic performance across countries has always been one of the great challenges in economics. Until recently, efforts to address this question relied mostly on aggregate data, often computing productivity as country residuals.\(^1\) The availability of firm-level data revolutionized the field by showing that productivity varies enormously even across firms within countries, and that this rich heterogeneity at the micro level has important consequences for aggregate statistics.\(^2\) Yet, due to the lack of comprehensive and comparable data, existing studies have been confined to a handful of countries only.\(^3\) As a result, to date there is still little systematic evidence on the role of firms in explaining country performance. In this paper, we overcome this difficulty by using detailed import data to compare firms from virtually all countries in the world competing in a single market. In doing so, we provide the most comprehensive account of how the distribution of firm-level characteristics shape aggregate sales and trade flows. We then use our findings to study how these distributions vary across countries and explore some of their determinants.

Following recent methodological advances in trade theory, we show that data on unit values and volumes of imports in a single destination market, together with few and commonly used assumptions on demand and supply, are sufficient to map the market shares captured by each country into the characteristics of the underlying firms.\(^4\) We apply this methodology to transaction-level data on US imports in 2002 and 2012 containing information on prices, volumes and the identity of exporting firms for 6-digit products from over 100 countries. As a first step, we decompose the variation in the value of imports into an extensive margin, the number of firm-products per country, and an intensive margin, the average sales per firm-product in a given country. This decomposition, which we implement across 4-digit industries, shows that each margin accounts on average for half of the overall variation in market shares.

In a second and more interesting step, we decompose average sales per firm-product in a country-industry-year triplet into two parts: the average “appeal” of the firm-product and a “heterogeneity” term, capturing the dispersion of appeal around its average. Intuitively, countries with more appealing firm-products can sell more and capture larger market shares. However, dispersion also affects the total value of sales because consumers can substitute low-appeal products for high-appeal products. We show that, when the elasticity of substitution between products is higher (lower) than two, more dispersion implies larger (smaller) average sales. We find that the heterogeneity term explains roughly half of the cross-country variation in average sales per firm-product in our data.

In a third step, we decompose appeal into two components: a demand shifter, often interpreted as

\(^1\)See, for instance, Hall and Jones (1999), Caselli (2005), Gancia, Mueller and Zilibotti (2013).

\(^2\)See, for instance, Hsieh and Klenow (2009), Restuccia and Rogerson (2008), Syverson (2011) and, more recently, Baqae and Farhi (2017).

\(^3\)For instance, Gennaioli et al. (2013) use firm-level data for 20 countries, Bartelsman, Haltiwanger and Scarpetta (2013) for 24 countries, Bloom, Sadun and Van Reenen (2016) for 34 countries and Poschke (2015) for 50 countries.

\(^4\)In particular, see Hottman, Redding and Weinstein (2016) and Redding and Weinstein (2017).
“quality” and identified from the variation in market shares conditional on prices, and “efficiency”. We find that quality explains between 75% and 100% of the variation in appeal across firm-products. As expected, quality and efficiency are negatively correlated, suggesting that higher quality products are more costly to produce. In sum, our exercise shows that countries capturing larger US market shares have more exporters, producing higher-quality products with a more dispersed distribution.

One reason for the importance of firm heterogeneity in explaining sales is the presence of exceptional firms in each country. Indeed, it is well-known that top exporters dominate trade flows. We explore the role of “superstar firms” by imposing additional theoretical restrictions. Assuming that attributes follow log-normal distributions, with parameters that can differ across countries and industries, we develop a novel and general decomposition that separates the role of heterogeneity, defined as smooth variation in attributes across a large number of firms, from that of granularity, defined as exceptional performance in a small sample. This decomposition helps us quantifying how much information is lost when using a continuum distribution to approximate sales. Surprisingly we find that, although top firms are quantitatively very important, the granular residual explains about 5% only of the observed variation in sales across countries and industries. This suggests that superstar firms are more a manifestation of export performance than a cause of it.

Implementing some of these decompositions, i.e., identifying the firm characteristics that explain exactly the observed sales, requires estimating the elasticity of substitution between products in any given industry. Given the importance of this parameter, we follow various empirical strategies to identify it. First, we use the recent “reverse-weighting” estimator pioneered by Redding and Weinstein (2017), which relies on restrictions on CES demand and exploits variation in prices and market shares over time. Time variation is however limited in our data. Hence, guided by our theoretical framework, we also develop a new approach to identify the elasticity of substitution from the cross-sectional variation in the dispersion of sales. The intuition is that a higher substitutability generates more dispersion in sales for a given distribution of attributes. As a last robustness check, we also use existing estimates taken form the literature. In all cases, we find that the median elasticity across industries is well above two and that our decompositions are remarkably stable across the different estimates of this elasticity.

The main lesson from our decompositions is that heterogeneity in firms’ attributes plays an important role in explaining economic performance, a role which is however masked in aggregate statistics. Yet, surprisingly little is known about the determinants of firm heterogeneity. In a first attempt to fill this gap, we uncover a number of novel patterns. First, by studying the correlations with country characteristics, we find that measures of market size, namely GDP per capita, population and distance from the US, are all associated with a higher dispersion of sales and firms’ attributes, especially due to heterogeneity in quality. We then ask whether these results are driven by superstar firms. We show that the incidence of superstar firms is indeed positively correlated with measures of market size. Yet, the correlation between the heterogeneity term and market size
is not driven by superstar firms, as it holds even when they are removed from the sample. Finally, we use our theoretical framework to draw some quantitative implications. We show that when attributes are log-normally distributed, the effect of firm heterogeneity on prices is summarized by the variance of log sales. We then use this simple statistic to show that changes in firm heterogeneity across countries and over time have a large impact on price indexes.

Our results have important implications. From a policy perspective, they point towards a so far underexplored benefit of market size: larger markets host more diverse firms and seem to be more fertile ground for superstars. While a few large companies can define the economic success of countries, how to breed them is still poorly understood (Freund, 2016). Our results offer some hints and our approach provides a starting point for a more in-depth analysis of this question. From a theoretical perspective, our results confirm that product differentiation, varieties and heterogeneity in quality are essential features to explain the data. Besides confirming the importance of modeling firm heterogeneity, as in Melitz (2003) and Melitz and Redding (2014), our results underscore the need for introducing differences in the distribution of attributes. Our finding that firm heterogeneity varies systematically with country characteristics is likely to have significant implications for the level and distribution of the gains form trade in quantitative models (e.g., Costinot and Rodriguez-Clare, 2014).

Our approach based on comparing firms in the US market has several advantages. The first is the quality, coverage and comparability of the data. Second, the US market is the largest in the world and one of the most competitive. This alleviates the concerns that the results be driven by differences in market power and/or domestic distortions affecting the size of firms in the source country. Market shares in the United States are instead more likely to reflect solely firm characteristics such as the price and quality of products. The disadvantage of this approach is that our results apply, strictly speaking, only to the set of firms exporting to the United States. Although decomposing trade flows is interesting in its own right, the well-known observation that market shares are highly proportional to GDP suggests that our findings are likely to be more general.

This paper is related to the literature on the role of firms for explaining trade flows. Some papers have studied the role of the extensive and intensive margin in explaining trade flows (e.g., Bernard et al. 2018, Fernandes et al., 2017, Chaney, 2008, Hummels and Klenow, 2005). Other contributions have focused on quality (e.g., Crinò and Ogliari, 2017, Feenstra and Romalis, 2014, Hallak and Schott, 2011, and Khandelwal, 2010). The most closely related paper is Redding and Weinstein (2018), who use a similar framework for aggregating transaction-level US import data.

\footnote{The rise of superstar firms is attracting considerable attention (see, for instance, Autor et al. 2017). Yet, there is still no consensus on the causes of this phenomenon.}

\footnote{Redding and Weinstein (2018) is part of a line of research by these authors aimed at studying the consequences of micro-level heterogeneity for aggregate outcomes (Hottman, Redding and Weinstein 2016, Redding and Weinstein 2017). The present paper derives from a parallel project aimed at exploring the origins of firm heterogeneity (Bonfiglioli, Crinò and Gancia, 2018a,b).}
We differ in several important ways. First, we ask a different question. Redding and Weinstein (2018) are interested in quantifying the contribution of prices, quality and variety for comparative advantage and price indexes. Instead, we focus on absolute advantage with the aim of identifying the firm-level determinants of economic performance. For this reason, we go beyond a mere accounting exercise by exploring how firm heterogeneity varies across countries. Second, we propose a different decomposition aimed at fully separating the effect of averages and dispersion in the level of attributes. Compared to our results, the log-linear decomposition in Redding and Weinstein (2018) overstates the contribution of heterogeneity in the level of firm characteristics. Third, we propose a novel and more general decomposition that separately accounts for the role of exceptional firms. In sum, compared to the existing literature, our paper provides the most comprehensive analysis of the role of firms in explaining US imports.

Our attempt at quantifying the contribution of granularity in explaining trade flows is part of a recent line of research studying the role of exceptional exporters. Freund and Pierola (2015) document that on average the top five firms account for 30% of exports in a sample of 32 countries. While this evidence indicates that firms are not infinitesimal, it does not necessarily imply that superstar firms are outliers. Gaubert and Itskhoki (2016) estimate a structural model with an integer number of firms and find that the granular residual, compared to a continuum model, accounts for 30% of the variation in export shares. Besides the approach, there are two important differences from our paper: they assume firm attributes to be Pareto distributed, and abstract from asymmetries in these distributions across sectors and countries. Our results suggest that assuming log-normal distributions, which provide a better fit of the data, and allowing for realistic asymmetries in these distributions, reduces significantly the role of the granular residual.

The idea to use trade data to estimate productivity is relatively old. Trefler (1993) computes factor-augmenting productivity to match Heckscher-Ohlin-Vanek equations. Eaton and Kortum (2002) estimate country-level productivity by fitting a quantitative Ricardian model. Fadinger and Fleiss (2011) back out industry-level productivity differences from bilateral trade data using a hybrid Ricardo-Heckscher-Ohlin model. All these papers focus on country-sector level data and hence are silent on how firm-level characteristics shape aggregate productivity and trade flows.

The remainder of the paper is organized as follows. In Section 2 we introduce the theoretical framework which guides us through the decomposition of countries’ market shares. Section 3 describes the firm-level data on US imports that we use in the empirical analysis. In Section 4, we perform the structural estimation of the elasticity of substitution, necessary for the decomposition exercise, following alternative approaches. Section 5 reports the main results from our decomposition exercise: the role of the intensive versus extensive margins, the role of average appeal versus its dispersion, the role of quality versus efficiency in explaining appeal and, finally, the role of granular-

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7Similarly, Gabaix (2011), di Giovanni, Levechenko and Méjean (2014) and Carvalho and Grassi (2015) show that idiosyncratic shocks to individual firms contribute to aggregate fluctuations.
In Section 6, we study how these contributions, and especially firm heterogeneity, vary across countries and draw some quantitative implications. Section 7 concludes.

2 Theoretical Framework

We now describe the set of assumptions needed to map import data into firm-level characteristics by industry and country of origin. On the demand side, CES preferences together with elasticities of substitution across varieties in an industry are sufficient to decompose market shares into the contribution of the number of exported varieties, average product appeal and deviations from this country-industry average.\(^8\) Imposing more structure on the supply side yields further insights. Assuming monopolistic competition is sufficient to further identify the contribution of differences in efficiency across products in explaining appeal. With stronger restrictions on technology, the results learnt on exporters can even be generalized to other firms operating in a given country.

2.1 Demand-Side Restrictions

We focus on a multi-industry model. Given that our data are not sufficiently disaggregated to fully capture the product scope of firms, we abstract from multi-product firms. Consistently, in the empirical section, we will take the firm-product pair (“variety”) as the basic unit of analysis.\(^9\)

2.1.1 Preferences and Demand

Consider consumers located in a destination \(d\). In the empirical section, the destination will be the US market. Preferences over consumption of goods produced in \(I\) industries are:

\[
U_d = \prod_{i=1}^{I} C_{di}^{\beta_i}, \quad \beta_i > 0, \quad \sum_{i=1}^{I} \beta_i = 1.
\]

Each industry \(i \in \{1, ..., I\}\) produces differentiated varieties and preferences over these varieties take the constant elasticity of substitution form:

\[
C_{di} = \left\{ \sum_{\omega \in \Omega_{di}} [\gamma_{di}(\omega)c_{di}(\omega)]^{\frac{\sigma_i}{\sigma_i - 1}} \right\}^{\frac{\sigma_i}{\sigma_i - 1}}, \quad \sigma_i > 1,
\]

where \(c_{di}(\omega)\) is quantity consumed of variety \(\omega\), \(\gamma_{di}(\omega)\) is a demand shifter, \(\Omega_{di}\) denotes the set of varieties available for consumption in market \(d\) in industry \(i\), and \(\sigma_i\) is the elasticity of substitution.

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\(^8\)CES preferences are a dominant paradigm in the literature. See Matsuyama and Ushchev (2017), Mrázová and Neary (2017) and Mrázová, Neary and Parenti (2017) for interesting discussions of more general demand systems.

\(^9\)In this way, we do not impose any exogenous nesting structure between varieties produced by the same firm and across different firms.
between varieties within industry \( i \). In general, we use lowercase letters for variables referring to a single variety and uppercase letters for more aggregate variables. The demand shifter \( \gamma_d(\omega) \) is often interpreted as "quality", in that it captures the appeal of a certain product and its value for a given quantity consumed. Note that \( \gamma \) captures both the intrinsic quality of the variety and its specific appeal in the destination market considered. Since we have data on one destination market only, we will not be able to distinguish between them. With this caveat in mind, from now on we refer to \( \gamma \) as quality.

We denote by \( p_d(\omega) \) the price of variety \( \omega \) in industry \( i \) and by \( P_d \) the minimum cost of one unit of the consumption basket \( C_d \):

\[
P_d = \left\{ \sum_{\omega \in \Omega_d \gamma_d(\omega)} \left[ \frac{p_d(\omega)}{\gamma_d(\omega)} \right]^{1-\sigma_i} \right\}^{\frac{1}{1-\sigma_i}}.
\]

(1)

Then, demand for a variety \( \omega \) can be expressed as:

\[
c_d(\omega) = p_d(\omega)^{-\sigma_i} \gamma(\omega)^{\sigma_i-1} P_d^{\sigma_i} C_d.
\]

(2)

As usual, demand is a negative function of the price, with an elasticity \( \sigma_i \). Conditional on prices, demand is increasing in quality, with an elasticity \( \sigma_i - 1 \).

2.1.2 Decomposing Market Shares

Using (2), the expenditure share for a single variety \( \omega \) can be written as:

\[
s_d(\omega) = \frac{p_d(\omega) c_d(\omega)}{\sum_{\omega \in \Omega_d} p_d(\omega) c_d(\omega)} = \frac{[\gamma_d(\omega)/p_d(\omega)]^{\sigma_i-1}}{\sum_{\omega \in \Omega_d} [\gamma_d(\omega)/p_d(\omega)]^{\sigma_i-1}}.
\]

(3)

Market shares are increasing in the quality-to-price ratio of a variety, \( \gamma_d(\omega)/p_d(\omega) \).\(^{10}\) More importantly, this equation illustrates that quality-to-price ratios and the demand elasticity are sufficient statistics to compute any market share. In particular, the market share captured by all varieties sold from any single country of origin \( o \) in industry \( i \), denoted by \( S_{doi} \), is:

\[
S_{doi} = \frac{\sum_{\omega \in \Omega_{doi}} [\gamma_d(\omega)/p_d(\omega)]^{\sigma_i-1}}{\sum_{\omega \in \Omega_d} [\gamma_d(\omega)/p_d(\omega)]^{\sigma_i-1}},
\]

(4)

\(^{10}\) The inverse of the quality-to-price ratio is commonly called "quality-adjusted price". Since our results confirm that variation in sales is driven mostly by quality and that even price variation reflects to a large extent differences in quality, we prefer to emphasize \( \gamma/p \) rather than its inverse.
where $\Omega_{doi}$ is the set of varieties sold in market $d$ from origin $o$ in industry $i$. Starting from this equation, we are interested in understanding what makes a country capture a larger market share.

To this end, define the quality-to-price ratio for any variety in $\Omega_{doi}$ as $\gamma_{doi}(\omega) \equiv \gamma_{di}(\omega)/p_{di}(\omega)$, then add and subtract its mean within the summations:

$$S_{doi} = \frac{\sum_{\omega \in \Omega_{doi}} \left[ \gamma_{doi}(\omega)^{\sigma_i-1} + \mathbb{E}(\gamma_{doi})^{\sigma_i-1} - \mathbb{E}(\gamma_{doi})^{\sigma_i-1} \right]}{\sum_{\omega \in \Omega_{di}} \left[ \gamma_{di}(\omega)^{\sigma_i-1} + \mathbb{E}(\gamma_{di})^{\sigma_i-1} - \mathbb{E}(\gamma_{di})^{\sigma_i-1} \right]},$$

where $\mathbb{E}(\gamma_{doi})$ is the arithmetic mean across $\gamma_{doi}(\omega)$ from a single origin $o$ and $\mathbb{E}(\gamma_{di})$ is the arithmetic mean from all origins.

This allows us to decompose countries’ market shares as follows:

$$S_{doi} = \frac{N_{doi} \cdot \bar{r}_{doi}}{N_{di} \cdot \bar{r}_{di}}, \quad (5)$$

where $N_{doi}$ and $N_{di}$ are the number of varieties from $o$ and from all origins, respectively, in destination $d$ and industry $i$, and

$$\bar{r}_{doi} \equiv \mathbb{E}(\gamma_{doi})^{\sigma_i-1} + \frac{1}{N_{doi}} \sum_{\omega \in \Omega_{doi}} \left[ \gamma_{doi}(\omega)^{\sigma_i-1} - \mathbb{E}(\gamma_{doi})^{\sigma_i-1} \right], \quad (6)$$

with an analogous expression for $\bar{r}_{di}$ (removing the origin index $o$). Note that $\bar{r}_{doi}$ normalizes average sales, or revenue, from country $o$, so as to make them scale independent and comparable across industries too.\footnote{Sales are $r_{di}(\omega) = [\gamma_{di}(\omega)]^{\sigma_i-1} P_{di} C_{di}$. Hence, $\bar{r}_{doi} = \mathbb{E}(r_{doi}) / P_{di} C_{di}$.}

Equation (5) decomposes market shares into the contribution of the number of products (extensive margin) versus average sales of each product (intensive margin). More interestingly, equation (6) shows that average sales can be further decomposed into two terms. The first term captures the average quality-to-price ratio of products from a given country. The second term captures the importance of heterogeneity in quality-to-price ratios. Clearly, equation (6) shows that the heterogeneity term is zero if all the quality-to-price ratios from a given country are identical. But what is the sign of this term if quality-to-price ratios do vary across products? It turns out that the answer to this question depends on the value of $\sigma_i$.

To see why, note from equation (3) that sales are a convex function of the quality-to-price ratio when $\sigma_i > 2$. In this case, products are sufficiently substitutable that the possibility to reallocate expenditure from less to more attractive products increases total sales when holding constant the average quality-to-price ratio. Hence, the contribution of heterogeneity in (6) is positive. When $\sigma_i = 2$, instead, sales are linear in the quality-to-price ratio, so that only its average, and not its distribution, matters. In this case, the second term in (6) collapses to zero. Finally, when $\sigma_i < 2$,
sales are a concave function of the quality-to-price ratio, so that more heterogeneity has a negative contribution to the overall market share.

Note also that equations (5)-(6) can be used to decompose the market share of any country \( o \) relative to any other country \( j \) or any other group of countries, such as the set of all exporters to destination \( d \). Hence, it can be used to study how the number of products, their average appeal and its heterogeneity determine the distribution of the total value of imports across all possible source countries and industries.

The final step in the decomposition of market shares into product characteristics is to study the contribution of quality and prices in explaining the variation in the quality-to-price ratios. Since sales are a power function of \( \gamma_{doi}(\omega)/p_{doi}(\omega) \) with exponent \( (\sigma_i - 1) \), decomposing the variance of \( \gamma_{doi}(\omega)/p_{doi}(\omega) \) allows us to explain variation in market shares, or equivalently sales, across products as:

\[
\mathbb{V}(\ln s_{doi}) = (\sigma_i - 1)^2 \left[ \mathbb{V}(\ln \gamma_{doi}) + \mathbb{V}(-\ln p_{doi}) + 2\text{Cov}(\ln \gamma_{doi}; -\ln p_{doi}) \right], \quad (7)
\]

where \( \mathbb{V}(\ln s_{doi}) \) is the variance of the log of sales computed across all varieties sold by country \( o \) in industry \( i \) and market \( d \), and \( \text{Cov} \) is the covariance. Intuitively, sales dispersion is a positive function of the dispersion of quality, the inverse of prices, and their correlation. Sales dispersion is also increasing in the elasticity of substitution, \( \sigma_i \), because differences in quality-to-price ratios map into larger differences in sales if products are more substitutable. Note that this decomposition of sales can be applied to any set of firms (e.g., from a single origin or from all) in an industry.

### 2.2 Supply-Side Restrictions

The decompositions in Section 2.1.2 hold irrespective of any supply-side assumptions, that is, for any production function, any distribution of product characteristics and any market structure. However, imposing more structure on the supply side of the model allows us to gain further insights. Here, we consider a minimal set of restrictions that are common in the literature.

In each industry, every variety \( \omega \) is produced by firms that are heterogeneous in their labor productivity, \( \varphi \), and quality, \( \gamma \). Since all firms with the same attributes \( (\varphi, \gamma) \) behave similarly, we index firms by \( (\varphi, \gamma) \) and identify firms with products. We do not need any restriction on the distribution of attributes, nor do we need to specify where these distributions come from.

Firms engage in Bertrand competition. Then, the equilibrium price of a firm with attributes \( (\varphi, \gamma) \) serving market \( d \) from country \( o \) is:

\[
p_{doi}(\varphi, \gamma) = \mu_{doi}(\varphi, \gamma) \frac{\tau_{doi} w_o}{\varphi},
\]

where \( w_o \) is the wage in country \( o \), \( \tau_{doi} \geq 1 \) is the iceberg cost of shipping from \( o \) to \( d \) in industry \( i \) and \( \mu_{doi}(\varphi, \gamma) \) is the markup over the marginal cost charged by the firm. With a discrete number of
firms, the markup depends on the market share of each firm. In particular, the perceived demand elasticity is \( \sigma_i - (\sigma_i - 1)s_{doi}(\varphi, \gamma) \), where \( s_{doi}(\varphi, \gamma) \) is the market share of a firm from origin \( o \), with attributes \( (\varphi, \gamma) \), selling to destination \( d \). In our empirical application, we consider foreign firms selling in the US market. Since we find that even the largest foreign firms account for a tiny fraction of the US market in any given industry, we safely approximate their perceived demand elasticity with \( \sigma_i \) so that \( ^{12} \)

\[
\mu_{doi}(\varphi, \gamma) = \frac{\sigma_i}{\sigma_i - 1}.
\]

In this case, since \( \mu, \tau, w \) do not vary across products sold in \( d \) from a given origin in a given industry, we can identify dispersion in efficiency from the variation in prices at the destination:

\[
\nabla(\ln \varphi_{doi}) = \nabla(- \ln p_{doi}). \tag{8}
\]

Revenue earned from selling to market \( d \) is:

\[
r_{doi}(\varphi\gamma) = P^{\sigma_i}C_{di}\left(\frac{\sigma_i - 1}{\sigma_i}\frac{\gamma\varphi}{\tau_{doi}w_o}\right)^{\sigma_i - 1}. \tag{9}
\]

Note that revenue is a power function of \( \varphi\gamma \), which captures the overall appeal of a firm. Profits earned in market \( d \) are a fraction \( \sigma_i \) of revenue minus any fixed cost of serving the market, \( w_o f_{doi} \).

Hence:

\[
\pi_{doi}(\varphi\gamma) = \frac{r_{doi}(\varphi\gamma)}{\sigma_i} - w_o f_{doi}. \tag{10}
\]

A firm finds it profitable to serve market \( d \) only if \( \varphi\gamma \) is sufficiently high. Define \( (\varphi\gamma)_{doi}^* \) as the minimum level of \( \varphi\gamma \) such that a firm breaks even: \( \pi_{doi}((\varphi\gamma)_{doi}^*) = 0 \). Then, revenue from market \( d \) of a firm located in country \( o \) and operating in industry \( i \) can be expressed as:

\[
r_{doi}(\varphi\gamma) = r_{doi}^*[\frac{\varphi\gamma}{(\varphi\gamma)_{doi}^*}]^{\sigma_i - 1}, \tag{11}
\]

where \( r_{doi}^* = \sigma_i w_o f_{doi} \). Note that export participation, quantities and the price index all depend on the composite variable \( \varphi\gamma \), which can be taken as a synthetic measure of firm heterogeneity.

The structure imposed on the supply side of the model teaches us a number of lessons. First, even if we cannot observe markups, variation in prices across products from a given country can be purged from the effect of market power using just information on market shares. If market shares are small in a given destination, variation in prices is likely to be driven by differences in costs solely. Second, for characterizing the equilibrium allocation, quality and efficiency can be collapsed into a single firm attribute, \( \varphi\gamma \). Third, selection into exporting of the most productive firms implies

\[^{12}\text{In our sample, the share of individual varieties in the total imports of the US equals 0.04% in the average industry and 0.6% in the industry at the 99th percentile of the distribution.}\]
that the distribution of sales in a foreign destination will reflect a truncated distribution of the characteristics of domestic firms in any country of origin. Imposing more restrictions allows us to draw even stronger conclusions. For example, if firm attributes, $\varphi \gamma$, are Pareto distributed, as it is often assumed, then the shape parameter of all firms can be inferred from the dispersion of sales in an export market.

3 The Data

To perform our empirical analysis, we need data on the sales of individual products in a single destination market by firms of different origin countries. We obtain this information using transaction-level data on US imports from Piers, a database administered by IHS Markit. Piers contains the complete detail of the bill of lading of any container entering the US by sea. IHS Markit collects, verifies and standardizes the information contained in the bills of lading, and makes the resulting data available for sale. We purchased from IHS Markit information on the universe of waterborne import transactions of the US, by exporting firm and product, in two years, 2002 and 2012. For each transaction, we know the complete name of the exporting firm, its country of origin, the exported product (according to the 6-digit level of the HS classification), the value (in US dollars) and the quantity (in kilograms) of the transaction.\(^{13}\)

Compared to the US Customs and Border Protection (CBP) database, the Piers database is not restricted and can be accessed by anyone at a fee. Moreover, the fact that all firms in Piers use the same export mode (by sea) favors comparability. Finally, while the Piers data are slightly less detailed than the CBP data in terms of product classification (6-digit vs. 10-digit), they contain the full name of each firm. This unique feature allows researchers using Piers to precisely identify firms, reducing the risk of over-counting them. We use a string matching algorithm to match and aggregate firms that appear in the data more than once with similar names. The algorithm first homogenizes standard expressions (e.g., it converts the extensions "Lim." and "LTD" in "Limited") and then exploits the Levenshtein edit distance to match firms.\(^{14}\) With the cleaned firm names at hand, we assign varieties to industries by mapping each HS6 product exported by a firm to a 4-digit SIC industry, using a correspondence table developed by the World Integrated Trade Solutions. We perform some further standard data cleaning to mitigate the risk of including transactions contaminated by reporting mistakes. In particular, we drop observations corresponding to firms that, in a given industry and year, have total exports to the US below $1,000. We also exclude observations corresponding to firms that, in a given industry and year, have unit values for their

\(^{13}\)In the case of firms with multiple shipments (bills of lading) of the same product in a year, we purchased from IHS Markit information on the total value and quantity of these shipments across all bills of lading, but not the detailed information on each bill of lading, which would have been prohibitively expensive.

\(^{14}\)In more detail, the algorithm computes the Levenshtein edit distance between all pairwise combinations of firm names sharing the same first character. The distance is then normalized by the length of the longest string and a match is formed if the normalized edit distance is below a 5% threshold.
Source: Piers (IHS Markit), US import data for 2002 and 2012. Darker colors indicate a higher number of manufacturing firms exporting to the US (map a) or a higher ratio between the value of total manufacturing exports to the US obtained from Piers and the value obtained from customs data (map b). All figures are averages between 2002 and 2012.

Figure 1: Data Coverage
products above the top or below the bottom 0.01% of the unit value distribution for that year. Finally, we exclude country-industry-year triplets with less than two varieties exported to the US, as the variance of sales is not defined for these triplets.

Our final data set comprises 1,350,574 observations at the firm-product-year level. Firms belong to 366 manufacturing industries and 104 origin countries spanning the five continents. Figure 1a shows that the number of firms exporting to the US is particularly high in neighboring countries (Canada and Mexico), in large Latin American economies such as Brazil, in Europe and in South-East Asia (especially China). Figure 1b describes the coverage of Piers in terms of export value rather than number of firms. Darker colors indicate a better coverage, as measured by the ratio between the value of total exports computed from Piers and the same value computed from customs data (Feenstra, Romalis and Schott, 2002). Although Piers registers waterborne transactions only, its coverage is remarkably good, exceeding 80% of the total exports to the US for the average country. Not surprisingly, the coverage of Piers is less extensive for Canada and Mexico, two countries for which maritime trade is not the main mode of export to the United States. Nevertheless, these countries have a large number of firms exporting to the US, as shown in Figure 1a. Because our decompositions are valid for any subset of firms and sales, we therefore keep Canada and Mexico in our main baseline sample. In Section 5.2, we find that excluding all countries for which the coverage of Piers is not very extensive (i.e., the first group of countries in Figure 1b) leaves the results essentially unchanged.

Table 1 provides further details on sample coverage and composition. Panel a) confirms the high coverage of Piers, showing that for the average (median) country in our sample, Piers accounts for 83% (77%) of its total exports to the US. These numbers are similar to the figures reported by Feenstra and Weinstein (2017) for an earlier and more limited vintage of the Piers database. Panel b) provides instead details on sample composition. All variables in this panel are computed separately for each country-industry-year triplet, and the reported statistics are calculated across all triplets in the data set. The average triplet has 44 firms and 55 varieties, a value of total exports to the US exceeding $60 million and average exports per variety slightly above $1 million.

15 We have compared the number of foreign firms exporting to the US in our sample with the corresponding number in the World Bank Exporter Dynamics Database (EDD), which uses information for the universe of export transactions obtained from each country’s government custom agency. Since our sample excludes firms selling less than $1,000 to the US, we have used the EDD statistics computed for firms with total exports above $1,000. In 2012, 34 out of the 48 countries covered by the EDD were also part of our sample. For the average or median country, the coverage rate of our sample was equal to 63% of the number of exporting firms registered in the EDD. Kamal, Krizan and Monarch (2015) perform the same exercise for the CBP database, finding that it overshoots the number of foreign firms exporting to the US for most countries, with an overcounting rate of 25% on average. While some of the firms in Piers could be trading companies, we do not find any such company among the top-10 exporting firms in any 2-digit industry, suggesting that the majority of firms in our sample are actual exporters. We have also compared the information on unit values contained in Piers with the unit values for maritime trade obtained from customs data at the product level (Feenstra, Romalis and Schott, 2002). Regressing the unit values in the custom data on the unit values in Piers, across exporting countries and 6-digit products in 2002 and 2012, yields a coefficient of 0.836 (s.e. 0.003) and an $R^2$ of 0.58.
Table 1: Descriptive Statistics on Sample Coverage and Composition

<table>
<thead>
<tr>
<th>a) Sample coverage</th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of PIERS exports in total exports to the US (based on customs data)</td>
<td>0.83</td>
<td>0.77</td>
<td>0.55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>b) Sample composition</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>N. of firms</td>
<td>44</td>
<td>8</td>
<td>249</td>
</tr>
<tr>
<td>N. of firm-product pairs (varieties)</td>
<td>55</td>
<td>9</td>
<td>316</td>
</tr>
<tr>
<td>Total exports ($1000)</td>
<td>60347</td>
<td>2360</td>
<td>536000</td>
</tr>
<tr>
<td>Average exports per variety ($1000)</td>
<td>1273</td>
<td>230</td>
<td>11058</td>
</tr>
</tbody>
</table>

Notes: The variable in panel a) is computed for each country in the years 2002 and 2012. Reported statistics are the mean, median and standard deviation of this variable across all countries and years. The variables in panel b) are computed for each country-industry-year triplet. Reported statistics are the mean, median and standard deviation of these variables across all triplets.

4 Structural Estimation

To implement the decompositions in Section 2.1.2, we need to estimate the quality of each variety and the elasticity of substitution between varieties in any industry. These estimates can be obtained using data on prices and market shares, together with the structural equations of the model. We now discuss how. The first step is the estimation of the elasticity of substitution, $\sigma_i$. We use alternative empirical strategies to identify this crucial parameter.

First, we exploit the time variation in the data and use the reverse-weighting (RW) estimator of Redding and Weinstein (2017). As detailed in Appendix A, the idea behind this estimator is to search for the value of $\sigma_i$ that minimizes the sum of squared deviations of the forward and backward differences of the price index, which measure the changes in the cost of living using initial period (2002) and final period (2012) expenditure shares as weights, from a money-metric price index, which depends solely on prices and expenditure shares and is independent of demand parameters. The identifying assumption is that changes in $\gamma$ over time average out. This assumption does not seem very restrictive if $\gamma$ is interpreted as a demand shock. In that case, as Redding and Weinstein (2017) emphasize, it amounts to requiring preferences to be stable over time. When $\gamma$ is interpreted as quality, however, this assumption seems more restrictive. Moreover, the RW estimator identifies $\sigma_i$ out of time variation in market shares. Redding and Weinstein (2017) show that, unless $\gamma$ is small for each variety, the RW estimator requires a large number of common goods to provide consistent estimates of $\sigma_i$. In our dataset, we only have two time observations, and in some industries the number of firm-products that are present in both years is limited. While our data offer sufficient variation to identify the elasticity of substitution in most industries, we recognize the potential limitations of this strategy.

Second, we can use the demand-side restrictions to identify the elasticity of substitution from the dispersion of sales. To this end, note that, from $r_{doi}(\omega) = \left[\tilde{\gamma}_{doi}(\omega)\right]^{\sigma_i-1} P_{di}^\sigma C_{di}$, we can write:

$$\nabla(\ln r_{doi}) = (\sigma_i - 1)^2 \nabla(\ln \tilde{\gamma}_{doi}).$$

13
Taking logs and adding time subscripts yields

\[ \ln V(\ln r_{doi,t}) = 2 \ln \left( \sigma_i - 1 \right) + \ln V(\ln \tilde{\gamma}_{doi,t}), \]  

(12)

which shows that \( \sigma_i \) can be retrieved after regressing sales dispersion per country-industry-year on industry fixed effects as follows:

\[ \sigma_i = \exp \left( \frac{\alpha_i}{2} \right) + 1. \]

Intuitively, a higher substitutability generates more dispersion in sales for a given distribution of attributes. The limitation of this strategy is that an industry fixed effect identifies any common component of sales dispersion across countries in a given industry, and not just the demand parameter we are interested in. On the other hand, the advantage is that purging sales dispersion of any common component across countries allows us to isolate the cross-country variation in attributes. Hence, it is a way to study heterogeneity in attributes relative to other countries, rather than its absolute level.

A difficulty in estimating equation (12) is that the second term is not observed. One solution is to treat it as a residual, i.e., to leave it in the error term. Another solution is to control for the second term in (12) using variables that can be observed. What to use for the purpose comes from the model and the literature. Recall that \( \tilde{\gamma} \) is the quality-to-price ratio. While it cannot be observed directly, we can proxy for it using the variance of log prices. While prices are just one component of \( \tilde{\gamma} \), controlling for them would be sufficient if there is a one-to-one mapping between quality and prices, as in several models of endogenous quality. This choice is also supported by the evidence that prices are indeed a good proxy for quality (see Hottman, Redding and Weinstein, 2016, and Johnson, 2012). Nevertheless, we can also include some additional variables. Since the variance of sales may vary systematically with the number of observations over which it is computed (Bonfiglioli, Crinò and Gancia, 2018a,b), we also control for the number of products per country-industry-year triplet. Finally, we also include country-time dummies, so that the industry fixed effects are identified from deviations of sales dispersion from its country-year means, and are not contaminated by time-varying country characteristics that could affect sales dispersion uniformly across industries. These characteristics would bias the estimates of \( \sigma_i \) if they systematically induced countries to specialize in high- or low-dispersion industries.

As a final robustness check, we will also perform our decompositions using the elasticities of substitution estimated in Broda and Weinstein (2006). To sum up, we will work with four alternative measures of \( \sigma_i \), three of them estimated using our micro data and the model structural equations, and the fourth one borrowed from an external study. Henceforth, we will use the following notation

---

16For each industry in our sample, we use the median value of the Broda and Weinstein (2006) elasticities across all 10-digit products associated with that industry.
Table 2: Descriptive Statistics on the Elasticity of Substitution

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
<th>N. of Industries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression, baseline (reg. base.)</td>
<td>2.44</td>
<td>2.48</td>
<td>0.26</td>
<td>366</td>
</tr>
<tr>
<td>Regression, controls (reg. contr.)</td>
<td>4.22</td>
<td>4.22</td>
<td>0.46</td>
<td>366</td>
</tr>
<tr>
<td>Reverse weighting (RW)</td>
<td>3.71</td>
<td>3.30</td>
<td>1.74</td>
<td>232</td>
</tr>
<tr>
<td>Broda-Weinstein (BW)</td>
<td>3.12</td>
<td>2.74</td>
<td>1.37</td>
<td>342</td>
</tr>
</tbody>
</table>

Notes. The statistics are computed across all industries with available information on a given elasticity of substitution. Elasticites smaller than 1 or greater than 10 are excluded.

to label the four elasticities: reg. base. will denote the estimate obtained from the baseline regression in (12); reg. contr. the estimate obtained from (12) after adding controls; RW the reverse-weighting estimate; and BW the Broda and Weinstein (2006) estimate. Table 2 provides descriptive statistics on the estimated $\sigma_i$. For the median industry, our estimates range from $2.5$ (reg. base.) to $4.2$ (reg. contr.), with RW falling in between (3.3); reassuringly, our results are close to the BW estimate (2.7), obtained using a different estimation approach and aggregate product-level US import data for earlier years.

With the estimates of $\sigma_i$ at hand, we can infer quality from variation in market shares conditional on prices. As in Khandelwal, Schott, and Wei (2013), we start by rewriting the expression for revenue as follow:

$$\ln r_{d,i,t}(\phi) + (\sigma_i - 1) \ln p_{d,i,t}(\phi) = \alpha_t \ln P_{d,i,t} + \ln C_{d,i,t} + (\sigma_i - 1) \ln \gamma_{d,i,t}. \quad (13)$$

Then we regress, separately for each industry, the left-hand side variable of (13) on time dummies, and obtain log quality, $\ln \gamma_{d,i,t}$, by dividing the residuals from these regressions by $\sigma_i - 1$.\footnote{The time dummies absorb the terms $P_{d,i,t}$ and $C_{d,i,t}$, which vary over time but are constant across varieties within an industry. Running a separate regression for each industry raises comparability across varieties, relative to the alternative approach of running a pooled regression with industry-time fixed effects.}

The resulting quality estimates vary across varieties and over time.

Using the estimated qualities, we compute $\nabla(\ln \gamma_{d,i,t})$ and use it as a measure of quality dispersion in each country-industry-year triplet. Similarly, by (8), we use $\nabla(- \ln p_{d,i,t})$ as a measure of dispersion in efficiency. Finally, with $\nabla(\ln \gamma_{d,i,t})$, $\nabla(- \ln p_{d,i,t})$ and $\sigma_i$, we use (7) to compute

$$\text{Cov}(\ln \gamma_{d,i,t} - \ln p_{d,i,t}) = \frac{1}{2} \left[ \frac{\nabla(\ln s_{d,i,t})}{(\sigma_i - 1)^2} - \nabla(\ln \gamma_{d,i,t}) - \nabla(- \ln p_{d,i,t}) \right].$$

Before proceeding, we stress once more an important remark. If efficiency is not the only driver of price dispersion in a country-industry-year cell, our decomposition is still entirely correct. The only difference is that it should be interpreted as an assessment of the relative importance of quality
Table 3: Descriptive Statistics on Key Moments

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Var. log sales</td>
<td>3.69</td>
<td>3.11</td>
<td>0.10</td>
</tr>
<tr>
<td>Var. log efficiency</td>
<td>0.41</td>
<td>0.68</td>
<td>-0.01</td>
</tr>
<tr>
<td>Var. log quality (sub. ela.: reg. base.)</td>
<td>2.46</td>
<td>2.46</td>
<td>0.06</td>
</tr>
<tr>
<td>Var. log quality (sub. ela.: reg. contr.)</td>
<td>0.96</td>
<td>1.19</td>
<td>0.03</td>
</tr>
<tr>
<td>Var. log quality (sub. ela.: RW)</td>
<td>3.08</td>
<td>10.16</td>
<td>0.11</td>
</tr>
<tr>
<td>Var. log quality (sub. ela.: BW)</td>
<td>3.24</td>
<td>7.38</td>
<td>0.03</td>
</tr>
<tr>
<td>Var. log qual.-to-price ratio (reg. base.)</td>
<td>1.66</td>
<td>1.48</td>
<td>0.11</td>
</tr>
<tr>
<td>Var. log qual.-to-price ratio (reg. contr.)</td>
<td>0.36</td>
<td>0.30</td>
<td>0.10</td>
</tr>
<tr>
<td>Var. log qual.-to-price ratio (RW)</td>
<td>2.38</td>
<td>9.71</td>
<td>0.18</td>
</tr>
<tr>
<td>Var. log qual.-to-price ratio (BW)</td>
<td>2.41</td>
<td>6.72</td>
<td>0.04</td>
</tr>
<tr>
<td>Cov. log eff.-log. quality (reg. base.)</td>
<td>-0.61</td>
<td>1.11</td>
<td>-0.01</td>
</tr>
<tr>
<td>Cov. log eff.-log. quality (reg. contr.)</td>
<td>-0.50</td>
<td>0.86</td>
<td>-0.01</td>
</tr>
<tr>
<td>Cov. log eff.-log. quality (RW)</td>
<td>-0.54</td>
<td>1.01</td>
<td>-0.07</td>
</tr>
<tr>
<td>Cov. log eff.-log. quality (BW)</td>
<td>-0.62</td>
<td>1.20</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Notes. All variables are computed separately for each country-industry-year triplet. The first two columns report the mean and standard deviation of each variable across all countries and industries in the year 2012. The third column reports the percentage change in the average value of each variable between 2002 and 2012.

versus prices in explaining sales.

5 Decomposing US Imports

Having estimated elasticities of substitutions at the industry level and computed firm attributes that rationalize observed sales, we now study the role of firms in shaping trade flows. We start by presenting some new stylized facts about how sales and firm attributes vary across industries and countries. In Table 3, we report summary statistics on a number of important moments. The first two columns show the mean and standard deviation of each variable across all country-industry pairs in 2012; the third column shows the change in the average value of each variable between 2002 and 2012. Sales dispersion is high, varies markedly across countries and industries, and has increased by 10% over the sample period.\(^{18}\) Given that we know the identity of firms, with our data we can also compute the change in sales dispersion driven by reallocations among firms active in both years. We find that, in the subsample of continuing varieties, sales dispersion has increased by 29%.\(^{19}\) In the rest of the sample, sales dispersion has increased by approximately 8%.

Quality dispersion shows similar patterns. Instead, efficiency (or, more generally, price) disper-

\(^{18}\)These results are in line, both qualitatively and quantitatively, with evidence based on US firm-level sales data and cross-country product-level export data (Bonfiglioli, Crinò and Gancia, 2018a,b).

\(^{19}\)Continuing varieties account for 28% of total exports to the US in the average country-industry pair in 2012. To save space, statistics on the subsample of continuing firms are not reported in Table 3.
Notes. The variance of log sales is computed separately for each country-industry-year triplet. The graphs plot the simple average of this variable across all industries and years for each country. Real per-capita GDP is the simple average of this variable between the years 2002 and 2012. Average exports to the US is the simple average of this variable across all industries and years for each country.

Figure 2: Sales Dispersion and Country Characteristics

... is relatively small, exhibits a low cross-sectional variation, and has remained stable over time. Interestingly, the variance of log quality is generally close to the variance of log sales on average, suggesting that quality dispersion may be a key determinant of sales dispersion. Consistent with these patterns, the table also documents a substantial dispersion in quality-to-price ratios, $\gamma/p$, as well as a tendency for it to increase over time. Finally, the correlation between quality and efficiency is negative, suggesting that higher-quality products are more costly to produce and more expensive. The covariance has also become stronger over time.

In Appendix B, we also perform two variance decomposition exercises, aimed at studying the sources of variation in our main variables. In the first exercise, we focus on one origin country at a time, and decompose the variance of log sales, quality, and efficiency for this country into within-industry and between-industry contributions. In the second exercise, we focus on one industry at a time, and decompose the variance of log sales, quality, and efficiency for this industry into within-country and between-country contributions. The results show that the within-industry component explains 71% of sales dispersion, and approximately two-thirds of quality dispersion,
in the representative country. Cross-country heterogeneity explains 13% of sales dispersion and 25-35% of quality dispersion in the representative industry. The existence of significant differences in the dispersion of sales and firm attributes across industries is not particularly surprising. After all, whether firms are more or less heterogeneous is likely to depend on technological characteristics that may well vary across industries. The existence of significant differences in the dispersion of firm characteristics across countries is instead more interesting, especially given our aim of comparing firms from different origins and understanding how they shape aggregate outcomes.

To have a first sense of how firm heterogeneity varies across countries and correlates with economic performance, Figure 2 shows how sales dispersion at the country level correlates with real per-capita GDP and with average exports to the US. To draw the figure, we first compute the variance of log sales separately for each triplet. Then, to neutralize compositional effects due to differences in the industrial structure of production, we take for each country the simple average of sales dispersion across all industries and years. The first graph shows that sales are significantly more dispersed in richer countries. The second graph shows that sales dispersion has a strong positive correlation with average exports to the US, computed as the mean value across all industries and years for each country. Having described the main features of the data, we now proceed with an exact decomposition of firms’ sales into the US market, which allows us to quantifying the importance of firms’ attributes, and especially their dispersion, for economic success. We will then explore more in depth the origin and consequences of firm heterogeneity.

5.1 Decomposing Sales: Firms, Attributes and Heterogeneity

We now implement the decompositions presented in Section 2.1.2. We start by decomposing countries’ market shares into the contribution of the extensive and intensive margins. To this purpose, we take logs of (5) and run separate regressions of \(\ln N_{doi,t} - \ln N_{di,t}\) and \(\ln \tilde{r}_{doi,t} - \ln \tilde{r}_{di,t}\) on \(\ln S_{doi,t}\) across all available triplets. The properties of OLS imply that the coefficients on \(\ln S_{doi,t}\) from these regressions add up to one and thus provide the percentage contribution of each margin to explaining variation in countries’ market shares. We similarly decompose the intensive margin into the contribution of average attributes and heterogeneity in attributes, by regressing each term in the right-hand side of (6) on \(\tilde{r}_{doi,t}\).

The results of these decompositions are reported in Table 4. Panel a) reports the contribution of the extensive and intensive margins to explaining countries’ market shares. Each column uses the sample of triplets for which the elasticity of substitution indicated in the column’s heading is non missing. The results indicate that each margin explains roughly half of the variation in countries’ market shares. Hence, this first decomposition implies that countries selling a larger number of varieties, and more of each of variety, to the US exhibit larger market shares in a given industry and year, and that the contribution of the two margins is roughly equivalent in our data.
Table 4: Decomposition of Countries’ Market Shares

<table>
<thead>
<tr>
<th></th>
<th>reg. base.</th>
<th>reg. contr.</th>
<th>RW’</th>
<th>BW’</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>a) First step - Decomposition of market shares</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N. of varieties</td>
<td>0.502***</td>
<td>0.502***</td>
<td>0.499***</td>
<td>0.505***</td>
</tr>
<tr>
<td></td>
<td>[0.003]</td>
<td>[0.003]</td>
<td>[0.003]</td>
<td>[0.003]</td>
</tr>
<tr>
<td>Average revenue per variety</td>
<td>0.498***</td>
<td>0.498***</td>
<td>0.501***</td>
<td>0.495***</td>
</tr>
<tr>
<td></td>
<td>[0.003]</td>
<td>[0.003]</td>
<td>[0.003]</td>
<td>[0.003]</td>
</tr>
<tr>
<td>b) Second step - Decomposition of average revenue per variety</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average quality-to-price ratio</td>
<td>0.487***</td>
<td>0.480***</td>
<td>0.481***</td>
<td>0.492***</td>
</tr>
<tr>
<td></td>
<td>[0.075]</td>
<td>[0.106]</td>
<td>[0.114]</td>
<td>[0.118]</td>
</tr>
<tr>
<td>Heterogeneity in quality-to-price ratios</td>
<td>0.513***</td>
<td>0.520***</td>
<td>0.519***</td>
<td>0.508***</td>
</tr>
<tr>
<td></td>
<td>[0.075]</td>
<td>[0.106]</td>
<td>[0.114]</td>
<td>[0.118]</td>
</tr>
<tr>
<td>Obs.</td>
<td>24754</td>
<td>24754</td>
<td>17660</td>
<td>23622</td>
</tr>
</tbody>
</table>

Notes. Panel a) performs the decomposition in eq. (5) and panel b) the decomposition in eq. (6). Each coefficient in panel a) is obtained from a separate regression, run across triplets, of the corresponding margin (in logs) on the log of countries’ market shares. Each coefficient in panel b) is obtained from a separate regression, run across triplets, of the corresponding margin on normalized average revenue per variety. The standard errors are corrected for heteroskedasticity. ***, **, *: indicate significance at the 1, 5 and 10% level, respectively.

Panel b) decomposes the intensive margin (average sales per variety) into the contribution of average attributes and heterogeneity in attributes. In this case, differences across columns arise not only because of the different sample used, but also because the estimate of the elasticity of substitution influences the computation of each margin. Nevertheless, the estimated contributions are remarkably similar across columns, suggesting that the results are largely insensitive to the measure of \( \sigma_i \). Interestingly, the results show that heterogeneity in attributes contributes at least as much as average attributes to explaining variation in average revenue per variety. This suggests that firm heterogeneity is an important factor for understanding countries’ economic performance.

We now turn to the next step of the decomposition, which consists of decomposing the variance of log sales across varieties within triplets into the contributions of quality and efficiency. Following (7) we compute, separately for each triplet, the contributions of quality and efficiency as

\[
\frac{(\sigma_i - 1)^2 \left[ \nabla (\ln \gamma_{d, t}) + \text{Cov}(\ln \gamma_{d, t}, -\ln p_{d, t}) \right]}{\nabla (\ln s_{d, t})}
\]

and

\[
\frac{(\sigma_i - 1)^2 \left[ \nabla (-\ln p_{d, t}) + \text{Cov}(\ln \gamma_{d, t}, -\ln p_{d, t}) \right]}{\nabla (\ln s_{d, t})}
\]

respectively, and then average each contribution across all triplets.\(^20\) The results are reported in

\(^20\) By apportioning the covariance equally between the two components of \( \nabla \ln s_{d, t} \), this approach is equivalent to a regression-based method like the one used in Table 4, whereby the average contribution of quality and efficiency would be obtained by regressing \((\sigma_i - 1) \ln \gamma_{d, t}\) and \((\sigma_i - 1) \ln p_{d, t} (\varphi \gamma)\) on \( \ln r_{d, t} (\varphi \gamma) \) separately for each triplet,
Table 5: Decomposition of Sales Dispersion across Varieties

<table>
<thead>
<tr>
<th></th>
<th>reg. base.</th>
<th>reg. contr.</th>
<th>RW</th>
<th>BW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contribution of quality (average, %)</td>
<td>1.06</td>
<td>1.13</td>
<td>0.75</td>
<td>1.10</td>
</tr>
<tr>
<td>Contribution of efficiency (average, %)</td>
<td>-0.06</td>
<td>-0.13</td>
<td>0.25</td>
<td>-0.10</td>
</tr>
</tbody>
</table>

Notes: The table performs the decomposition in eq. (7). The contribution of quality is defined as the ratio between the variance of log quality, plus the covariance of log quality and log efficiency, times the square of the elasticity of substitution minus 1, over the variance of log sales. The contribution of efficiency is defined analogously. Reported figures are averages across all country-industry-year triplets.

Table 5. Strikingly, quality dispersion accounts for the lion’s share of sales dispersion, its contribution ranging from 75% to more than 100% across different estimates of $\sigma_i$. The contribution of efficiency is accordingly much smaller.

At this stage, we pause to briefly discuss the relationship between these results and the existing literature. That the extensive margin explains about half of the variation in trade flows is consistent with the findings in Hummels and Klenow (2005) using product-level data, Fernandes et al. (2017) using firm-level data for 50 countries, and Redding and Weinstein (2017), using US Census import data. The fact that firm appeal depends mostly on quality is consistent with the findings in Hottman, Redding and Weinstein (2016), using highly disaggregated barcode data on US household purchases. The contribution of firm heterogeneity in affecting the volume of trade has received less attention. The notable exception is Redding and Weinstein (2018), who develop a similar decomposition of US imports and find that the dispersion of firm attributes accounts for 36% of the variation in measures of revealed comparative advantage. However, their log-linear decomposition holds constant the mean of the log of firm attributes, which is negatively affected by the dispersion in the level of attributes. Our aim at separating the effect of the mean and dispersion in the level of attributes motivated our alternative (still exact) decomposition. Holding constant the average level, we find that the contribution of dispersion, while still important, is significantly reduced to around 25%.

5.2 Decomposing Sales: Superstars and Granularity

One reason for the importance of firm heterogeneity in explaining sales is the presence of exceptional firms in each country. It is known that top firms can define the export performance of a sector. As long as these "superstar firms" are exceptional, i.e., they have significantly better attributes compared to the remaining firm in a sector, their presence would be associated to both high sales and high dispersion. As in previous studies, the top firm in each country plays a dominant role and then averaging the corresponding coefficients across all triplets.
also in our sample, accounting for 25% of total exports to the US, on average. But are these firms really "exceptional"? Answering this question poses a difficulty. Without taking a stand on the distribution of attributes, it is hard to say whether superstar firms are really outliers or not. Similarly, it is difficult to separate the role of heterogeneity, i.e., smooth variation in attributes across a large number of firms, from that of granularity, i.e., exceptional performance in a small sample of firms. Yet, distinguishing them is a crucial challenge in both the fields of trade and macroeconomics. Although models of firm heterogeneity are by now a standard tool, only recently have economists started to explore the effects of “granular” firms. As of now, however, there are still very few attempts at quantifying the importance of these firms. We now show that, by imposing more structure on the data, we can implement a novel decomposition that disentangles the role of granularity from that of heterogeneity.

Our approach is based on two premises. First, following Gaubert and Itskhoki (2018), we define "granularity" as exceptional deviations from a benchmark continuous distribution. This specific definition is meant to capture the failure of the law of large numbers inherent to a granular world. Second, we need to choose an appropriate benchmark distribution. To do so, we follow a large set of papers showing that firm sales are well approximated by log-normal distributions (see, for instance, Cabral and Mata, 2003, Head, Mayer and Thoenig, 2014, Bas, Mayer and Thoenig, 2017), an observation that will be confirmed in our data. Hence, we now assume that the quality-to-price ratio, \( \tilde{\gamma} \), is log-normally distributed. Importantly, however, to be consistent with the heterogeneity in the data just documented, we allow \( \tilde{\gamma} \) in each country-industry-year to be drawn from log-normal distributions with possibly different parameters. One difficulty of such a flexible approach is that it implies that, in general, the overall distribution of imports will no longer be log-normal, though it may still be approximately so. Despite this, we can use the properties of these distributions to obtain a formula that decomposes market shares across country pairs, in any given industry, into the contribution of the characteristics of their populations of firms.

To see this, we start from (4) and take the log of the market share in industry \( i \) captured by country \( o \) relative to another country \( x \):

\[
\ln \frac{S_{doi}}{S_{dxi}} = \ln \sum_{\omega \in \Omega_{doi}} \tilde{\gamma}_{doi}(\omega)\sigma_i^{-1} - \ln \sum_{\omega \in \Omega_{dxi}} \tilde{\gamma}_{dxi}(\omega)\sigma_i^{-1}.
\]

Next, we add and subtract \((\ln N_{doi} - \ln N_{dxi})\):

\[
\ln \frac{S_{doi}}{S_{dxi}} = \ln \mathbb{E}\left(\tilde{\gamma}_{doi}^{\sigma_i^{-1}}\right) - \ln \mathbb{E}\left(\tilde{\gamma}_{dxi}^{\sigma_i^{-1}}\right) + \ln N_{doi} - \ln N_{dxi}.
\]

These findings are consistent with results obtained by Freund and Pierola (2015) for a sample of developing countries.
Using the properties of the log-normal distribution, this equation can be rewritten as:\(^{22}\)

\[
\ln \frac{S_{doi}}{S_{dxi}} = (\sigma_i - 1) [E(\ln \tilde{\gamma}_{doi}) - E(\ln \tilde{\gamma}_{dxi})] + \frac{(\sigma_i - 1)^2}{2} [V(\ln \tilde{\gamma}_{doi}) - V(\ln \tilde{\gamma}_{dxi})] + [\ln N_{doi} - \ln N_{dxi}].
\]

(14)

This formula is a special case of our exact decompositions. Once again, it shows that country \(o\) captures a higher market share than country \(x\) if it has better, more heterogeneous and more numerous exporting firms. Compared to (6), (14) illustrates that, in the case of log-normal distributions, the theory-based measure of heterogeneity is the variance of the log of attributes. Moreover, applied to country pairs, (14) uses the information in the data more efficiently.

Another advantage of (14) is that it provides a decomposition that is independent of the estimate of \(\sigma_i\). This surprising result depends on our empirical strategy for identifying quality and CES demand. To see this, using the equation for the estimation of quality, (13), to substitute the expected values and using \(V(\ln \tilde{\gamma}_{doi}) = V(\ln r_{doi}) / (\sigma_i - 1)^2\), from demand functions, to substitute the variances, yields:

\[
\ln \frac{S_{doi}}{S_{dxi}} = [E(\ln r_{doi}) - E(\ln r_{dxi})] + \frac{[V(\ln r_{doi}) - V(\ln r_{dxi})]}{2} + [\ln N_{doi} - \ln N_{dxi}].
\]

(15)

Comparing (14) to (15) simply illustrates the mapping from sales to the unobservable firm attributes generating them. It also makes clear that a log-normal distribution of \(\tilde{\gamma}\) generates log-normally distributed sales. Remarkably, to take (15) to the data, one can use readily observable variables only.

The advantages of this new decomposition come however at a cost, namely, that it does not isolate the effect of heterogeneity in attributes across firms. There are two reasons for this. Since the logarithm is a concave function, by Jensen’s inequality dispersion in sales also affects the mean of log sales, that is, the first term of the decomposition. Second, we know that dispersion in attributes generates dispersion in sales only when \(\sigma_i \neq 2\).\(^{23}\) Yet, for the current purpose, these are not concerns, because we have already examined an exact decomposition, (6), that does not suffer from these problems. Nevertheless, the decomposition in (15) is still informative of the importance of dispersion in sales across countries and sectors.

So, does the log-normal distribution provide a good description of the data? Standard tests applied to our sample reject log normality when the number of observation is sufficiently high. This is of no surprise. In fact, we are interested precisely in studying the deviations from log normality

\(^{22}\)Recall that, if \(x \sim \text{log Normal}\), then \(\ln \mathbb{E}(x^n) = n \mathbb{E}(\ln x) + \frac{n^2 \text{var}(\ln x)}{2}\).

\(^{23}\)To see this, use the properties of the log-normal distribution to substitute \(\mathbb{E}(\ln \tilde{\gamma}) = \mathbb{E}(\tilde{\gamma}) - \mathbb{V}(\ln \tilde{\gamma}) / 2\) into (14):

\[
\ln \frac{S_{doi}}{S_{dxi}} = (\sigma_i - 1) [E(\ln \tilde{\gamma}_{doi}) - E(\ln \tilde{\gamma}_{dxi})] + \frac{(\sigma_i - 1)(\sigma_i - 2)}{2} [V(\ln \tilde{\gamma}_{doi}) - V(\ln \tilde{\gamma}_{dxi})] + [\ln N_{doi} - \ln N_{dxi}].
\]

This formula makes it clear that market shares are increasing in the variance of firm attributes only when \(\sigma_i > 2\).
that are needed to match the data. To this end, we compute the residual, \( g_{dxi,t} \), from the exact equation:

\[
\ln \frac{S_{dxi,t}}{S_{doi,t}} = [\mathbb{E}(\ln r_{dxi,t}) - \mathbb{E}(\ln r_{doi,t})] + \frac{[\mathbb{V}(\ln r_{dxi,t}) - \mathbb{V}(\ln r_{doi,t})]}{2} + [\ln N_{dxi,t} - \ln N_{doi,t}] + g_{dxi,t},
\]

where we have added the time subscript \( t \) referring to the years 2002 and 2012. Our strategy is to study the importance of \( g_{dxi,t} \) as a way to assess the importance of outliers reflecting the granularity of the data. To see why, imagine an hypothetical sample in which (i) sales are log-normal and (ii) the Law of Large Numbers (LLN) applies. In this case, the residual \( g_{dxi,t} \) would be zero. If we maintain the assumption of log normality, but relax the LLN, the residual \( g_{dxi,t} \) would capture entirely the effect of outliers in small samples. Hence, \( g_{dxi,t} \) can be interpreted as a measure of granularity relative to a world of continuum log-normal distributions. While this interpretation is legitimate, in that granularity is hard to measure without a reference distribution, we will nevertheless conduct some diagnostic tests suggesting that the residual \( g_{dxi,t} \) is indeed driven by exceptional firms.

To assess what fraction of the variation in \( \ln(S_{doi,t}/S_{dxi,t}) \) is explained by each component of (16), we regress each term in the right-hand side on \( \ln(S_{doi,t}/S_{dxi,t}) \). The results are reported in Table 6. Columns (1)-(3) broadly confirm, using a different decomposition, the findings in Section 5.1: the number of firms accounts for about half of the total variation in market shares, while average and variance play a comparable role. The novelty is column (4), showing that the residual, \( g_{dxi,t} \), explains less than 5% of the overall variation in market shares. These results are consistent across a number of robustness checks, presented in the remaining panels of Table 6. In panel b), we exclude countries for which Piers covers less than 45% of total exports to the US (i.e., the first group of countries in Figure 1b). The coefficients are essentially unchanged, suggesting that our results are not driven by countries for which the coverage of Piers is not very extensive. In panels c) and d), we instead exclude small and large market shares, respectively. The former (latter) are market shares falling below the 5th (above the 95th) percentile of the distribution of market shares in each industry and year. These exercises show that our results are not driven by either small or large exporters. Finally, in panel e), we exclude industries for which the share of imports of parts and components in total US imports is above 25%.\(^{24}\) The results are virtually unchanged, suggesting that our evidence is not driven by industries that are particularly involved in global value chains.

We now perform a test aimed at shedding light on whether the contribution of the residual, \( g_{dxi,t} \), captures the role of exceptional firms. To this end, we propose various criteria to identify superstar firms in our sample. Then, we compute \( g_{dxi,t} \) from (16) on a restricted sample that excludes the superstar firms and implement our usual decomposition. If the importance of the residual falls significantly after the exclusion of these firms, it suggests that the fit of the log-normal

\(^{24}\)We use data on imports of parts and components from Schott (2004) for the pre-sample period 1972-2001.
Table 6: Decomposition of Countries’ Market Shares under Log Normality

<table>
<thead>
<tr>
<th></th>
<th>Difference in av. log sales</th>
<th>Difference in var. of log sales</th>
<th>Difference in log n. of varieties</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>a) Baseline</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log relative market share</td>
<td>0.236***</td>
<td>0.228***</td>
<td>0.487***</td>
<td>0.048***</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.001]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>Obs.</td>
<td>1078915</td>
<td>1078915</td>
<td>1078915</td>
<td>1078915</td>
</tr>
<tr>
<td>b) Excluding small countries</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log relative market share</td>
<td>0.231***</td>
<td>0.214***</td>
<td>0.498***</td>
<td>0.057***</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.001]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>Obs.</td>
<td>793086</td>
<td>793086</td>
<td>793086</td>
<td>793086</td>
</tr>
<tr>
<td>c) Excluding small market shares</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log relative market share</td>
<td>0.191***</td>
<td>0.227***</td>
<td>0.518***</td>
<td>0.064***</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.001]</td>
<td>[0.000]</td>
<td>[0.001]</td>
</tr>
<tr>
<td>Obs.</td>
<td>969795</td>
<td>969795</td>
<td>969795</td>
<td>969795</td>
</tr>
<tr>
<td>d) Excluding large market shares</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log relative market share</td>
<td>0.262***</td>
<td>0.236***</td>
<td>0.466***</td>
<td>0.036***</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.001]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td>Obs.</td>
<td>972707</td>
<td>972707</td>
<td>972707</td>
<td>972707</td>
</tr>
<tr>
<td>e) Excluding industries with high shares of imported inputs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log relative market share</td>
<td>0.241***</td>
<td>0.233***</td>
<td>0.482***</td>
<td>0.044***</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.001]</td>
<td>[0.000]</td>
<td>[0.001]</td>
</tr>
<tr>
<td>Obs.</td>
<td>863789</td>
<td>863789</td>
<td>863789</td>
<td>863789</td>
</tr>
</tbody>
</table>

Notes. The table performs the decomposition in eq. (16). Each coefficient is obtained from a different regression. The dependent variables are indicated in the columns’ headings and are: the difference in average log sales between country o and country x in each industry and year (column 1); the difference in the variance of log sales, times one half, between country o and country x in each industry and year (column 2); the difference in the log number of varieties exported to the US between country o and country x in each industry and year (column 3); the difference between the actual and the predicted (according to eq. 15) log relative market share between country o and country x in each industry and year (column 4). The explanatory variable is the actual log relative market share between country o and country x in each industry and year. In panel b), small countries are those for which the share of Piers exports in total exports to the US is smaller than 45% (i.e., the first group of countries in map b of Figure 1). In panel c), small market shares are those falling below the 5th percentile of the distribution in each industry and year. In panel d), large market shares are those falling above the 95th percentile of the distribution in each industry and year. In panel e), industries with high shares of imported inputs are those for which the average share of imports of parts and components in total US imports over 1972-2001 is above 25%. The standard errors are corrected for heteroskedasticity. ***, **, *: indicate significance at the 1, 5 and 10% level, respectively.
distribution is worsened by infrequent and influential observations, as it would be expected in a granular world.

To implement this test we adopt three alternative definitions of superstar firms. First, we consider as superstars all firms whose total exports (across all products) to the US in a given year are at least two standard deviations above the average exports for their country-industry-year triplet. This is a conservative criterion to identify outliers: the presence of exceptional firms, through its direct effect on the mean, makes it harder to detect them, especially in triplets with few observations. Second, we simply define superstars as the top firm in any triplet. While commonly used in the literature, this criterion has the obvious disadvantages of including some top firms that are not exceptional and of excluding some non-top firms that are exceptional. Finally, we consider firms whose total exports are at least two standard deviations above the average for their country-industry-year triplet computed after excluding the top firm. Compared to the first definition, the exclusion of the top firm when computing the cutoff size makes it easier to detect outliers. Across all triplets in our sample, the average frequency of superstar firms equals 3% according to the first definition, 19% according to the second and 12% according to the third. Superstar firms account for 28%, 55% and 51%, respectively, of total exports to the US in the average triplet.

The results obtained by performing our decomposition without superstar firms are reported in Table 7. Removing exceptional firms according to our first and most conservative criterion leads to a substantial reduction in the contribution of the residual $g_{doxi,t}$, which becomes very close to zero (panel a). A similar pattern emerges for the two other criteria, suggesting that the residual is indeed driven by influential observations (panels b and c). As expected, the contribution of $g_{doxi,t}$ falls relatively less when removing only the top firm in each triplet, consistent with the fact that these firms not always represent outliers. In panel d)-e), we repeat the decomposition using 3-digit, rather than 4-digit, industries to define the triplets. Pooling firms from different 4-digit industries might reduce the role of influential observations, but also creates more noise, because a single distribution may approximate the data from different industries less well. Either way, it is instructive to know if the results are very sensitive to the level of aggregation. Reassuringly, they are not. As shown in panel d), the residual $g_{doxi,t}$ now explains around 7%, of the overall variation in market shares, suggesting that imposing a single distribution across different industries worsens the fit of the data. Yet, once superstar firms are removed from the sample (panel e), the contribution of the residual drops to just about 1%.

Finally, an alternative way to gauge the importance of $g_{doxi,t}$ is to regress the actual relative market shares, $\ln(S_{doi,t}/S_{dxi,t})$, on the predicted values according to (15). Leaving all the coefficients in the regression unconstrained, this strategy purges the residual of any systematic deviations from

\[\text{25 To save space, we use only the first definition of superstar firms. Results using the alternative definitions are very similar.}\]
Table 7: Decomposition of Countries’ Market Shares under Log Normality: The Role of Superstar Firms

<table>
<thead>
<tr>
<th>Difference in av. log sales</th>
<th>Difference in var. of log sales</th>
<th>Difference in log n. of varieties</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
</tbody>
</table>

| Log relative market share   | 0.272***                        | 0.215***                          | 0.527*** | -0.014*** |
|                            | [0.000]                         | [0.001]                           | [0.000]  | [0.001]   |
| Obs.                       | 1078909                        | 1078909                           | 1078909  |           |

| Log relative market share   | 0.249***                        | 0.200***                          | 0.525*** | 0.026***  |
|                            | [0.000]                         | [0.000]                           | [0.000]  | [0.000]   |
| Obs.                       | 809621                         | 809621                            | 809621   |           |

| Log relative market share   | 0.284***                        | 0.191***                          | 0.547*** | -0.022*** |
|                            | [0.000]                         | [0.001]                           | [0.000]  | [0.000]   |
| Obs.                       | 1077933                        | 1077933                           | 1077933  |           |

| Log relative market share   | 0.190***                        | 0.203***                          | 0.532*** | 0.074***  |
|                            | [0.000]                         | [0.001]                           | [0.000]  | [0.000]   |
| Obs.                       | 681672                         | 681672                            | 681672   |           |

| Log relative market share   | 0.217***                        | 0.185***                          | 0.587*** | 0.011***  |
|                            | [0.001]                         | [0.001]                           | [0.001]  | [0.001]   |
| Obs.                       | 681646                         | 681646                            | 681646   |           |

Notes: The table performs the decomposition in eq. (16). Each coefficient is obtained from a different regression. The dependent variables are indicated in the columns’ headings and are: the difference in average log sales between country $o$ and country $x$ in each industry and year (column 1); the difference in the variance of log sales, times one half, between country $o$ and country $x$ in each industry and year (column 2); the difference in the log number of varieties exported to the US between country $o$ and country $x$ in each industry and year (column 3); the difference between the actual and the predicted (according to eq. 15) log relative market share between country $o$ and country $x$ in each industry and year (column 4). The explanatory variable is the actual log relative market share between country $o$ and country $x$ in each industry and year. In panel a), superstar firms are firms whose total exports to the US in a given year are at least two standard deviations above the average exports for their country-industry-year triplet. In panel b), a superstar firm is the top firm in its country-industry-year triplet. In panel c), superstar firms are defined as in panel a), but average and standard deviation are computed without the top firm in each triplet. In panel d) and e), all variables are computed for 3-digit rather than 4-digit industries. In panel e), the superstar firms are accordingly identified within country-(3-digit) industry-year triplets. The standard errors are corrected for heteroskedasticity. ***, **, *: indicate significance at the 1, 5 and 10% level, respectively.
the log-normal distribution that are explained by the components of (15). In the full sample, this exercise yields an $R^2$ of 0.961, suggesting again that outliers only explain less than 4% of the relative market shares. When removing superstar firms as in panels a)-c) of Table 7, the $R^2$ increases to 0.975, 0.980 and 0.986, respectively. Overall, these remarkable results suggest that, while individual firms are quantitatively very important, heterogeneity in attributes with continuous distributions plays a larger role in explaining US imports than deviations from such distribution.

6 Understanding Firm Heterogeneity

Having isolated the exact contribution of firm heterogeneity for aggregate sales and found that it plays an important role, we now explore more in depth this result. We ask: What are the characteristics of the distribution of firm attributes that drive our theory-based measure of heterogeneity? And, how does this measure vary across countries?

6.1 Heterogeneity and Dispersion

As a first step, we examine what the term measuring heterogeneity in attributes in (6) captures. While this term can be interpreted as a measure of dispersion, its variation across triplets could also reflect variation in average attributes or in the number of varieties. To shed light on this question, we now study how the heterogeneity term correlates with a direct measure of dispersion in attributes, $\mathbb{V}(\bar{\gamma}_{doi,t})$, as well as with average attributes, $\mathbb{E}(\bar{\gamma}_{doi})^{\sigma_t-1}$, and with the number of varieties, $N_{doi,t}$.

The results are reported in Table 8, where each panel refers to a different estimate of $\sigma_t$ and where we report beta coefficients for comparability. Note that the heterogeneity term is positively correlated both with the variance of attributes (columns 1 and 6) and with average attributes (columns 2 and 7). However, the correlation with the variance is much stronger than that with average attributes. Moreover, the variance alone accounts for more than 70% of the variation in the heterogeneity term across triplets, as indicated by the $R^2$ in columns (1) and (6). These patterns are robust to including the three terms jointly in the same specification (columns 4 and 9), as well as to controlling for country, industry, and year fixed effects (columns 5 and 10) to absorb time invariant characteristics and common trends. Hence, Table 8 shows that the term measuring heterogeneity in attributes in (6) does to a large extent capture the role of dispersion.

These results are largely insensitive to the choice of $\sigma_t$. We now ask to what extent this robustness is driven by the fact that different estimation strategies produce similar measures of heterogeneity, as captured by the second term of (6). To answer this question, we compute the pairwise correlations

---

26 Constraining the slope to be one and the intercept to be zero, this exercise yields an $R^2$ of 0.803, which indicates that log-normal distributions explain 80% of the variation in sales to the US across country pairs. This is a remarkable result, especially since comparing sales across country pairs is likely to introduce significant noise.

27 The small negative coefficient on the number of varieties (columns 3 and 8) likely reflects a mechanical correlation, due to the presence of $N_{doi,t}$ at the denominator of the heterogeneity term in (6).
### Table 8: Determinants of Heterogeneity in Firm Attributes

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Var. q-to-p ratios</strong></td>
<td>0.909***</td>
<td>0.772***</td>
<td>0.768***</td>
<td>0.887***</td>
<td>0.785***</td>
<td>0.774***</td>
<td>0.881***</td>
<td>0.779***</td>
<td>0.765***</td>
<td>0.842***</td>
</tr>
<tr>
<td></td>
<td>[0.132]</td>
<td>[0.073]</td>
<td>[0.071]</td>
<td>[0.228]</td>
<td>[0.211]</td>
<td>[0.216]</td>
<td>[0.245]</td>
<td>[0.221]</td>
<td>[0.224]</td>
<td>[0.240]</td>
</tr>
<tr>
<td><strong>Av. q-to-p ratios</strong></td>
<td>0.653***</td>
<td>0.295***</td>
<td>0.297***</td>
<td>0.588**</td>
<td>0.213*</td>
<td>0.177</td>
<td>0.581**</td>
<td>0.220*</td>
<td>0.182</td>
<td>0.548***</td>
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<td></td>
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<td>[0.061]</td>
<td>[0.065]</td>
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<td>[0.122]</td>
<td>[0.120]</td>
<td>[0.246]</td>
<td>[0.127]</td>
<td>[0.123]</td>
<td>[0.197]</td>
</tr>
<tr>
<td><strong>N. of varieties</strong></td>
<td>-0.002***</td>
<td>0.000*</td>
<td>-0.001*</td>
<td>-0.003***</td>
<td>-0.001***</td>
<td>-0.002</td>
<td>-0.003***</td>
<td>-0.001***</td>
<td>-0.001***</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>[0.001]</td>
<td>[0.000]</td>
<td>[0.001]</td>
<td>[0.001]</td>
<td>[0.000]</td>
<td>[0.002]</td>
<td>[0.001]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.002]</td>
</tr>
<tr>
<td><strong>Obs.</strong></td>
<td>24754</td>
<td>24754</td>
<td>24754</td>
<td>24754</td>
<td>24754</td>
<td>24754</td>
<td>24754</td>
<td>24754</td>
<td>24754</td>
<td>24754</td>
</tr>
<tr>
<td><strong>R2</strong></td>
<td>0.83</td>
<td>0.43</td>
<td>0.00</td>
<td>0.90</td>
<td>0.90</td>
<td>0.79</td>
<td>0.35</td>
<td>0.00</td>
<td>0.82</td>
<td>0.83</td>
</tr>
</tbody>
</table>

---

**Notes.** The dependent variable is the term measuring heterogeneity in quality-to-price ratios (see eq. (6)) in each country-industry-year triplet. The variance of quality-to-price ratios is the variance of quality-to-price ratios raised to the power of the elasticity of substitution minus 1, computed separately for each triplet. The average quality-to-price ratio is the mean quality-to-price ratio in each triplet raised to the power of the elasticity of substitution minus 1. The regressions in columns (5) and (10) also include country, industry and year fixed effects. All coefficients are beta coefficients. The standard errors are corrected for heteroskedasticity. ***, **, *: indicate significance at the 1, 5 and 10% level, respectively.

---

6.2 **Heterogeneity, Superstar Firms and Market Size**

We now provide evidence on a number of important correlates of firm heterogeneity. We show, in particular, that firm heterogeneity varies systematically with country characteristics associated with market size: dispersion is sales and attributes tends to be greater in richer, more populous and less distant markets. We also document that the relation between firm heterogeneity and market size is almost entirely driven by product quality: quality dispersion is systematically higher in larger markets, and this explains most of the correlation between market size and sales dispersion.

To start off, we show that our data replicate a number of known facts about how trade flows vary with market size. In the first column of panel a) of Table 9, we regress log exports from each country to the US on the log of countries’ real per-capita GDP. All trade measures used in this and subsequent tables are computed as country-level averages across all industries.
Table 9: Trade, Firm Heterogeneity and Country Characteristics

<table>
<thead>
<tr>
<th></th>
<th>a) Total exports</th>
<th>b) N. of exported varieties</th>
<th>c) Average exports per variety</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real per-capita GDP</td>
<td>0.444*** 0.562*** 0.538***</td>
<td>0.137*** 0.221*** 0.210***</td>
<td>0.307*** 0.341*** 0.328***</td>
</tr>
<tr>
<td></td>
<td>[0.097] [0.072] [0.072]</td>
<td>[0.064] [0.045] [0.043]</td>
<td>[0.052] [0.049] [0.048]</td>
</tr>
<tr>
<td>Population</td>
<td>0.580*** 0.608***</td>
<td>0.414*** 0.428***</td>
<td>0.166*** 0.181***</td>
</tr>
<tr>
<td></td>
<td>[0.056] [0.058]</td>
<td>[0.048] [0.046]</td>
<td>[0.047] [0.047]</td>
</tr>
<tr>
<td>Distance</td>
<td>-0.405* 0.055</td>
<td>-0.459***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.225] [0.118]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td>102 102 101</td>
<td>102 102 101</td>
<td>102 102 101</td>
</tr>
<tr>
<td>R2</td>
<td>0.18 0.59 0.61</td>
<td>0.04 0.58 0.61</td>
<td>0.24 0.33 0.39</td>
</tr>
</tbody>
</table>

Notes. The dependent variables (indicated in the panels’ headings) are constructed separately for each country-industry-year triplet and then averaged at the country level. These variables are: total exports (panel a); number of exported varieties (panel b); average exports per variety (panel c); the term measuring heterogeneity in quality-to-price ratios (see eq. (6)), computed using the reg. base. (panel d) or the reg. contr. (panel e) estimate of the elasticity of substitution; and the variance of log exports (panel f). Real per-capita GDP and population are simple averages of these variables between the years 2002 and 2012. All variables are expressed in logs. The standard errors are corrected for heteroskedasticity. ***, **, *: indicate significance at the 1, 5 and 10% level, respectively.

precisely estimated. In the second column we add log population, whose correlation with exports is also positive and highly statistically significant. In the third column we finally include log distance from each country to the US, which enters with a negative and precisely estimated coefficient.29 The three variables jointly explain over 60% of the cross-country variation in exports. In panels b) and c) we decompose export flows into an extensive margin (number of exported varieties) and an intensive margin (average exports per variety). Both margins are positively associated with market size. Accordingly, larger countries export more varieties and sell more of each of them to the US, thereby having larger exports to this market than poorer countries. Note that the coefficients from the regressions of the individual margins add up to those from the regressions of total exports. Accordingly, the results show that the contribution of the intensive margin is particularly important for explaining the correlation of exports with GDP and distance, whereas the extensive margin dominates in the case of population.

Next, we turn to studying variation in firm heterogeneity across countries. To this purpose, in panels d)-f) of Table 9, we repeat the previous regressions using alternative measures of heterogeneity and years. Distance is the population-weighted number of kilometers between each exporting country and the US. Except for distance, the other measures of market size are computed as averages between the years 2002 and 2012.

29 The size of the distance coefficient is slightly smaller than the distance elasticity of trade flows estimated in previous papers. This is not surprising given that the Piers data include maritime trade flows, the bulk of which occur with more distant countries.

29
Table 10: Firm Heterogeneity and Country Characteristics (Robustness Checks)

<table>
<thead>
<tr>
<th></th>
<th>a) Share of superstar firms in total exports</th>
<th>b) Heterogeneity (reg. base.), no superstar firms</th>
<th>c) Heterogeneity (reg. contr.), no superstar firms</th>
<th>d) Variance of log sales, no superstar firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real per-capita GDP</td>
<td>0.029*** [0.007]</td>
<td>0.038*** [0.005]</td>
<td>0.038*** [0.005]</td>
<td>0.595*** [0.087]</td>
</tr>
<tr>
<td></td>
<td>0.047*** [0.004]</td>
<td>0.047*** [0.004]</td>
<td>0.201** [0.086]</td>
<td>0.214** [0.085]</td>
</tr>
<tr>
<td>Distance</td>
<td>0.004 [0.016]</td>
<td>-0.791*** [0.254]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td>102</td>
<td>102</td>
<td>101</td>
<td>102</td>
</tr>
<tr>
<td>R2</td>
<td>0.13</td>
<td>0.61</td>
<td>0.62</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Notes. The dependent variables (indicated in the panels’ headings) are computed separately for each country-industry-year triplet and then averaged at the country level. These variables are: the share of superstar firms in total exports (panel a); the term measuring heterogeneity in quality-to-price ratios (see eq. (6)), computed using the reg. base. (panel b) or the reg. contr. (panel c) estimate of the elasticity of substitution; and the variance of log sales (panel d). Superstar firms are defined as firms whose total exports to the US in a given year are at least two standard deviations above the average exports for their country-industry-year triplet. Real per-capita GDP and population are simple averages of these variables between the years 2002 and 2012. All variables, except the share of superstar firms in total exports, are expressed in logs. The standard errors are corrected for heteroskedasticity. ***, **, * indicate significance at the 1, 5 and 10% level, respectively.

As already discussed, a possible explanation for this result is that the correlation between market size and the dispersion in attributes and sales is driven by exceptional firms in each country. To study this possibility, panel a) of Table 10 shows how the share of superstar firms in a country’s exports to the US varies with market size, using our more conservative definition of superstars (sales two standard deviations above the triplet mean). The importance of these firms in a country is positively correlated with its per-capita GDP and population. Hence, larger markets seem to be more fertile ground for superstar firms. Nevertheless, the correlation between firm heterogeneity and market size is not driven by superstar firms alone. Indeed, in panels b)-d) of Table 10, we regress the three measures of dispersion in sales and attributes computed on the subsample of non-superstar firms on market size, and find that the coefficients are all precisely estimated and only
Notes. Each curve corresponds to the kernel density distribution of log exports to the US for a different group of exporting countries. Rich (poor) countries are those whose real per-capita GDP (averaged between the years 2002 and 2012) is above (below) the 75th (25th) percentile. Each distribution is drawn by pooling together all the varieties exported by a group of countries to the US over the two sample periods, and is centered around zero by deviating the log exports of each variety from the average log exports of the corresponding exporting country.

Figure 3: Distribution of Log Exports to the United States by Group of Exporting Countries

slightly smaller than those obtained in Table 9 using the whole sample of firms.

Figure 3 provides a graphical illustration of the previous results, by plotting the kernel density distribution of log exports to the US for different groups of exporting countries, classified according to their real per-capita GDP. Note that the distribution of log exports across all exporting countries (red curve) resembles a normal distribution. Moreover, the distribution of log exports is evidently more spread out for richer exporting countries (dashed grey curve) than for poorer exporting countries (solid black curve). The graph confirms that this finding is not driven by superstar firms, as the distribution of log exports for richer countries exhibits substantially lower mass around the mean compared to the distribution for poorer countries.

We now study how market size relates to the individual components of sales dispersion according

---

30 Rich (poor) countries are those whose real per-capita GDP is above (below) the 75th (25th) percentile. We obtain similar results when classifying countries according to their population or distance from the US. Each kernel density distribution is drawn by pooling together all the varieties exported by a given group of countries to the US over the two sample periods, and is centered around zero by deviating the log exports of each variety from the average log exports of the corresponding exporting country.
Notes. Variance and covariance components are computed as in eq. (7), separately for each country-industry-year triplet. The variance components are the products between the variance and the square of the elasticity of substitution minus 1. The covariance component is equal to twice the covariance, times the square of the elasticity of substitution minus 1. All graphs use the reg. contr. estimate of the elasticity of substitution. Superstar firms are defined as firms whose total exports to the US in a given year are at least two standard deviations above the average exports for their country-industry-year triplet. Each graph plots the country-level average of the corresponding variable on the log of average real per-capita GDP between the years 2002 and 2012.

Figure 4: Components of Sales Dispersion and Country Characteristics

to (7). As a preliminary step, in Figure 4 we plot each term on the right-hand-side of (7) on the log of countries’ real per-capita GDP. Richer countries exhibit more dispersion of both quality and efficiency compared to poorer countries. Interestingly, the correlation of per-capita GDP with quality dispersion is much stronger than its correlation with the dispersion of efficiency, suggesting quality dispersion to be a key driver of the correlation between market size and firm heterogeneity documented before. The figure also shows that quality and efficiency are more negatively correlated in richer than in poorer countries. This pattern is consistent with richer countries being specialized in higher-quality varieties, which are more costly to produce than lower-quality goods. Finally, the figure confirms that the share of superstar firms in total exports is higher in richer countries.

To better appreciate the role of quality and efficiency in explaining variation in sales dispersion

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31 To save space, we report the results obtained for the reg. contr. estimate of \( \sigma_1 \). Results for the other estimates of \( \sigma_1 \) are similar and available upon request.
Table 11: Components of Sales Dispersion and Country Characteristics

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
<td>(9)</td>
<td></td>
</tr>
<tr>
<td>Real per-capita GDP</td>
<td>0.259**</td>
<td>0.297***</td>
<td>-0.038***</td>
<td>0.287***</td>
<td>0.333***</td>
<td>-0.046***</td>
<td>0.267***</td>
<td>0.311***</td>
<td>-0.043***</td>
</tr>
<tr>
<td></td>
<td>[0.100]</td>
<td>[0.103]</td>
<td>[0.013]</td>
<td>[0.106]</td>
<td>[0.110]</td>
<td>[0.013]</td>
<td>[0.104]</td>
<td>[0.107]</td>
<td>[0.013]</td>
</tr>
<tr>
<td>Population</td>
<td>0.137***</td>
<td>0.177***</td>
<td>-0.039***</td>
<td>0.160***</td>
<td>0.202***</td>
<td>-0.042***</td>
<td>-0.605***</td>
<td>-0.680***</td>
<td>0.074***</td>
</tr>
<tr>
<td></td>
<td>[0.049]</td>
<td>[0.052]</td>
<td>[0.011]</td>
<td>[0.047]</td>
<td>[0.050]</td>
<td>[0.011]</td>
<td>[0.130]</td>
<td>[0.137]</td>
<td>[0.029]</td>
</tr>
<tr>
<td>Distance</td>
<td>0.17***</td>
<td>-0.087***</td>
<td>-0.100***</td>
<td>0.252***</td>
<td>-0.092***</td>
<td>0.160***</td>
<td>0.162***</td>
<td>-0.767***</td>
<td>0.062***</td>
</tr>
<tr>
<td></td>
<td>[0.049]</td>
<td>[0.058]</td>
<td>[0.021]</td>
<td>[0.047]</td>
<td>[0.056]</td>
<td>[0.022]</td>
<td>[0.130]</td>
<td>[0.150]</td>
<td>[0.060]</td>
</tr>
<tr>
<td>Obs.</td>
<td>102</td>
<td>102</td>
<td>102</td>
<td>102</td>
<td>102</td>
<td>102</td>
<td>101</td>
<td>101</td>
<td>101</td>
</tr>
<tr>
<td>R2</td>
<td>0.12</td>
<td>0.14</td>
<td>0.07</td>
<td>0.17</td>
<td>0.21</td>
<td>0.17</td>
<td>0.24</td>
<td>0.28</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Notes. The dependent variables (indicated in the columns’ headings) are computed separately for each country-industry-year triplet and then averaged at the country level. The quality component is defined as the variance of log quality, plus the covariance between log quality and log efficiency, times the square of the elasticity of substitution minus 1 (see eq. (7)). The efficiency component is defined analogously. All regressors are expressed in logs. The standard errors are corrected for heteroskedasticity. ***, **, *: indicate significance at the 1, 5 and 10% level, respectively.

across countries, we now turn to regression analysis. In particular, we follow (7) and regress sales dispersion, quality dispersion and efficiency dispersion on the three market size proxies. We allocate the covariance term in (7) equally between the variance of log quality and the variance of log efficiency, so that the coefficients estimated from the regressions for quality dispersion and efficiency dispersion add up to the coefficients estimated from the regression for sales dispersion. In this way, we obtain an exact decomposition of the effect of market size on sales dispersion into the contribution of quality and efficiency dispersion.

The results are reported in Table 11, where the two panels refer to a different estimate of the elasticity of substitution, reg. base. in panel a) and reg. contr. in panel b). Columns (1), (4) and (7) show the correlations between sales dispersion and market size. The remaining columns decompose these correlations into the contribution of quality and efficiency dispersion. Strikingly, the coefficients of the quality dispersion regressions are close in size to those of the sales dispersion regressions, whereas the coefficients of the efficiency dispersion regressions are small. This suggests that quality dispersion is the main driver of variation in firm heterogeneity across countries: larger markets have a more dispersed sales distribution because their firms produce goods of more heterogeneous quality levels.
6.3 Quantitative Implications of Firm Heterogeneity

We now use our theoretical framework to perform some simple quantifications. In particular, we are interested in evaluating the effect of firm heterogeneity on economic performance and welfare. Building on the observation that log-normal distributions provide a good approximation of the data, we derive simple formulas mapping few, easy-to-compute, statistics about firms into sales and prices. These formulas allow us to compare alternative scenarios in the spirit of a counterfactual exercise.

In particular, we now study the effect of heterogeneity on the price index, which is in turn informative about welfare. We start by rewriting the price index in destination $d$ and industry $i$ as:

$$P_{di} = \left( \sum_{o} P_{doi}^{1-\sigma_i} \right)^{1/\sigma_i},$$

where the summation is taken across all possible origins (including $d$) and

$$P_{doi} = \left[ \sum_{\omega \in O_{doi}} (\tilde{\gamma}_{doi})^\sigma_i \right]^{1/\sigma_i}$$

is the price index of the basket of goods imported from $o$.

Under the assumption of log normality and following a decomposition similar to that in (14), we obtain:

$$\ln(1/P_{doi}) = \mathbb{E}(\ln \tilde{\gamma}_{doi}) + \frac{\sigma_i - 1}{2} \mathbb{V}(\ln \tilde{\gamma}_{doi}) + \ln N_{doi} \sigma_i^{-1}.$$  

Using again the properties of the log-normal distribution to substitute $\mathbb{E}(\ln \tilde{\gamma}_{doi})$ yields:

$$\ln(1/P_{doi}) = \mathbb{E}(\tilde{\gamma}_{doi}) + \frac{\sigma_i - 2}{2} \mathbb{V}(\ln \tilde{\gamma}_{doi}) + \frac{\ln N_{doi}}{\sigma_i - 1}. \quad (17)$$

This expression shows that heterogeneity lowers price indexes, and hence is welfare improving, when $\sigma_i > 2$.\(^{32}\)

These formulas allow us to evaluate the welfare effect of changes in heterogeneity, as captured by the variance of log attributes, holding constant average attributes. Consider for example two countries, $o$ and $x$, differing only in the variance of firms’ attributes. Then, their relative price index can be read from the difference in $\mathbb{V}(\ln \tilde{\gamma})$ as:

$$\frac{P_{doi}}{P_{dxi}} = \exp \left\{ \frac{\sigma_i - 2}{2} \left[ \mathbb{V}(\ln \tilde{\gamma}_{dxi}) - \mathbb{V}(\ln \tilde{\gamma}_{doi}) \right] \right\}.$$

Recalling that $\mathbb{V}(\ln \tilde{\gamma}_{doi}) = \mathbb{V}(\ln r_{doi}) / (\sigma_i - 1)^2$, we can rewrite this expression in terms of variables

\(^{32}\)See Epifani and Gancia (2011) for a related result.
that are easier to observe and measure:
\[
P_{\text{doi}} = \exp\left\{ \frac{\sigma_i - 2}{2(\sigma_i - 1)^2} \left[ \mathbb{V}(\ln r_{\text{doi}}) - \mathbb{V}(\ln r_{\text{dxi}}) \right] \right\}.
\] (18)

This formula shows that the effect of changes in heterogeneity on price indexes can be computed from the elasticity of substitution and the difference in the variance of log sales. We now use this handy result to perform some quantification.

Our estimates of \( \sigma_i \) range from 2.44 to 4.22 for the average industry. In this interval, the factor \( \frac{(\sigma_i - 2)}{2(\sigma_i - 1)^2} \) is around 0.11 and it turns out not to be very sensitive to the exact value of \( \sigma_i \).\(^{33}\) Regarding a reasonable range for \( \mathbb{V}(\ln r_{\text{doi}}) \), the average value in our data is 3.69, with a standard deviation of 3.11. Figure 2 shows that the average \( \mathbb{V}(\ln r_{\text{doi}}) \) exhibits indeed significant variation across countries. Recall also that the average \( \mathbb{V}(\ln r_{\text{doi}}) \) increased by 10% from 2002 to 2012. With these numbers in mind, according to (18), a difference of 3 in \( \mathbb{V}(\ln r_{\text{doi}}) \) implies a price index about 40% higher in the low-variance case. Moreover, an increase in \( \mathbb{V}(\ln r_{\text{doi}}) \) of 0.3, comparable to the observed change over the decade covered by our data, implies a fall in the price index of more than 3%. Overall, these numbers suggest that variations in micro-level firm heterogeneity have quantitatively important effects both in the cross section and over time.

Note that these formulas allow us to evaluate the effect of changes in the variance of log attributes holding constant the number of firms selling in a market. When the number of operating firms is endogenous, changes in heterogeneity have additional effects. Unfortunately, it is difficult to study them without imposing additional restrictions on the supply side of the model. To get a sense of how these additional effects may change the results obtained so far, in Appendix C we study a version of the model in which attributes are drawn from a Pareto distribution and \( N_{\text{doi}} \) is endogenous. This special case, which is analytically tractable, shows that selection makes the beneficial effect of dispersion stronger, so that an increase in the variance of log attributes, holding constant the unconditional mean, can lower the price index even for \( \sigma_i < 2 \).\(^{34}\)

7 Conclusions

In this paper, we used highly-disaggregated, transaction-level, US import data to compare firms from virtually all countries in the world competing in a single destination market. With the help of commonly-made assumptions on demand and supply, we decomposed the economic performance of countries into the contribution of the number of firm-products, their average attributes (quality and efficiency) and the dispersion around the mean. The most important and novel lessons from

\(^{33}\)The term \( \frac{\sigma_i - 2}{2(\sigma_i - 1)^2} \) reaches a maximum of 0.125 for \( \sigma_i = 3 \). It takes value 0.106 for \( \sigma_i = 2.44 \) and 0.107 for \( \sigma_i = 4.22 \).

\(^{34}\)Yet, the overall effect still changes sign for \( \sigma_i \) sufficiently low.
our analysis are that variation in firm-level heterogeneity is very important for explaining countries’ aggregate economic performance, and that firm-level heterogeneity correlates systematically with country characteristics. In particular, proxies for market size are associated to a higher dispersion of firm attributes.

These results beg the question of what mechanism might be generating the observed variation in firm heterogeneity. We conclude by discussing briefly some candidate explanations. First, it seems natural to conjecture that differences in attributes may depend on differences in innovation. For instance, richer and larger markets may be more conducive to drastic innovation with more dispersed outcomes than the adoption of existing technologies (e.g., Bonfiglioli, Crinò and Gancia 2018a,b) or imitation (Benhabib, Perla and Tonetti, 2017, König, Lorenz and Zilibotti, 2016). Another possibility is that agglomeration economies, or more in general increasing returns, may explain the effect of market size. It could also be that richer and thicker markets facilitate a stronger sorting between firms, suppliers and workers, which amplify pre-existing productivity differences (e.g., Bonfiglioli and Gancia, 2016, Sampson, 2014). Identifying the exact mechanism explaining how the distribution of attributes across firms is generated and evolves seems an important direction for future research.

Appendix A The Reverse-Weighting Estimator

Following Redding and Weinstein (2017), we start by obtaining three equivalent expressions for the change in the price index of industry \(i\) between 2002 \((t - 1)\) and 2012 \((t)\). Dropping the industry and destination subscripts to save on notation, these expressions read as follow

\[
\frac{P_t}{P_{t-1}} = \left( \frac{\lambda_{t,t-1}}{\lambda_{t-1,t}} \right)^{\frac{1}{\sigma-1}} \left( \Theta_{t-1,t} \right)^{\frac{1}{\sigma-1}} \left\{ \sum_{\omega \in \Omega_{t,t-1}} s_{t-1}^* (\omega) \left[ \frac{p_t(\omega)}{p_{t-1}(\omega)} \right]^{\frac{1}{1-\sigma}} \right\}^{\frac{1}{1-\sigma}},
\]

(19)

\[
\frac{P_t}{P_{t-1}} = \left( \frac{\lambda_{t,t-1}}{\lambda_{t-1,t}} \right)^{\frac{1}{\sigma-1}} \left( \Theta_{t-1,t} \right)^{-\frac{1}{\sigma-1}} \left\{ \sum_{\omega \in \Omega_{t,t-1}} s_t^* (\omega) \left[ \frac{p_t(\omega)}{p_{t-1}(\omega)} \right]^{-(1-\sigma)} \right\}^{\frac{1}{1-\sigma}},
\]

(20)

\[
\frac{P_t}{P_{t-1}} = \left( \frac{\lambda_{t,t-1}}{\lambda_{t-1,t}} \right)^{\frac{1}{\sigma-1}} \frac{\tilde{P}_t^*}{P_{t-1}^*} \left( \frac{\tilde{S}_t^*}{S_{t-1}^*} \right)^{\frac{1}{\sigma-1}},
\]

(21)

where \(\Omega_{t,t-1}\) denotes the set of common varieties in both years; \(s^* (\omega)\) denotes the share of common variety \(\omega\) in expenditure on all common varieties; \(\tilde{S}^*\) and \(\tilde{P}^*\) denote the geometric averages of \(s^* (\omega)\) and \(p (\omega)\), respectively, computed on common varieties; \((\lambda_{t,t-1}/\lambda_{t-1,t})^{1/(\sigma-1)}\) is the variety-adjustment term, which adjusts the common varieties price index for entering and exiting varieties;
and

\[
\Theta_{t-1,t}^F = \left\{ \sum_{\omega \in \Omega_{t-1}} s_{t-1}^* (\omega) \left[ \frac{p_t(\omega)}{p_{t-1}(\omega)} \right]^{-\sigma} \left[ \frac{\gamma_{t-1}(\omega)}{\gamma_{t-1}(\omega)} \right]^{-\sigma} \right\}^{-1} = \left\{ \sum_{\omega \in \Omega_{t-1}} s_{t-1}^* (\omega) \left[ \frac{\gamma_{t-1}(\omega)}{\gamma_{t-1}(\omega)} \right]^{-\sigma} \right\}^{-1}
\]

\[
\Theta_{t,t-1}^B = \left\{ \sum_{\omega \in \Omega_{t-1}} s_{t-1}^* (\omega) \left[ \frac{p_{t-1}(\omega)}{p_t(\omega)} \right]^{-\sigma} \left[ \frac{\gamma_{t-1}(\omega)}{\gamma_{t-1}(\omega)} \right]^{-\sigma} \right\}^{-1} = \left\{ \sum_{\omega \in \Omega_{t-1}} s_{t-1}^* (\omega) \left[ \frac{\gamma_{t-1}(\omega)}{\gamma_{t-1}(\omega)} \right]^{-\sigma} \right\}^{-1}
\]

are the forward and backward differences of the price index, which evaluate its change using varieties’ expenditure shares in \( t-1 \) and \( t \), respectively.

The three ways of expressing the change in the price index are equivalent. However, the formulation in (21) is the only one that exclusively depends on prices and expenditure shares, and not also on the demand parameters \( \gamma \) (i.e., this formulation is money-metric). Note also that the three expressions depend on the elasticity of substitution, \( \sigma \). Hence, the idea of the RW estimator is to look for the value of \( \sigma \) that renders the three expressions for the change in the price index consistent with the same money-metric utility function. This requires imposing the following identifying assumption:

\[
\Theta_{t-1,t}^F = (\Theta_{t,t-1}^B)^{-1} = 1,
\]  

(22)

which means that changes in \( \gamma \) over time average out.

Combining (19)-(21) and using (22), one can construct a generalized method of moment estimator for \( \sigma \). In particular, the following moment functions obtain:

\[
M(\sigma) = \begin{pmatrix} m_t^F(\sigma) \\ m_t^B(\sigma) \end{pmatrix}
\]

\[
= \begin{pmatrix}
\frac{1}{1-\sigma} \ln \left\{ \sum_{\omega \in \Omega_{t-1}} s_{t-1}^* (\omega) \left[ \frac{p_t(\omega)}{p_{t-1}(\omega)} \right]^{-\sigma} - \ln \left[ \frac{\tilde{p}_t}{\tilde{p}_{t-1}} \left( \frac{\tilde{s}_t}{\tilde{s}_{t-1}} \right) \right] ^{1\sigma} \right\} \\
- \frac{1}{1-\sigma} \ln \left\{ \sum_{\omega \in \Omega_{t-1}} s_t^* (\omega) \left[ \frac{p_{t-1}(\omega)}{p_t(\omega)} \right]^{-1(\sigma)} - \ln \left[ \frac{\tilde{p}_{t-1}}{\tilde{p}_t} \left( \frac{\tilde{s}_{t-1}}{\tilde{s}_t} \right) \right] ^{1\sigma} \right\}
\end{pmatrix} = \begin{pmatrix} \ln \Theta_{t-1,t}^F \\ \ln \Theta_{t,t-1}^B \end{pmatrix}.
\]

The RW estimator \( \hat{\sigma}^{RW} \) solves:

\[
\hat{\sigma}^{RW} = \arg \min \left\{ M(\hat{\sigma}^{RW})' \times I \times M(\hat{\sigma}^{RW}) \right\},
\]

where \( I \) is the identity matrix. Weighting the two moments by the identity matrix implies that the RW estimator minimizes the sum of squared deviations of the aggregate demand parameters \( ((-\ln \Theta_{t-1,t}^F)^2 + (-\ln \Theta_{t,t-1}^B)^2) \) from zero. Hence, the RW estimator selects the value of \( \sigma \) that
minimizes the squared deviations of the forward and backward differences of the price index from a money-metric utility function.

**Appendix B  Sales and Firm Attributes across Countries and Sectors**

In this section, we perform two variance decomposition exercises, with the aim of studying the main sources of variation in the variance of sales. In the first exercise, we focus on one origin country at a time, and decompose the variance of log sales, quality, and efficiency for this country into within-industry and between-industry contributions. We use the following decomposition, adapted from Helpman et al. (2017):

\[
\text{V} \left( \ln x_{do,t} \right) = \frac{1}{N_{do,t}} \sum_{i} \sum_{\omega \in \Omega_{do,t}} (x_{doi,t} (\omega) - \mathbb{E}(x_{doi,t}))^2 + \frac{1}{N_{do,t}} \sum_{i} N_{doi,t} (\mathbb{E}(x_{doi,t}) - \mathbb{E}(x_{do,t}))^2, \tag{23}
\]

where \( x \) denotes sales, efficiency, or quality; \( N_{do,i,t} \) and \( \mathbb{E}(x_{doi,t}) \) are the number of varieties exported from country \( o \) to the US in industry \( i \) and the mean of \( x \) computed across these varieties, respectively; and \( N_{do,t} \) and \( \mathbb{E}(x_{do,t}) \) denote the total number of varieties exported from country \( o \) to the US and the overall mean of \( x \), respectively. The first term in (23) measures the part of the overall variance of \( x \) that is due to variety-specific deviations from each industry’s average (within-industry contribution). The second term measures instead the part that is due to deviations of each industry’s average from the overall mean of \( x \) (between-industry contribution). We perform this decomposition separately for each of the 104 countries in our sample. In panel a) of Table 12, we report the simple averages and the standard deviation of the within-industry and between-industry contributions across all countries for the year 2012.\(^{35}\) The results show that the within-industry component explains 71\% of sales dispersion, and approximately two-thirds of quality dispersion, in the representative country. The relative importance of the two contributions varies little across countries as shown by the low values of the standard deviation. Hence, although cross-industry differences play an important role, as expected the lion’s share of the overall dispersion in sales and quality in a country is due to within-industry heterogeneity.

In the second exercise, we focus on one industry at the time, and decompose the variance of log sales, quality, and efficiency for this industry into within-country and between-country contributions. We use the following decomposition:

\[
\text{V} \left( \ln x_{di,t} \right) = \frac{1}{N_{di,t}} \sum_{o} \sum_{\omega \in \Omega_{do,i,t}} (x_{doi,t} (\omega) - \mathbb{E}(x_{doi,t}))^2 + \frac{1}{N_{do,t}} \sum_{o} N_{doi,t} (\mathbb{E}(x_{doi,t}) - \mathbb{E}(x_{di,t}))^2, \tag{24}
\]

where the variables are defined similarly to (23). The first term in (24) measures the part of the

\(^{35}\)Note that the standard deviations of the within-industry and between-industry contributions are equal to each other by construction.
### Table 12: Variance Decompositions

<table>
<thead>
<tr>
<th></th>
<th>a) Within/Between Industry</th>
<th></th>
<th>b) Within/Between Country</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Within (mean)</td>
<td>Between (mean)</td>
<td>Std. Dev.</td>
<td>Within (mean)</td>
</tr>
<tr>
<td>Log sales</td>
<td>0.71</td>
<td>0.29</td>
<td>0.19</td>
<td>0.87</td>
</tr>
<tr>
<td>Log efficiency</td>
<td>0.26</td>
<td>0.74</td>
<td>0.16</td>
<td>0.48</td>
</tr>
<tr>
<td>Log quality (reg. base.)</td>
<td>0.66</td>
<td>0.34</td>
<td>0.21</td>
<td>0.77</td>
</tr>
<tr>
<td>Log quality (reg. contr.)</td>
<td>0.61</td>
<td>0.39</td>
<td>0.21</td>
<td>0.66</td>
</tr>
<tr>
<td>Log quality (RW)</td>
<td>0.66</td>
<td>0.34</td>
<td>0.23</td>
<td>0.71</td>
</tr>
<tr>
<td>Log quality (BW)</td>
<td>0.64</td>
<td>0.36</td>
<td>0.21</td>
<td>0.75</td>
</tr>
</tbody>
</table>

**Notes.** Panel a) decomposes the variance of each variable into within-industry and between-industry contributions according to eq. (23). Each contribution is computed separately for the 104 countries in the sample; reported figures are the simple averages and the standard deviation of the two contributions across all countries. Panel b) decomposes the variance of each variable into within-country and between-country contributions according to eq. (24). Each contribution is computed separately for the 366 industries in the sample; reported figures are the simple averages and the standard deviation of the two contributions across all industries. By construction, the standard deviations of the two contributions are equal.

overall variance of $x$ that is due to variety-specific deviations from each country’s average (within-country contribution). The second term measures instead the part that is due to deviations of each country’s average from the overall mean of $x$ (between-country contribution). We perform this decomposition separately for each of the 366 industries in our sample. In panel b) of Table 12, we report simple averages of the within-country and between-country contributions across all industries for the year 2012. Note that, although the within-country contribution generally dominates, cross-country heterogeneity explains a sizable share of sales dispersion (13%) and quality dispersion (25-35%) in the representative industry.

### Appendix C Firm Heterogeneity and Welfare with Pareto Distributions

To study the effect of heterogeneity in firms’ attributes on prices and welfare in general equilibrium we now focus on a special case of the model that admits analytical solutions. To close the model, we need to study the entry stage and how firms attributes are determined. Following the literature, we assume that, upon paying an entry cost $w_o F_{oi}$, entering firms can draw their attributes form some known distribution. Although attributes are two-dimensional $(\varphi, \gamma)$, tractability is preserved by the fact that, for the purpose of determining the equilibrium allocation, quality and efficiency can be collapsed into a one-dimensional object, the product $\varphi \gamma$, which can be taken as a single measure of performance. We simplify the notation by defining this variable $\phi \equiv \varphi \gamma$. Next, we assume that $\phi$ is drawn from a Pareto distribution with support on $[\phi_{oi}^{\min}, \infty)$, shape parameter $1/v_{oi}$ and c.d.f. $G_{oi}(\phi) = 1 - (\phi/\phi_{oi}^{\min})^{-1/v_{oi}}$. Notice that $v_{oi}$, i.e., the inverse of the shape parameter, is equal to the standard deviation of the log of $\phi$ and hence can be interpreted as a measure of dispersion. Moreover, $v_{oi}$ also affects the expected value of $\phi$, which is equal to $\phi_{oi}^{\min}(1 - v_{oi})^{-1}$, so that mean and variance are linked, consistently with the data. A Pareto distribution has been used extensively in the literature because of its convenient analytical properties and because it has been shown to be
a reasonable approximation of the data, especially in the right tail. We also assume that there is a continuum of firms.

Firms enter until expected profits are equal to the entry cost. Using (10) and (11), expected profits from selling to market \( d \) can be expressed as:

\[
E[\pi_{doi}] = \int_0^\infty \pi_{doi} (\varphi^*) dG_{oi} (\varphi) = w_o f_{oi} \int_0^\infty \left[ \left( \frac{\sigma_{oi}}{\phi_{doi}^*} \right)^{-1} - 1 \right] dG_{oi} (\phi),
\]

where \( \phi_{doi}^* \) is the minimum attribute below which firms from \( o \) would make losses in market \( d \) and hence exit it. Expected profits from selling in all potential markets are \( E[\pi_{oi}] = \sum_d E[\pi_{doi}] \). Using \( G_{oi} (\phi) = 1 - (\phi / \phi_{oi}^{min})^{-1/\sigma_{oi}} \), the expected value of entry can be solved (provided that \( 1/\sigma_{oi} + 1 > \sigma_i \)) as:

\[
E[\pi_{oi}] = \frac{(\sigma_i - 1)w_o}{1/\sigma_{oi} - (\sigma_i - 1)} \left( \frac{\phi_{oi}^{min}}{\phi_{ooi}^*} \right)^{1/\sigma_{oi}} \sum_d f_{doi} \rho_{doi}^{1/\sigma_{oi}},
\]

where

\[
\rho_{doi} \equiv \frac{\varphi_{ooi}^*}{\varphi_{doi}^*} = \frac{\tau^{-1}_{ooi} \left( \frac{w_i L d P_{oi}^{\sigma_i-1} f_{ooi}^{1-\sigma_i}}{w_o L_o P_{oi}^{\sigma_i-1} f_{doi}^{1-\sigma_i}} \right)^{1/(\sigma_i - 1)}}
\]

is a measure of export opportunities in destination \( d \). In particular, in a given industry \( i \), \( \rho_{doi}^{1/\sigma_{oi}} \in (0, 1) \) is the fraction of country \( o \) firms selling to market \( d \).

Setting \( E[\pi_{oi}] \) equal to the entry cost \( w_o F_{oi} \) yields the solution for the domestic cutoff \( \phi_{ooi}^* \):

\[
\left( \frac{\phi_{ooi}^*}{\phi_{oi}^{min}} \right)^{1/\sigma_{oi}} = \frac{\sigma_i - 1}{1/\sigma_{oi} - (\sigma_i - 1)} \frac{\sum_d f_{doi} \rho_{doi}^{1/\sigma_{oi}}}{F_{oi}}.
\]

Next, note that the domestic price index in an industry is a function of \( \phi_{ooi}^* \). Combining the break-even condition for the marginal firm, \( r (\phi_{ooi}^*) = \sigma_i w_o f_{ooi} \), with (9) and substituting \( C_{oi} = \beta_i w_o L_o / P_{oi} \) yields:

\[
P_{oi} = \frac{w_o \sigma_i}{\phi_{ooi}^* \sigma_i - 1} \left( \frac{\sigma_i f_{ooi}}{\beta_i L_o} \right)^{\frac{1}{\sigma_i - 1}}.
\]

This expression shows that the effect of the distribution of firms’ attributes on prices is entirely summarized by its effect on the domestic cutoff \( \phi_{ooi}^* \) and that a higher cutoff, i.e., more selection, lowers prices.

What is the effect of more dispersion in attributes on prices? To simplify the analysis, we now focus on the case in which countries are symmetric. This implies that \( \rho_{doi} \) is just a parameter. Then, if dispersion is simply captured by the inverse of the shape parameter, \( \sigma_{oi} \), (25) shows that more dispersion increases the cutoff and hence lowers prices. However, this is the result of the effect of \( \sigma_{oi} \) on both the variance and the mean of \( \phi \). To isolate the former, we can consider a mean-preserving
spread of the distribution $G_{oi}(\phi)$. We obtain this by setting $\phi_{oi}^{\min} = \bar{\phi}_{oi} (1 - v_{oi})$ so that the mean of the unconditional distribution is fixed at $\bar{\phi}_{oi}$. In this case, the expression for the cutoff becomes:

$$\phi_{oi}^* = \left[ \frac{\sigma_i - 1}{1/v_{oi} - (\sigma_i - 1) \frac{\sum_d f_{doi} \rho_{doi}}{F_{oi}}} \right]^{v_{oi}} \bar{\phi}_{oi} (1 - v_{oi}).$$

Then, we can compute the effect on prices of changing $v_i$ while keeping the unconditional mean constant as:

$$\frac{d \ln(1/P_{oi})}{dv_{oi}} = \frac{d \ln \phi_{oi}^*}{dv_{oi}} = \frac{(\sigma_i - 2)}{(1 - v_{oi}) (1 - \sigma_i + 1/v_{oi})} + \frac{1}{v_{oi}} \ln \left( \frac{\phi_{oi}}{\phi_{oi}^{\min}} \right) + \frac{\sum_d f_{doi} \rho_{doi}^{1/v_{oi}} \ln \rho_{doi}^{1/v_{oi}}}{\sum_d f_{doi} \rho_{doi}^{1/v_{oi}}}. \tag{26}$$

The first term in (26) yields a familiar result: for a given set of firms, dispersion lowers prices when $\sigma_i > 2$, precisely as in (17). However, the second term shows that, when the number of operating firms in endogenous, there is an additional positive effect of dispersion through selection. The positive effect is even stronger when firms can also sell to foreign markets, as captured by the third term. The intuition is that a higher dispersion increases the probability of drawing an attribute above the domestic and export cutoffs, thereby increasing the value of entry. In turn, more entry means lower prices. These results extend the findings in Bonfiglioli, Crinò and Gancia (2018a,b) who show how firm heterogeneity affects the value of entry through similar channels and develop a model in which the extent of heterogeneity depends on the choice of innovation.

**References**


