Time-Varying Wage Risk, Incomplete Markets, and Business Cycles*

Shuhei Takahashi†

Abstract

Micro-level data indicate cyclical variation in idiosyncratic earnings risk. This paper shows that the risk variation is the key to resolving the counterfactually strong comovement of total hours worked with average labor productivity (output per labor hour) commonly found in equilibrium business cycle models. I develop a model economy in which individuals make employment choices each period in the presence of time-varying person-specific wage (productivity) risk. In the economy, fluctuations in wage risk shift labor supply and the effect varies with individuals’ wealth and productivity. This leads to a negative correlation between hours and productivity in equilibrium. Specifically, a rise in wage risk initially increases and then decreases the employment of individuals with low productivity. The employment of high-productivity individuals moves in the opposite direction to a lesser extent. As a result, total hours worked first increase and then decrease. Average labor productivity initially falls and later rises.

I quantify cyclical variation in idiosyncratic wage risk using individual wage data in the Panel Study of Income Dynamics. My estimate is that idiosyncratic wage risk varies by an annual standard deviation of 4.4%. When driven by shocks to both aggregate total factor productivity (TFP) and idiosyncratic wage risk, the calibrated model reproduces the weakly negative correlation between total hours worked and average labor productivity found in the U.S. data (–0.38 in the model compared with –0.32 in the data). Since aggregate TFP shocks primarily shift labor demand, in the absence of risk fluctuations, the model generates a strong positive correlation of 0.83.

*I am grateful to Aabhik Khan, Bill Dupor, and Julia Thomas for their comments, encouragements, and guidance. I also thank Paul Evans, seminar and conference participants at Development Bank of Japan, Kyoto, Bank of Japan, Adelaide, Osaka, Tokyo, Fall 2012 Midwest Macroeconomics Meetings, 2012 Asian Meeting of the Econometric Society, 2013 AEI Four-Joint Workshop, and 2013 Annual Conference of the Royal Economic Society for their comments and suggestions. Any remaining errors are my own.

†Assistant Professor, Institute of Economic Research, Kyoto University, Yoshida-Honmachi, Sakyo-ku, Kyoto 606-8501, Japan. Tel:+81-75-753-7153, Email: takahashi@kier.kyoto-u.ac.jp
1 Introduction

Micro-level data suggest cyclical fluctuations in person-specific earnings risk. Since insurance is incomplete in practice, such changes in idiosyncratic risk could have important impacts on macroeconomic fluctuations. A large amount of literature have studied the possibility.\(^1\) For example, Krusell and Smith (1999), Storesletten, Telmer, and Yaron (2001), Mukoyama and Sahin (2006), and Krusell, Mukoyama, Sahin, and Smith (2009) calculate the welfare cost of business cycles in the presence of countercyclical idiosyncratic income risk. Implications for asset pricing are also studied, for example, by Krusell and Smith (1997), Pijoan-Mas (2007), and Storesletten, Telmer, and Yaron (2007). However, little is known about how cyclical variation in idiosyncratic earnings risk affects overall business cycles, including labor market dynamics, because previous general-equilibrium studies assume inelastic labor supply or exogenous earnings.\(^2\) They exclude the possibility that individuals change labor supply, responding to fluctuations in idiosyncratic wage risk, which is more primitive than earnings risk.

In contrast, empirical studies find that changes in idiosyncratic wage risk affect labor supply. For example, analyzing the PSID data, Parker, Belghitar, and Barmby (2005) conclude that an increase in wage risk raises labor supply of self-employed workers. Further, using a two-period, single-agent model, Flodén (2006) theoretically shows that greater uncertainty on the second-period wage increases the labor supply in the first period and decreases the labor supply in the second period. This result suggests that assuming endogenous labor supply might not only capture reality more accurately, but also provide new insights concerning the impact of varying idiosyncratic earnings risk on business cycles.

This paper analyzes how cyclical variation in idiosyncratic wage risk affects business cycles under endogenous labor supply. The model studied here is a generalization of an incomplete asset markets model used by recent macroeconomic studies (e.g., Chang and Kim

\(^{1}\)Krusell and Smith (1998), the seminal paper on income and wealth inequality across individuals and business cycles, also assume variation in individual-level income risk.

\(^{2}\)An exception is Lopez (2010). The paper is discussed in the latter part of this section.
(2006, 2007) and Krusell, Mukoyama, Roserson, and Sahin (2010, 2011)). Individuals make discrete labor supply choices and consumption-saving decisions each period. They face idiosyncratic wage risk because person-specific labor productivity changes stochastically. Asset markets are incomplete and there is only one asset (physical capital). Hence, individuals cannot fully insure against this idiosyncratic wage risk. I introduce risk fluctuations into this environment using uncertainty shocks à la Bloom (2009), that is, assuming time-varying volatility of idiosyncratic productivity shocks. Such risk variation influences employment decisions of individuals, who differ in both their assets and labor productivity, and thereby generates the comovement of total hours worked and average labor productivity (output per labor hour) that might differ from that observed in representative-agent models.

Toward a quantitative evaluation of the impact of time-varying idiosyncratic wage risk, I examine properties of actual risk fluctuations using the individual wage data in the Panel Study of Income Dynamics (PSID). As consistent with the model analyzed here, idiosyncratic wage risk is identified using the cross-sectional dispersion of wage shocks, and its time variation is studied. The result indicates that idiosyncratic wage risk varies by an annual standard deviation of 4.4%. The risk variation also shows some persistence with an annual first-order autocorrelation coefficient of 0.26. Based on these findings, I calibrate the model’s parameters governing the stochastic process of idiosyncratic wage risk.

The main finding of this paper is that cyclical variation in idiosyncratic wage risk is the key to explaining the dynamics of the U.S. labor market. In particular, I find that when driven by exogenous shocks to both aggregate total factor productivity (TFP) and idiosyncratic wage risk, the model in this paper reproduces the weakly negative correlation between total hours worked and average labor productivity observed in the U.S. (−0.38 in the model compared to −0.32 in the data). This solves the so-called hours-productivity puzzle, fixing the counterfactually strong comovement of the two found in typical equilibrium business cycle models (e.g., Kydland and Prescott (1982) and Hansen (1985)). The positive

\[3\] Ohanian and Raffo (2011) report a negative correlation between total hours worked and average labor productivity found in OECD countries.
correlation arises in those models because shocks to aggregate TFP primary shift labor demand. In fact, when only aggregate TFP shocks are fed, the heterogeneous-agent model in this paper also produces a strong, positive correlation between hours and productivity (0.83).\(^4\)

The varying risk model resolves the hours-productivity puzzle because shocks to idiosyncratic wage risk produce a negative correlation between total hours worked and average labor productivity, dominating the positive one generated by aggregate TFP shocks. The negative correlation arises because a change in wage risk affects labor supply and the effect depends on individuals’ wealth and productivity. In equilibrium, the employment of low-productivity individuals moves inversely to the employment of high-productivity individuals, and the former shows larger fluctuations. This leads to a negative comovement of hours with labor productivity at the aggregate level.

To provide an intuition for the above result, I classify the impact of an increase in idiosyncratic wage risk into the following two effects. First, the precautionary effect is the current response of individuals to greater uncertainty on their future wages. Since individuals make discrete labor supply choices, changes in aggregate employment are caused by responses of individuals who were close to indifferent towards working before the rise in risk. Those individuals with low productivity hold low (negative) wealth near their borrowing limit. Hence, they have strong incentive to self-insure against elevated risk by working and saving more in the current period. Further, the rise in the interest rate reduces their wealth, which enhances their incentive to work. By contrast, since individuals with high productivity are wealthy, there is little effect on their labor supply. The raised interest rate also makes them wealthy. In equilibrium, the employment of individuals with low productivity increases, whereas because of the decline in the wage rate, the employment of individuals with high productivity increases.

\(^4\)Although Chang and Kim (2007) report a weakly positive correlation between total hours worked and average labor productivity in the constant risk model, their solution method is largely responsible for the result. Takahashi (2013) provides a detailed explanation for the problems in the Chang and Kim (2007) solution method and shows that eliminating those problems leads to a strong, positive correlation between hours and productivity in their model, as found in this paper.
decreases, although to a lesser degree. Total hours worked increase. Average labor productivity falls, especially by more than the decline in the marginal product of labor, because the employment share of individuals with low productivity rises. Hence, the precautionary effect generates a strong, negative comovement of hours with labor productivity.

The second effect of increased idiosyncratic wage risk is the *distribution effect*. It arises when the cross-sectional productivity distribution becomes more dispersed in the following period. Overall, there is a movement of individuals from the middle of the productivity distribution to the lower and higher levels of productivity. Crucially, since idiosyncratic productivity is persistent, those who had productivity near the middle of the distribution tended to be wealthier than those with lower levels of productivity. Hence, the shift in the productivity distribution increases the number of individuals with a relatively high level of wealth among low-productivity groups. As a result, the labor supply of individuals with low productivity decreases relative to before. The opposite mechanism applies to individuals with high productivity, and their labor supply increases, which in equilibrium lowers the wage rate. The employment of low-productivity individuals decreases, while the employment of high-productivity individuals increases to a lesser extent. Total labor hours decrease. Average labor productivity rises because this time the employment share of individuals with high productivity increases. Thus, the distribution effect also yields a negative correlation between hours and labor productivity.

These heterogeneous responses of labor supply are unique to this paper and generate a strong, negative comovement of total hours worked with average labor productivity. In an effort to resolve the hours-productivity puzzle, researchers have included labor supply shocks into equilibrium business cycle models (e.g., Benhabib, Rogerson, and Wright (1991) and Christiano and Eichenbaum (1992)). However, those models assume a representative agent, and when calibrated to the U.S. economy, they generate a relatively strong positive correlation between hours and productivity.5

---

5Benhabib, Rogerson, and Wright (1991) suggest shocks to home-production technology and find a correlation between total hours worked and average labor productivity of 0.49 under their benchmark para-
Importantly, the resolution of the hours-productivity puzzle comes at no cost to the empirical performance of the model analyzed in this paper. The volatilities and comovements of GDP, consumption, and investment are essentially unaltered by the introduction of cyclical variation in idiosyncratic wage risk. Crucially, in addition to resolving the above puzzle, risk variation raises the variability of total hours worked, and this moves the model’s result closer to the data.

Further, the present model indicates that total hours worked recover more slowly following a recession in which idiosyncratic wage risk rises compared to a recession without such elevated risk. This result is consistent with the U.S. experience. My analysis using the PSID data finds a rise in idiosyncratic wage risk during the early 1990s recession and a decline in risk during the 1981–1982 recession. At the aggregate level, total hours worked recovered much more slowly after the 1990s recession than the 1980s. I take this finding as additional evidence in support of the mechanisms developed in this paper.

Lopez (2010) assumes endogenous labor supply and examines how fluctuations in idiosyncratic wage risk affect the labor wedge measured at the aggregate level. However, his model assumes divisible labor and a borrowing constraint different from the one assumed in this paper. As a result, changes in idiosyncratic wage risk affect labor supply quite differently from my model. At the aggregate level, his model generates a correlation between total hours worked and average labor productivity close to one and hence leaves the hours-productivity puzzle unresolved.

---

6 The labor wedge is computed dividing the marginal rate of substitution of leisure for consumption by the marginal product of labor.

7 The model of Lopez (2010) produces a positive correlation between output and total hours worked (0.98) and a low volatility of hours relative to output (0.32). These imply a strong comovement of hours with average labor productivity. Although Lopez (2010) does not report the correlation, it can be computed using other business cycle moments: Specifically, $corr(\ln H, \ln Y/H) = (cov(\ln Y, \ln H) - var(\ln H))/\sqrt{var(\ln H)var(\ln Y/H)}$. Notice also that $cov(\ln Y, \ln H) = corr(\ln Y, \ln H)\sqrt{var(\ln Y)var(\ln H)}$ and $var(\ln Y/H) = var(\ln Y) + var(\ln H) - 2cov(\ln Y, \ln H)$. With the statistics reported in the paper, $corr(\ln H, \ln Y/H) = 0.96$. 

---
There are also general-equilibrium studies on uncertainty shocks to firms’ idiosyncratic productivity. Examples include Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), Bachmann and Bayer (2009), and Arellano, Bai, and Kehoe (2010). Analyzing a labor-market search model with heterogeneous firms, Schaal (2010) finds that changes in idiosyncratic productivity risk that firms face generate a positive correlation between unemployment and productivity (output per person). His mechanism mainly relies on hiring, entry, and exit decisions of firms and differs from that of this paper, which highlights labor supply of individuals with different levels of productivity and wealth.

The rest of this paper proceeds as follows. Section 2 provides evidence for cyclical variation in idiosyncratic wage risk using the PSID data. Section 3 lays out the model, while Section 4 determines parameter values and discusses the steady-state result. Section 5 assumes acyclical idiosyncratic wage risk and analyzes its impact on the model business cycle. Section 6 examines the implication of countercyclical risk. Section 7 concludes. Appendices explain the data and solution methods.

2 Evidence for Time-Varying Idiosyncratic Wage Risk

This section analyzes data in the PSID and provides evidence for cyclical variation in idiosyncratic wage risk. First, for each person-year observation, the hourly wage is computed as the ratio of annual labor income to annual total labor hours. Next, those individual wage data are fitted to the wage process of the model analyzed in this paper and the residuals are recorded. Idiosyncratic wage risk each year is defined as the cross-sectional dispersion of the residuals and its year-by-year variation is examined.

Specifically, the model’s wage process is derived using the following two equations. The first equation means that in the model, an individual hourly wage \( w_{i,t} \) (\( i \): individual and \( t \): time) is equal to \( w_t x_{i,t} \), where \( w_t \) is the equilibrium wage rate per efficiency unit of labor and \( x_{i,t} \) is person-specific labor productivity:

---

8 Appendix A1 explains details of the data and sample selection.
\[ \ln w_{i,t} = \ln w_t + \ln x_{i,t}. \] (1)

The second equation is the stochastic process of \( x_{i,t} \):

\[ \ln x_{i,t} = \rho_x \ln x_{i,t-1} + \varepsilon_{x,i,t}, \varepsilon_{x,i,t} \sim N(0, \sigma_{\varepsilon_x}^2). \] (2)

As shown by Chang and Kim (2006), (1) and (2) imply the following individual wage process:

\[ \ln w_{i,t} = \rho_x \ln w_{i,t-1} + (\ln w_t - \rho_x \ln w_{t-1}) + \varepsilon_{x,i,t}. \] (3)

It is well known that in practice, variables such as years of education and experience influence individual wages and wage growth (e.g., Card (1999), Heathcote, Perri, and Violante (2010)). This implies that individuals can forecast their wages using those variables, at least partially. By contrast, since the aim of this paper is to evaluate the impact of cyclical risk variation on business cycles, the model economy should be calibrated to changes in the pure risk that individuals face. This consideration leads me to identify idiosyncratic wage risk by estimating the following equation:

\[ \ln w_{i,t} = \rho_{x,t} \ln w_{i,t-1} + (\ln w_t - \rho_{x,t} \ln w_{t-1}) + Z_{i,t} \beta_t + \varepsilon_{x,i,t}, \] (4)

where \( Z_{i,t} \) includes education, experience, and experience-squared, defining experience as age minus education minus six. The regression is done each year, replacing the term \( (\ln w_t - \rho_{x,t} \ln w_{t-1}) \) with a constant. Idiosyncratic wage risk in each year is then calculated by \( \hat{\sigma}_{\varepsilon_x,t} = \text{std}(\varepsilon_{x,i,t}) \).

Table 1 presents the estimation result. As consistent with existing estimates obtained

---

\( (4) \) is estimated using ordinary least squares. A potential problem of endogenous selection is mitigated because only data of male heads of households are used. Heckman (1979)’s method is not directly applicable here because the equation includes a lagged variable and \( \sigma_{\varepsilon_x} \) changes each period.

The pooled estimation of (4) is also conducted, assuming that \( \rho_x \) is constant throughout. The estimated
by using an equation like (4), the identified idiosyncratic wage risk $\tilde{\sigma}_{\varepsilon_{x,t}}$ show an upward trend. Thus, to isolate its cyclical variation, the linear trend in $\tilde{\sigma}_{\varepsilon_{x,t}}$ is removed and then the percentage deviation from the trend is computed. Figure 1 shows this detrended result. Four empirical regularities characterize the cyclical component of idiosyncratic wage risk. First, idiosyncratic wage risk varies over time. The largest deviation from the trend is around 10%, and 5% fluctuations are frequent. The annual standard deviation of the detrended $\tilde{\sigma}_{\varepsilon_{x,t}}$ is 4.4%. Second, risk changes are persistent. Idiosyncratic wage risk typically stays above or below the trend for about two years, showing an annual first-order autocorrelation coefficient of 0.26. Third, risk variation is almost symmetric. Whether wage risk is above or below the trend does not substantially affect the size and persistence of changes in risk. Fourth, fluctuations in idiosyncratic wage risk show neither clear procyclicality nor countercyclicality. For example, wage risk declined during the 1981-1982 recession, whereas it increased during the 1990-91 recession. Based on these findings, Section 4 determines risk variation introduced into the model described below.

3 Model

To analyze the impact of changes in idiosyncratic wage risk on business cycles, including labor market dynamics, I augment a prototype incomplete asset markets model that incorporates a large number of individuals and a representative firm as follows. First, it is assumed that person-specific labor productivity changes stochastically. Second, individuals make employment choices each period. Hence, individuals endogenously choose labor supply $\tilde{\sigma}_{\varepsilon_{x,t}}$ are essentially the same as those obtained from the year-by-year estimation. The result makes sense because as shown in Table 1, the year-by-year estimation finds that $\rho_x$ rarely varies over time.  

11 This is not necessarily inconsistent with countercyclical income risk documented by Storesletten, Telmer, and Yaron (2004) because income and wage risk could move differently. Concerning this point, Heathcote, Perri, and Violante (2010) find that although the cross-sectional dispersion of income is countercyclical, the cross-sectional dispersion of wages does not show a clear cyclical pattern. This leads them to conclude that labor supply is the main driving force of fluctuations in the earnings’ dispersion. As idiosyncratic wage and income risk are essentially identified using the cross-sectional dispersion of wages and income respectively, Heathcote, Perri, and Violante (2010)’s finding indicates countercyclicality of income risk and no clear cyclicity of wage risk. However, it is interesting to see how countercyclical wage risk affects business cycles, and Section 6 examines the question.
under the presence of idiosyncratic wage risk, as in the models of Flodén and Linde (2001), Chang and Kim (2006, 2007), and Krusell, Mukoyama, Roserson, and Sahin (2010, 2011). Third, I introduce uncertainty shocks à la Bloom (2009) into this setting, and thus shocks to idiosyncratic labor productivity have a time-varying second moment. The combination of person-specific productivity and uncertainty shocks generates time-varying idiosyncratic wage risk in a parsimonious way. Below, I describe individuals, the representative firm, and equilibrium.

3.1 Individuals Facing Time-Varying Uninsured Idiosyncratic Wage Risk

There are a continuum of individuals of measure one in the model economy. Those individuals have the identical momentary utility function \( u(c, h) \), where \( c \) is consumption and \( h \) is labor hours. Labor is indivisible, and individuals choose to work for fixed hours or not to work at all: \( h \in \{h, 0\} \).\(^\text{12}\) Individuals earn labor income of \( wxh \), where \( w \) is the equilibrium wage rate per efficiency unit of labor and \( x \) is random, person-specific labor productivity.

Crucially, individuals face time-varying idiosyncratic wage risk because of uncertainty shocks: the volatility of shocks to idiosyncratic productivity, \( \sigma_{e_x} \), which is common to all individuals, changes over time. Following the convention of the literature on uncertainty shocks, I use \( \sigma_{e_x} \) to represent the volatility of shocks not to \( x \), but to \( x' \), and assume that individuals learn of the size of \( \sigma_{e_x} \) one period ahead.\(^\text{13,14}\) As asset markets are incomplete, individuals are unable to insure themselves against this varying wage risk. They save using a single asset \( k \) and earn (or pay if \( k < 0 \)) the equilibrium rental rate of capital \( r \). There is also a borrowing constraint, \( k \geq \bar{k} \ (\bar{k} < 0) \).

At the beginning of each period, individuals are characterized by \((k, x)\). Let \( z \) be ag-

\(^{12}\)See Hansen (1985) and Rogerson (1988) for a discussion of individual employment decisions with lotteries.

\(^{13}\)Variables with a prime are next period values.

\(^{14}\)This timing assumption captures the concept of risk. See, for example, Bloom (2009) and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012).
aggregate total factor productivity (TFP) and $\mu$ represent the joint distribution of $k$ and $x$ across individuals. The aggregate state includes $(z, \mu, \sigma_{\varepsilon})$. Observing these idiosyncratic and aggregate state, individuals choose $c, h, \text{ and } k'$.

Define $V(k, x; z, \mu, \sigma_{\varepsilon})$ as the beginning-of-the-period value for an individual characterized by $(k, x)$. This beginning-of-the-period value reflects the individual’s current employment choice:

$$V(k, x; z, \mu, \sigma_{\varepsilon}) = \max \left\{ V^E(k, x; z, \mu, \sigma_{\varepsilon}), V^N(k, x; z, \mu, \sigma_{\varepsilon}) \right\}. \tag{5}$$

The individual’s within-the-period value conditional on working (i.e., $h = \bar{h}$) is $V^E(k, x; z, \mu, \sigma_{\varepsilon})$, which satisfies:

$$V^E(k, x; z, \mu, \sigma_{\varepsilon}) = \max_{k'} \left\{ u(c, \bar{h}) + \beta E \left[ V(k', x'; z', \mu', \sigma'_{\varepsilon}) \mid x, z, \mu, \sigma_{\varepsilon} \right] \right\}, \tag{6}$$
subject to $c = w(z, \mu, \sigma_{\varepsilon})x\bar{h} + [1 + r(z, \mu, \sigma_{\varepsilon})]k - k'$ and $k' \geq \bar{k}$.

Here, $\beta$ is the discount factor and $\mu$ is assumed to evolve according to the equilibrium mapping, $\mu' = \Gamma(z, \mu, \sigma_{\varepsilon})$.

Similarly, $V^N(k, x; z, \mu, \sigma_{\varepsilon})$ is the individual’s within-the-period value conditional on not working (i.e., $h = 0$), and it is defined as:

$$V^N(k, x; z, \mu, \sigma_{\varepsilon}) = \max_{k'} \left\{ u(c, 0) + \beta E \left[ V(k', x'; z', \mu', \sigma'_{\varepsilon}) \mid x, z, \mu, \sigma_{\varepsilon} \right] \right\}, \tag{7}$$
subject to $c = [1 + r(z, \mu, \sigma_{\varepsilon})]k - k'$ and $k' \geq \bar{k}$.

### 3.2 Representative Firm

A representative firm produces the final good $Y$ using capital $K$ and labor $L$ as inputs. The production function is $Y = zF(K, L)$, and it exhibits constant returns to scale. Given
\( r(z, \mu, \sigma_{\varepsilon_x}) \) and \( w(z, \mu, \sigma_{\varepsilon_x}) \), the firm maximizes static profits by choosing \( K(z, \mu, \sigma_{\varepsilon_x}) \) and \( L(z, \mu, \sigma_{\varepsilon_x}) \). The first-order conditions are

\[
\begin{align*}
\frac{r(z, \mu, \sigma_{\varepsilon_x})}{z} &= F_K(K, L) - \delta, \\
\frac{w(z, \mu, \sigma_{\varepsilon_x})}{z} &= F_L(K, L),
\end{align*}
\]

where \( \delta \) is the capital depreciation rate.

### 3.3 General Equilibrium

A recursive competitive equilibrium is a set of functions,

\[
\left( w, r, V^E, V^N, V, c, k', h, K, L, \Gamma \right),
\]

that satisfy the following conditions.

1. Individuals’ Optimization:

\[
V(k, x; z, \mu, \sigma_{\varepsilon_x}), V^E(k, x; z, \mu, \sigma_{\varepsilon_x}), \text{ and } V^N(k, x; z, \mu, \sigma_{\varepsilon_x}) \text{ satisfy } (5), (6), \text{ and } (7), \text{ while }
\]

\[
c(k, x; z, \mu, \sigma_{\varepsilon_x}), k'(k, x; z, \mu, \sigma_{\varepsilon_x}), \text{ and } h(k, x; z, \mu, \sigma_{\varepsilon_x}) \text{ are the associated policy functions.}
\]

2. Firms’ Optimization:

The representative firm chooses \( K(z, \mu, \sigma_{\varepsilon_x}) \) and \( L(z, \mu, \sigma_{\varepsilon_x}) \) to satisfy (8) and (9).

3. Labor Market Clearing:

\[
L(z, \mu, \sigma_{\varepsilon_x}) = \int x h(k, x; z, \mu, \sigma_{\varepsilon_x}) \mu([dk \times dx])
\]

4. Capital Market Clearing:
\[
K(z, \mu, \sigma_{\epsilon_x}) = \int k\mu([dk \times dx])
\]

5. Goods Market Clearing:
\[
\int \{k'(k, x; z, \mu, \sigma_{\epsilon_x}) + c(k, x; z, \mu, \sigma_{\epsilon_x})\} \mu([dk \times dx]) = zF(K(z, \mu, \sigma_{\epsilon_x}), L(z, \mu, \sigma_{\epsilon_x})) + (1 - \delta) \int k\mu([dk \times dx])
\]

6. Evolution of Individual Distribution:
\[
\Gamma(z, \mu, \sigma_{\epsilon_x}) \text{ is consistent with individual decisions and the stochastic processes of } x, \ z, \ \text{and } \sigma_{\epsilon_x}. \text{ Specifically, for all } B \subseteq K,
\mu'(B, x') = \int \{k'(k, x; z, \mu, \sigma_{\epsilon_x}) \in B\} \pi_x(x'|x, \sigma_{\epsilon_x}) \mu([dk' \times dx']),
\text{where } \pi_x(x'|x, \sigma_{\epsilon_x}) \text{ is the transition probability from } x \text{ to } x' \text{ under } \sigma_{\epsilon_x}.
\]

4 Calibration and the Steady State

This section discusses parameter values and the steady-state result. First, I determine values of parameters not concerning idiosyncratic labor productivity \( x \). Those values are chosen so that the model economy replicates several aspects of the U.S. economy, and therefore they are comparable to the values used in existing incomplete asset markets models (e.g., Krusell and Smith (1998) and Chang and Kim (2007)). Next, parameter values on \( x \) are chosen so that moments of individual wages generated in the model match those found in the PSID data. The steady-state result is briefly presented at the end of this section.

4.1 Parameter Values Other Than Idiosyncratic Labor Productivity

A period corresponds to one quarter. The discount factor \( \beta \) is 0.97962, and together with other parameter values, this implies a one percent quarterly rental rate of capital at the steady state. The momentary utility when individuals work is \( u(c, h) = \ln c - B \). When they
do not work, $u(c, h) = \ln c$. The disutility parameter is set to $B = 0.9897$, generating a 60% steady-state employment rate. This target is chosen based on the U.S. average employment-population ratio during the period of 1948Q1–2009Q3. Individuals use one third of their time when working ($\bar{h} = 1/3$). They need to hold assets $k \geq \bar{k} = -2.0$. As shown by Chang and Kim (2007) and also discussed below, the model economy with this borrowing limit produces a wealth distribution close to that found in the data.\footnote{The average wealth is 11.59 and average annual labor income is 3.74 at the steady state. Therefore, individuals can borrow up to 17% of the mean wealth and 53% of the mean annual income. The latter is comparable to the borrowing limit set by Krusell and Smith (1998) when they allow borrowing: 50% of the mean annual income.}

The production side of the model is essentially that of the neoclassical stochastic growth model, and the calibration follows previous studies, such as Kydland and Prescott (1982) and Prescott (1986). In particular, the production function is $Y = zK^{1-\alpha}L^\alpha$ and labor’s share $\alpha$ is 0.64. The capital depreciation rate $\delta$ is set to 0.025. Aggregate TFP $z$ follows an AR(1) process, $\log z' = \rho_z \log z + \epsilon_z$, where $\epsilon_z \sim N(0, \sigma_{\epsilon_z}^2)$. As in Cooley and Prescott (1995), $\rho_z = 0.95$ and $\sigma_{\epsilon_z} = 0.007$. Table 2 lists these parameter values.

### 4.2 Parameter Values on Idiosyncratic Labor Productivity

There are four parameters concerning idiosyncratic labor productivity $x$. Since $x$ follows an AR(1) process as in (2), $\rho_x$ (the persistence of $x$) and $\bar{\sigma}_x$ (the steady-state volatility of shocks to $x$) need to be determined. The other two parameters involve fluctuations in $\sigma_{\epsilon_x}$. Specifically, the following three risk states are introduced: high (H), middle (M), and low (L). They are a Markov chain. As discussed in Section 2, $\sigma_{\epsilon_x}$ does not show strong cyclicality, and thus as a benchmark, it is assumed that $\sigma_{\epsilon_x}$ evolves independently of aggregate TFP $z$. Further, the symmetric nature of risk variation suggests that a symmetric transition matrix is a reasonable assumption. Hence, a risk state stays unchanged with a quarterly probability of $\rho_{\sigma_x}$ and moves to each of the other two states with a probability of $(1 - \rho_{\sigma_x})/2$. Also, the size of risk variation should be symmetric around the mean: the high risk $\sigma_{\epsilon_x,H} = (1+\lambda)\bar{\sigma}_x$, the middle risk $\sigma_{\epsilon_x,M} = \bar{\sigma}_x$, and the low risk $\sigma_{\epsilon_x,L} = (1-\lambda)\bar{\sigma}_x$. These four parameter values
\((\rho_x, \sigma_{\varepsilon_x}, \rho_{\sigma_{\varepsilon_x}}, \lambda)\) are determined, so that annualized individual wages in the model economy reproduce the persistence and cross-sectional dispersion found in the PSID individual wage data.\(^{16}\)

The first column of Table 3A lists four moments compared between the PSID and model wage data. The two moments, \(\hat{\rho}_x\) and \(\hat{\sigma}_{\varepsilon_x}\), are estimated from (3), and they are mainly used to pin down the values of \(\rho_x\) and \(\sigma_{\varepsilon_x}\).\(^{17}\) The other two moments are the annual standard deviation \(\text{std}(\hat{\sigma}_{\varepsilon_x,t})\) and first-order autocorrelation coefficient \(\text{corr}(\hat{\sigma}_{\varepsilon_x,t}, \hat{\sigma}_{\varepsilon_x,t-1})\) of the estimated idiosyncratic wage risk. As for the PSID moments, the ones presented in Section 2 are used. Those moments represent time variation in idiosyncratic wage risk that individuals actually face because they are computed using the risk remained even after controlling for the ability of demographic variables to forecast individual wages. By contrast, the model moments are computed by estimating (3) each year. The values of \(\rho_{\sigma_{\varepsilon_x}}\) and \(\lambda\) are determined so that \(\text{std}(\hat{\sigma}_{\varepsilon_x,t})\) and \(\text{corr}(\hat{\sigma}_{\varepsilon_x,t}, \hat{\sigma}_{\varepsilon_x,t-1})\) of the model data match those of the PSID data.

The second and third columns of Table 3A compare the above four moments of the PSID (“PSID Data”) and varying risk model (“Varying Risk”). The varying risk model is parameterized using the values in the third column of Table 3B. As shown, the moments of the calibrated varying-risk model closely match those of the PSID data. Specifically, the PSID data imply that the annual persistence of idiosyncratic labor productivity \(\hat{\rho}_x\) is 0.822, and the model replicates this persistence with a quarterly value of \(\rho_x = 0.915.\(^{18}\) Further, the estimate of the average annual value of idiosyncratic wage risk \(\hat{\sigma}_{\varepsilon_x}\) is 0.287 in the PSID data, and the model replicates this size of wage risk with a quarterly value of \(\sigma_{\varepsilon_x} = 0.242\). These values of \(\rho_x\) and \(\sigma_{\varepsilon_x}\) are comparable to those set in existing models assuming constant wage risk. For example, Chang and Kim (2007) use \(\rho_x = 0.929\) and \(\sigma_{\varepsilon_x} = 0.227\) quarterly,

\(^{16}\)It is not straightforward to choose the model’s quarterly parameter values using the annual PSID data because the conventional transformation yields inaccurate quarterly estimates. For example, it is common to calculate the quarterly \(\rho_x\) by raising the estimated annual value to the one-fourth power. However, model simulation shows that the model parameterized in this manner implies individual wages that are more persistent than those of the PSID data. See footnote 18. To overcome such a problem, a simulation-based method is used and all the four parameter values \((\rho_x, \sigma_{\varepsilon_x}, \rho_{\sigma_{\varepsilon_x}}, \lambda)\) are jointly chosen.

\(^{17}\)The pooled OLS is applied to both the model and PSID data.

\(^{18}\)This is lower than the value implied by the conventional transformation: \(0.822^{1/4} = 0.952\).
whereas Krusell, Mukoyama, Roserson, and Sahin (2011) target $\rho_x = 0.92$ and $\sigma_{\varepsilon_x} = 0.21$ annually for their monthly model.

Concerning risk fluctuations, idiosyncratic wage risk estimated with the PSID data varies by an annual standard deviation $std(\hat{\sigma}_{\varepsilon_x,t})$ of 4.4% and shows an annual first-order autocorrelation $corr(\hat{\sigma}_{\varepsilon_x,t}, \hat{\sigma}_{\varepsilon_x,t-1})$ of 0.262. These findings indicate that risk states stay unchanged with a quarterly probability of $\rho_{\sigma_{\varepsilon_x}} = 0.52$ in the model, while the quarterly risk variation is 10% of the steady-state level ($\lambda = 0.10$). Under these parameter values, the varying risk model successfully produces fluctuations in idiosyncratic wage risk comparable to those seen in the PSID data.

In contrast, the constant risk model fails to replicate the risk variation of the PSID data. As shown in the last column of Table 3A (“Constant Risk”), the risk fluctuations estimated using the data of the constant risk model are substantially smaller and less persistent compared to those of the PSID data. This finding provides further evidence for cyclical variation in idiosyncratic wage risk. In Section 5, I compare the business cycles of these varying and constant risk models.

### 4.3 Steady State

As Chang and Kim (2007) show, the heterogeneous-agent model analyzed here generates inequality of wealth and labor income comparable to that found in the U.S. economy. The Gini coefficient of annual labor income is 0.54 in the model. The PSID data suggests that it is 0.65 at the individual level in 1991. Further, analyzing the Survey of Consumer Finances (SCF) in 1992, Díaz-Giménez, Quadrini, and Ríos-Rull (1997) find that the household-level Gini coefficient is 0.63. As for the wealth inequality, the Gini coefficient in the model is 0.69 at the individual level. Since it is difficult to define individual wealth in the actual economy,

---

19 Because of endogeneous employment choice, $\hat{\sigma}_{\varepsilon_x,t} = std(\hat{\varepsilon}_{x,i,t})$ obtained by estimating (3) shows some variation even in the constant risk model.

20 Appendix B1 explains the solution method for the steady state. As the model frequency is quarterly, the steady state is simulated with 3,000 individuals to generate the distribution of annual labor income.

21 Appendix A2 describes the PSID data used to compute the Gini coefficient.
this individual-level wealth inequality is compared with an existing finding on household-level inequality. According to Díaz-Giménez, Quadrini, and Ríos-Rull (1997), the Gini coefficient of wealth is 0.78 in the 1992 SCF. Therefore, the model analyzed here replicates the degree of inequality in wealth and labor income in the data reasonably well.

5 Business Cycle Results

In this section, I first compare business cycle statistics of the varying and constant risk models calibrated in the last section and show that the hours-productivity puzzle can be resolved by introducing time variation in idiosyncratic wage risk identified from the PSID wage data.\footnote{To analyze the model’s business cycle, I extend the solution method developed by Krusell and Smith (1997, 1998). Appendix B2 explains the extended solution method.} Next, to trace the result, I analyze how the varying risk model responds to a rise in idiosyncratic wage risk. It is found that elevated wage risk generates a negative comovement of total hours worked with average labor productivity. A further analysis shows that the key to this negative comovement is opposing employment responses between high- and low-productivity individuals. Specifically, following a rise in idiosyncratic wage risk, the employment of individuals with low productivity first increases and then decreases. The employment of high-productivity individuals moves inversely to a lesser extent. As a result, total labor hours first increase and then decrease. Average labor productivity initially declines and later rises.

Moreover, I show that a decline in total factor productivity (TFP) generates a positive correlation between hours worked and labor productivity at the aggregate level. This is because such a shock reduces the demand for labor without much affecting labor supply, and thus it decreases the employment of individuals with both high and low productivity. In the varying risk model, the negative comovement produced by variation in idiosyncratic wage risk dominates the positive one generated by aggregate TFP shocks, and thus a weakly negative correlation of hours and productivity is observed. By contrast, the constant risk model.
which only incorporates aggregate TFP shocks, generates a strong, positive correlation.

5.1 Time-Varying Idiosyncratic Wage Risk and Business Cycles

Table 4 compares business cycle properties of the U.S. economy and those of the two model economies parametrized in Section 4. The column labeled “U.S. Economy” shows the business cycle statistics of the U.S. In the column labeled “Varying Risk,” I present the result of the model incorporating variation in both aggregate TFP and idiosyncratic wage risk. The column “Constant Risk” shows the result when including only aggregate TFP shocks and assuming constant idiosyncratic wage risk. These two model results are obtained through 3,500-period simulations (discarding the first 500 periods) using the same sequence of aggregate TFP. Hence, the difference between the two sets of results shows the impact of time-varying idiosyncratic wage risk on aggregate fluctuations.

As shown, fluctuations in idiosyncratic wage risk are essential to replicating the key labor market regularity of the U.S. Most importantly, as consistent with the data, the varying risk model generates a weakly negative correlation found between total hours worked and average labor productivity (output per labor hour) (–0.38 in the model compared with –0.32 in the data). On the other hand, the constant risk model produces a counterfactually strong positive correlation (0.83). Further, when compared to the constant risk model, the varying risk model shows the volatility of total hours worked and the correlation of output with labor productivity that are closer to the U.S. data. Also, in the presence of risk variation, the volatility of labor productivity and the correlation between output and hours move in the right direction, although they overshoot the data.

In contrast, the volatilities and comovements of output, consumption, and investment are largely the same in the varying and constant risk models. Based on these results, I

\[ H \equiv \int h(k, x; z, \mu, \sigma_{\epsilon_x}) \mu([dk \times dx]). \]

This is different from efficiency-weighted total labor \[ L \equiv \int xh(k, x; z, \mu, \sigma_{\epsilon_x}) \mu([dk \times dx]). \]

---

23 The U.S. macroeconomic data are taken from the source listed in Appendix A3.
24 Total hours worked are computed by \[ H \equiv \int h(k, x; z, \mu, \sigma_{\epsilon_x}) \mu([dk \times dx]). \] This is different from efficiency-weighted total labor \[ L \equiv \int xh(k, x; z, \mu, \sigma_{\epsilon_x}) \mu([dk \times dx]). \]
conclude that introducing time variation in idiosyncratic wage risk improves the model’s overall performance, especially concerning the labor market dynamics.

5.2 Responses to Idiosyncratic Wage Risk Shocks

To obtain an intuition for the above results, I analyze how the varying risk model responds to an increase in idiosyncratic wage risk $\sigma_{\varepsilon x}$. As the upper-left panel of Figure 2 shows, the experiment assumes that $\sigma_{\varepsilon x}$ rises by 10% in period 0 relative to its pre-shock (steady-state) level, stays at that level until period 2, and then returns to the pre-shock level in period 3.\textsuperscript{25} Aggregate TFP $z$ is held constant at its steady-state level.

5.2.1 Responses of Aggregate Variables

The remaining panels of Figure 2 show the responses of output, total hours worked, and average labor productivity to the rise in idiosyncratic wage risk introduced above. Output increases from period 0 to period 3 and then gradually decreases. Total hours worked increase in period 0 and then start to decrease in period 1. The trough of hours occurs in period 3. Therefore, average labor productivity (output per labor hour) first falls slightly, and then starts to rise in period 1. It reaches the peak in period 3, thereafter gradually returning to its pre-shock level.

These results provide explanations for the difference in business cycles between the varying and constant risk models summarized in Table 4. In particular, they reveal that changes in idiosyncratic wage risk generate a negative comovement of total hours worked with average labor productivity. This breaks the strong positive comovement found in the constant risk model, and a weakly negative correlation is observed in the varying risk model.

\textsuperscript{25}Idiosyncratic wage risk $\sigma_{\varepsilon x}$ is in the middle state until period –1 and then enters the high state during periods 0–2. Afterward, $\sigma_{\varepsilon x}$ returns to the middle state.
5.2.2 **Micro-level Responses**

Next, I explain the micro-level changes in behavior generating the macroeconomic results explained above. To do so, I first show how individuals’ employment choices in equilibrium vary with individual productivity $x$ and wealth $k$ (Figure 3).\(^{26}\) They follow a threshold rule. Given $k$, individuals whose $x$ exceeds a certain level choose to work. Also, given $x$, individuals who hold $k$ less than a certain level elect to work. A rise in $\sigma_{\varepsilon_x}$ shifts those thresholds and the distribution of $k$ and $x$ across individuals. This leads to changes in the number and composition of individuals who work, thereby moving total hours worked and average labor productivity.

In particular, I highlight two distinct mechanisms through which increased idiosyncratic wage risk $\sigma_{\varepsilon_x}$ influences individual employment choices. The first is the *precautionary effect*. When individuals become more uncertain about their future wages by learning that $\sigma_{\varepsilon_x}$ in the next period is unusually high, individuals change their current labor supply. In equilibrium, this and the resulting price changes switch current employment decisions of some individuals, suggesting shifts in the thresholds of Figure 3. The second is the *distribution effect*. In the following period, the elevated $\sigma_{\varepsilon_x}$ increases the cross-sectional dispersion of shocks to idiosyncratic productivity. As a result, the distribution of productivity and wealth across individuals shifts, and in equilibrium the price movements shift the thresholds. Hence, the distribution effect also overturns employment decisions of some individuals.

First, the precautionary effect increases the employment of individuals currently having low productivity and to a lesser extent, decreases the employment of individuals with high productivity. Because individual employment choices follow the threshold rule described above, these employment responses are attributed to changes in the employment choices of individuals who were close to indifferent towards working before the rise in idiosyncratic wage risk. When uncertainty on their future wages rises, individuals increase their savings in

\(^{26}\)Aggregate state $(z, \mu, \sigma_{\varepsilon_x})$ is held constant at their steady-state values.
an effort to self-insure. This itself increases current labor supply. However, in equilibrium, such an increase in labor supply raises the current rental rate of capital and decreases the current wage rate, both of which affect individual employment choices.

I find that changes in employment decisions differ between individuals with high and low productivity. Figure 4 illustrates this by showing how individuals around the threshold wealth change their employment choices (0: not work and 1: work), asset holdings, and consumption in equilibrium, responding to the rise in idiosyncratic wage risk introduced as in the upper-left panel of Figure 2. The left panel shows the response of individuals with relatively low current labor productivity (25th percentile in idiosyncratic labor productivity), whereas the right panel depicts the response of individuals with relatively high productivity (75th percentile). Note that the responses in period 0 arise solely from the precautionary effect because the distribution starts to shift in period 1.

As shown, in period 0, the employment of these two individuals move in opposite directions. Low-productivity individuals close to the threshold have low levels of wealth and are close to their borrowing limit. Hence, when uncertainty on their future wages increases, the volatility of their future consumption also rises substantially, strengthening incentive to self-insure. Further, since those individuals are borrowers \((k < 0)\), the rise in the rental rate reduces their wealth. Hence, they start to work when risk rises in period 0. By contrast, high-productivity individuals close to the threshold are wealthy and thus well-insured. Moreover, those individuals are savers \((k > 0)\), and the rise in the rental rate increases their wealth. Since the wage rate falls in equilibrium, they stop working.

At the aggregate level, total labor hours increase in period 0. Average labor productivity falls, especially by more than the decline in the marginal product of efficiency-weighted labor, because the employment share of low-productivity individuals increases. Hence, hours and productivity move in opposite directions in period 0, as shown in Figure 2. This completes

---

\(^{27}\) As discussed below, the wage rate in the following period falls, which further increases current labor supply.

\(^{28}\) The rental rate continues to be higher than usual in the following periods.
the explanation for the precautionary effect.

Second, as opposed to the precautionary effect, the distribution effect decreases the employment of individuals with low current productivity and to a lesser degree, it increases the employment of individuals with high productivity. As Figure 5 shows, the productivity distribution gradually becomes more dispersed during periods 1–3 because elevated $\sigma_{\varepsilon}$ increases the cross-sectional dispersion of shocks to idiosyncratic productivity. Overall, there is a tendency for individuals to move from the middle of the distribution to the lower and higher levels of productivity. Importantly, since the persistence in idiosyncratic productivity generates a positive correlation between productivity and wealth at the individual level, this shift in the productivity distribution substantially alters the joint distribution of productivity and wealth.

Specifically, individuals with low productivity in period 3 tend to hold more wealth than those in period 0.\textsuperscript{29} To illustrate this, Figure 6 compares the cumulative wealth distribution in period 0 and that in 3. It uses individuals at the 25th productivity percentile as an example and focuses on the asset level close to the threshold for employment. As shown, the cumulative asset distribution shifts down, implying that there is a decrease in the number of individuals who hold assets less than the threshold.\textsuperscript{30} Hence, the employment of individuals with this productivity decreases from period 0 to 3. This effect is present for any productivity below the mean level. The opposite mechanism works for individuals whose productivity is above the mean. Thus, more of those individuals choose to work during periods 1–3 compared to period 0.

In equilibrium, the increase in the labor supply of individuals with high productivity pushes down the wage rate. The employment of those individuals increases, but it is less than the decline in the employment of individuals with low productivity. Hence, total labor hours decreases, while average labor productivity rises, generating a negative comovement of

\textsuperscript{29}This is partly because those individuals accumulated wealth during periods 0–2 through the precautionary effect.

\textsuperscript{30}The threshold itself declines relative to period 0 because the equilibrium wage rate falls.
the two. This is the explanation for the distribution effect. In periods 1–3, the distribution effect dominates the precautionary effect.\(^{31}\) Thus, hours decrease and labor productivity rises.

To summarize, following a rise in idiosyncratic wage risk, the employment of individuals with low productivity first increases and then decreases. The employment of individuals with high productivity shows the opposite pattern and also smaller fluctuations. Hence, total hours worked first increase and then decrease. In contrast, labor productivity measured at the aggregate level initially falls and later rises. The result is a negative correlation between hours and productivity.

5.3 Responses to Aggregate TFP Shocks

This subsection shows how the varying risk model responds to an exogenous decline in aggregate TFP. The result explains why the constant risk model, essentially that of Chang and Kim (2007), generates a strong, positive comovement of total hours worked with average labor productivity.\(^{32}\)

Figure 7 presents the result of a simulation in which aggregate TFP \(z\) exogenously drops by 1.67\% in period 0, as shown in the upper-left panel. Idiosyncratic wage risk \(\sigma_{\epsilon_x}\) stays constant throughout. The other panels show the responses of output, total hours worked, and average labor productivity. In contrast to the responses to the rise in \(\sigma_{\epsilon_x}\) presented in Figure 2, hours and labor productivity move in the same direction.

Figure 8 illustrates the responses of individuals close to the employment threshold in equilibrium. The left panel shows the response of individuals with relatively low current productivity (25th percentile in idiosyncratic labor productivity), whereas the response of individuals with relatively high productivity (75th percentile) is presented in the right panel. Since the decline in aggregate TFP primarily reduces labor demand and rarely affects labor supply, both individuals stop working in equilibrium, contrasting with the opposing employ-

\(^{31}\) The precautionary effect of the elevated risk disappears in period 3.

\(^{32}\) Further analysis of the model is in Takahashi (2013).
ment responses to the rise in idiosyncratic wage risk shown in Figure 4. Hence, the TFP shock rarely changes the employment shares of individuals with high and low productivity. The aggregate result is that total hours worked and average labor productivity fall together.

To sum up, a change in aggregate TFP generates a positive correlation between total hours worked and average labor productivity in the model analyzed here. A strong comovement of the two is maintained in the constant risk model because aggregate TFP shocks are solo exogenous disturbances there. By contrast, the varying risk model generates a weakly negative correlation because under the proper calibration, a negative comovement generated by variation in idiosyncratic wage risk dominates the positive one produced by aggregate TFP shocks.

6 Implications of Countercyclical Wage Risk

Up to this point, I have assumed that idiosyncratic wage risk and aggregate TFP evolve independently. Although the finding in Section 2 motivates this assumption, countercyclical risk is also an interesting case. First, idiosyncratic wage risk in fact rose in the early 1990 recession. It would be interesting to see how such a recession is different from one without elevated risk in the model analyzed here. Second, previous studies typically assume countercyclical income risk. As discussed earlier, this is consistent with acyclical wage risk under endogenous labor supply.\textsuperscript{33} In order to explore the impact of cyclicality of risk, however, I calculate business cycle statistics of the model assuming countercyclical wage risk and compare them with those of the model with acyclical risk.

\textsuperscript{33}See footnote 11.
6.1 Recessions With and Without Elevated Idiosyncratic Wage Risk

As shown in Figure 1, idiosyncratic wage risk rose during the early 1990s recession in the U.S., while it stayed low in the 1981–1982 recession. This subsection examines the model’s prediction for the output and employment dynamics in these two types of recessions and compares it with the actual data.

To analyze a smooth change in idiosyncratic wage risk $\sigma_{\varepsilon_x}$ as seen in practice, I compute the average response of the varying risk model to the average change in $\sigma_{\varepsilon_x}$ across 500 simulations. In each simulation, $\sigma_{\varepsilon_x}$ moves to the next highest risk state in period 251, and at the same time, aggregate TFP $z$ drops by one grid point. At other times, $\sigma_{\varepsilon_x}$ and $z$ evolve according to their independent stochastic processes. The average response across those 500 simulations is first taken and then the percentage deviation from the pre-shock average (230–250 periods) is computed. As shown in the upper two panels of Figure 9, the initial increase in $\sigma_{\varepsilon_x}$ is 4.59%, while the decline in $z$ is 1.5%. I compare the responses of output and total hours worked of this experiment with those when only $z$ declines in period 251.

The result is presented in the lower panels of Figure 9. Compared with when only aggregate TFP declines, output recovers faster when idiosyncratic wage risk also rises simultaneously. However, the recovery of total hours worked is delayed. Thus, the recession characterized with elevated risk shows a jobless recovery when compared to the recession without such increased risk.

This prediction of the model is largely consistent with the U.S. experience. Figure 10 shows cyclical components of output and total hours worked in the U.S. between 1975Q1 and 2009Q3. During the recovery following the early 1990s recession, hours lagged output. In contrast, output and hours recovered together after the 1981–1982 recession. These results

\footnote{If $\sigma_{\varepsilon_x}$ was in the high state in period 250, then it stays in that state in period 251. Similarly, if $z$ was the lowest in period 250, then it stays the same in period 251.}
also show that cyclical variation in idiosyncratic wage risk plays an important role for labor market dynamics over the business cycle.

6.2 Countercyclical Idiosyncratic Wage Risk and Business Cycles

Next, I evaluate the impact of countercyclical idiosyncratic wage risk on overall business cycles. Countercyclical risk is introduced to the model economy by assuming that idiosyncratic wage risk \( \sigma_{\varepsilon_x} \) moves with aggregate TFP \( z \) as follows. When \( z \) is around its mean (approximately \( \pm 1.7\% \) relative to the steady-state level), \( \sigma_{\varepsilon_x} = \sigma_{\varepsilon_x,M} = \bar{\sigma}_{\varepsilon_x} \). When \( z \) is lower than this range, \( \sigma_{\varepsilon_x} = \sigma_{\varepsilon_x,H} = (1 + \lambda) \bar{\sigma}_{\varepsilon_x} \). When \( z \) exceeds the range, \( \sigma_{\varepsilon_x} = \sigma_{\varepsilon_x,L} = (1 - \lambda) \bar{\sigma}_{\varepsilon_x} \).

All the parameter values are the same as those set in Section 4, including \( \bar{\sigma}_{\varepsilon_x} = 0.242 \) and \( \lambda = 0.10 \), except that here \( \rho_{\sigma_{\varepsilon_x}} \) (the persistence of \( \sigma_{\varepsilon_x} \)) is determined by \( z \)’s process. The business cycle statistics of this countercyclical risk model are shown in the column labeled “Countercyclical Risk” of Table 5.

Importantly, the result of the countercyclical risk model constructed above is not directly comparable with the result of the varying risk model calibrated in Section 4. This is because the movements of idiosyncratic wage risk \( \sigma_{\varepsilon_x} \) are quite different between the two models. In particular, \( \sigma_{\varepsilon_x} \) in the countercyclical risk model is substantially less volatile and more persistent than \( \sigma_{\varepsilon_x} \) in the varying risk model. To ensure the comparability of the two models, I reset \( \lambda \) and \( \rho_{\sigma_{\varepsilon_x}} \) of the varying risk model, so that \( \text{std}(\sigma_{\varepsilon_x}) \) and \( \text{corr}(\sigma_{\varepsilon_x}, \sigma_{\varepsilon_x,-1}) \) match those implied by the countercyclical risk model. The result is \( \lambda = 0.067 \) and \( \rho_{\sigma_{\varepsilon_x}} = 0.9225 \).

All other parameter values are the same as those of the countercyclical risk model. In Table 5, the column labeled “Recalibrated Varying Risk” presents the result of this recalibrated model with acyclical wage risk. For comparison, the result of the constant risk model is listed again in the column labeled “Constant Risk.”

As shown, both acyclical and countercyclical wage risk move the model’s labor market statistics in the same direction, although countercyclical risk has a smaller impact. When compared to the constant risk model, the volatilities of total hours worked and average
labor productivity both increase, while the correlations among output, hours, and labor productivity all decrease. In particular, the correlation between hours and productivity is substantially reduced in both the recalibrated varying and countercyclical risk models. These findings suggest that the key to the result of this paper is not acyclicity of wage risk, but fluctuations in wage risk.

7 Conclusion

Individuals face time-varying idiosyncratic wage risk. This paper evaluates the impact of such risk variation on business cycles using a model in which individuals make discrete labor supply choices each period in the presence of person-specific wage risk. Toward a quantitative assessment, I identify actual fluctuations in idiosyncratic wage risk using the PSID data. I find that the introduction of cyclical variation in risk improves the model’s performance in several dimensions. Most notably, it resolves the counterfactually strong comovement of total hours worked with average labor productivity typically found in equilibrium business cycle models. The calibrated varying risk model replicates the weakly negative correlation between the two found in the U.S. data, whereas the constant risk model generates a counterfactually strong positive correlation.

References


<table>
<thead>
<tr>
<th>Year</th>
<th>$\hat{\rho}_{x,t}$</th>
<th>$\hat{\sigma}_{\varepsilon,t}$</th>
<th>Obs.</th>
<th>Adj. $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1975</td>
<td>0.741 (0.017)</td>
<td>0.247</td>
<td>1604</td>
<td>0.690</td>
</tr>
<tr>
<td>1976</td>
<td>0.813 (0.016)</td>
<td>0.258</td>
<td>1742</td>
<td>0.700</td>
</tr>
<tr>
<td>1977</td>
<td>0.730 (0.014)</td>
<td>0.251</td>
<td>1858</td>
<td>0.689</td>
</tr>
<tr>
<td>1978</td>
<td>0.749 (0.016)</td>
<td>0.269</td>
<td>1957</td>
<td>0.657</td>
</tr>
<tr>
<td>1979</td>
<td>0.744 (0.017)</td>
<td>0.302</td>
<td>2011</td>
<td>0.611</td>
</tr>
<tr>
<td>1980</td>
<td>0.728 (0.015)</td>
<td>0.284</td>
<td>2055</td>
<td>0.662</td>
</tr>
<tr>
<td>1981</td>
<td>0.762 (0.014)</td>
<td>0.271</td>
<td>2083</td>
<td>0.693</td>
</tr>
<tr>
<td>1982</td>
<td>0.795 (0.014)</td>
<td>0.270</td>
<td>2039</td>
<td>0.716</td>
</tr>
<tr>
<td>1983</td>
<td>0.751 (0.013)</td>
<td>0.267</td>
<td>2066</td>
<td>0.724</td>
</tr>
<tr>
<td>1984</td>
<td>0.749 (0.013)</td>
<td>0.288</td>
<td>2197</td>
<td>0.690</td>
</tr>
<tr>
<td>1985</td>
<td>0.764 (0.013)</td>
<td>0.295</td>
<td>2289</td>
<td>0.694</td>
</tr>
<tr>
<td>1986</td>
<td>0.771 (0.013)</td>
<td>0.299</td>
<td>2365</td>
<td>0.705</td>
</tr>
<tr>
<td>1987</td>
<td>0.805 (0.012)</td>
<td>0.282</td>
<td>2372</td>
<td>0.748</td>
</tr>
<tr>
<td>1988</td>
<td>0.754 (0.012)</td>
<td>0.292</td>
<td>2420</td>
<td>0.725</td>
</tr>
<tr>
<td>1989</td>
<td>0.781 (0.012)</td>
<td>0.276</td>
<td>2432</td>
<td>0.752</td>
</tr>
<tr>
<td>1990</td>
<td>0.774 (0.012)</td>
<td>0.297</td>
<td>2437</td>
<td>0.719</td>
</tr>
<tr>
<td>1991</td>
<td>0.776 (0.013)</td>
<td>0.309</td>
<td>2411</td>
<td>0.703</td>
</tr>
</tbody>
</table>

Note: This table reports the result from year-by-year OLS regressions of (4). The estimated coefficients on the past wage, $\hat{\rho}_{x,t}$, are shown with their standard errors in parentheses. Idiosyncratic wage risk is calculated by $\hat{\sigma}_{\varepsilon,t} = std(\hat{\varepsilon}_{x,i,t})$. 
Table 2: Values of Parameters Other Than Idiosyncratic Labor Productivity $x$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>discount factor</td>
<td>0.9836</td>
</tr>
<tr>
<td>$B$</td>
<td>disutility parameter</td>
<td>0.9897</td>
</tr>
<tr>
<td>$\bar{h}$</td>
<td>working hours</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>$\bar{k}$</td>
<td>borrowing limit</td>
<td>$-2.0$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>labor’s share</td>
<td>0.64</td>
</tr>
<tr>
<td>$\delta$</td>
<td>capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>persistence in aggregate TFP</td>
<td>0.950</td>
</tr>
<tr>
<td>$\sigma_{\varepsilon}$</td>
<td>std of aggregate TFP shocks</td>
<td>0.007</td>
</tr>
</tbody>
</table>

Note: This table lists the model’s parameter values not concerning idiosyncratic labor productivity $x$. A period in the model corresponds to one quarter.
Table 3: Moments for Calibration and Values of Parameters on Idiosyncratic Labor Productivity $x$

<table>
<thead>
<tr>
<th></th>
<th>U.S. Economy</th>
<th>Model Economy</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PSID Data</td>
<td>Varying Risk</td>
<td>Constant Risk</td>
<td></td>
</tr>
<tr>
<td>A. Moments (Annual)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\rho}_x$</td>
<td>0.822</td>
<td>0.822</td>
<td>0.822</td>
<td></td>
</tr>
<tr>
<td>$\hat{\sigma}_{\epsilon_x}$</td>
<td>0.287</td>
<td>0.288</td>
<td>0.288</td>
<td></td>
</tr>
<tr>
<td>$std(\hat{\sigma}_{\epsilon, t})$</td>
<td>0.044</td>
<td>0.043</td>
<td>0.017</td>
<td></td>
</tr>
<tr>
<td>$corr(\hat{\sigma}<em>{\epsilon, t}, \hat{\sigma}</em>{\epsilon, t-1})$</td>
<td>0.262</td>
<td>0.254</td>
<td>-0.001</td>
<td></td>
</tr>
<tr>
<td>B. Parameters (Quarterly)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>-</td>
<td>0.915</td>
<td>0.915</td>
<td></td>
</tr>
<tr>
<td>$\overline{\sigma}_{\epsilon_x}$</td>
<td>-</td>
<td>0.242</td>
<td>0.242</td>
<td></td>
</tr>
<tr>
<td>$\rho_{\sigma_{\epsilon_x}}$</td>
<td>-</td>
<td>0.520</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>-</td>
<td>0.100</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Note: Panel A lists moments of individual wages used for calibration. The column labeled “PSID Data,” “Varying Risk,” and “Constant Risk” show the moments of the U.S. economy, the varying risk model, and the constant risk model, respectively. The model moments are computed by using the data generated through a simulation with 3,000 individuals for 1,500 periods (discarding the first 500 periods) and then converted to an annual basis. Panel B shows the parameter values chosen for those model economies.
Table 4: Business Cycle Statistics of the U.S. and Model Economy

<table>
<thead>
<tr>
<th></th>
<th>U.S. Economy</th>
<th>Model Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Varying Risk</td>
</tr>
<tr>
<td>$\sigma_Y$</td>
<td>1.69</td>
<td>1.41</td>
</tr>
<tr>
<td>$\sigma_C$</td>
<td>0.54</td>
<td>0.32</td>
</tr>
<tr>
<td>$\sigma_I$</td>
<td>2.85</td>
<td>3.14</td>
</tr>
<tr>
<td>$\sigma_H$</td>
<td>1.00</td>
<td>0.81</td>
</tr>
<tr>
<td>$\sigma_{Y/H}$</td>
<td>0.63</td>
<td>0.97</td>
</tr>
<tr>
<td>$Corr(Y, C)$</td>
<td>0.78</td>
<td>0.88</td>
</tr>
<tr>
<td>$Corr(Y, I)$</td>
<td>0.80</td>
<td>0.99</td>
</tr>
<tr>
<td>$Corr(Y, H)$</td>
<td>0.80</td>
<td>0.44</td>
</tr>
<tr>
<td>$Corr(Y, Y/H)$</td>
<td>0.31</td>
<td>0.66</td>
</tr>
<tr>
<td>$Corr(H, Y/H)$</td>
<td><strong>-0.32</strong></td>
<td><strong>-0.38</strong></td>
</tr>
</tbody>
</table>

Note: This table shows the volatilities and major contemporaneous correlations of aggregate variables in the U.S. and model economy. The column labeled “U.S. Economy” presents those statistics of the U.S. For the model economy, two sets of results are presented. The column labeled “Varying Risk” shows the result of the model economy including shocks to both aggregate total factor productivity (TFP) and idiosyncratic wage risk, whereas the column “Constant Risk” is the model that assumes shocks to only aggregate TFP.

The U.S. macroeconomic data are taken from the source listed in Appendix A3. Model data are generated through 3,500-period simulations (discarding the first 500 periods) using the solution method described in Appendix B2. All the data are logged and then detrended by the Hodrick-Prescott (HP) filter with a smoothing parameter of 1,600. The volatility of output $\sigma_Y$ is the standard deviation of output (multiplied by 100). Other volatilities are presented as their ratio to $\sigma_Y$. 

35
### Table 5: Cyclicality of Idiosyncratic Wage Risk and Business Cycles

<table>
<thead>
<tr>
<th></th>
<th>Model Economy</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant Risk</td>
<td>Countercyclical Risk</td>
<td>Recalibrated Varying Risk</td>
</tr>
<tr>
<td>$\sigma_Y$</td>
<td>1.35</td>
<td>1.29</td>
<td>1.37</td>
</tr>
<tr>
<td>$\sigma_C$</td>
<td>0.32</td>
<td>0.37</td>
<td>0.33</td>
</tr>
<tr>
<td>$\sigma_I$</td>
<td>3.09</td>
<td>3.00</td>
<td>3.13</td>
</tr>
<tr>
<td>$\sigma_H$</td>
<td>0.57</td>
<td>0.60</td>
<td>0.71</td>
</tr>
<tr>
<td>$\sigma_{Y/H}$</td>
<td>0.48</td>
<td>0.66</td>
<td>0.83</td>
</tr>
<tr>
<td>$\text{Corr}(Y,C)$</td>
<td>0.90</td>
<td>0.90</td>
<td>0.87</td>
</tr>
<tr>
<td>$\text{Corr}(Y,I)$</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>$\text{Corr}(Y,H)$</td>
<td>0.96</td>
<td>0.77</td>
<td>0.57</td>
</tr>
<tr>
<td>$\text{Corr}(Y,Y/H)$</td>
<td>0.95</td>
<td>0.82</td>
<td>0.71</td>
</tr>
<tr>
<td>$\text{Corr}(H,Y/H)$</td>
<td><strong>0.83</strong></td>
<td><strong>0.25</strong></td>
<td><strong>-0.17</strong></td>
</tr>
</tbody>
</table>

Note: This table shows the volatilities and major contemporaneous correlations of aggregate variables in the model economy. Three sets of results are presented. The column labeled “Constant Risk” shows the result when only aggregate TFP shocks are included and constant idiosyncratic wage risk is assumed. The column “Countercyclical Risk” presents the result when the model includes countercyclical idiosyncratic wage risk in addition to aggregate TFP shocks. The column “Recalibrated Varying Risk” shows the result obtained when the varying risk model is parameterized so that its risk fluctuations match those of the countercyclical risk model.

Model data are generated through 3,500-period simulations (discarding the first 500 periods) using the solution method described in Appendix B2. All the data are logged and then detrended by the Hodrick-Prescott (HP) filter with a smoothing parameter of 1,600. The volatility of output $\sigma_Y$ is the standard deviation of output (multiplied by 100). Other volatilities are presented as their ratio to $\sigma_Y$. 

---

36
Figure 1: Cyclical Variation in Estimated Idiosyncratic Wage Risk $\hat{\sigma}_{\epsilon_x, t}$

Note: This figure shows the percentage deviation of the estimated idiosyncratic wage risk $\hat{\sigma}_{\epsilon_x, t}$ (Table 1) relative to its linear trend.
Figure 2: Responses of Aggregate Variables to a Rise in Idiosyncratic Wage Risk $\sigma_{\varepsilon x}$

Note: The upper-left panel shows the movement of idiosyncratic wage risk $\sigma_{\varepsilon x}$ considered in this simulation. The remaining three panels present how output $Y$, total hours worked $H$, and average labor productivity $Y/H$ respond to this elevated risk. All the responses show the percentage deviation from the steady-state levels of the variables.
Figure 3: Individual Employment Decisions at the Steady State

Note: This figure shows individual employment decisions at the steady state as a function of idiosyncratic labor productivity $x$ and asset holding $k$. 
Figure 4: Responses of Individuals to a Rise in Idiosyncratic Wage Risk $\sigma_{\varepsilon_x}$

Note: This figure shows individual-level responses to a rise in idiosyncratic wage risk $\sigma_{\varepsilon_x}$ that occurs during period 0–2 as shown in the upper-left panel of Figure 2. The responses presented here are those of individuals who were close to indifferent towards working before the rise in $\sigma_{\varepsilon_x}$. The left panel shows those with relatively low current labor productivity (25th percentile in the productivity distribution), whereas the right one is those with relatively high productivity (75th percentile). For each, the responses of employment (1: work and 0: not work), asset holding, and consumption are presented.
Figure 5: Shift in the Distribution of Idiosyncratic Labor Productivity $x$ across Individuals

Note: This figure plots the distribution of idiosyncratic labor productivity $x$ across individuals in period 0 (before the rise in $\sigma_{\varepsilon_x}$ increases the productivity dispersion) and that in period 3 (after the rise in $\sigma_{\varepsilon_x}$ increases the productivity dispersion). The arrows in the figure indicate the direction of net individual movements.
Figure 6: Shift in the Wealth Distribution across Individuals with Low Labor Productivity

Note: This figure shows the cumulative asset distribution across individuals with the 25th percentile productivity in period 0 (before the rise in $\sigma_{\varepsilon_x}$ increases the productivity dispersion) and that in period 3 (after the rise in $\sigma_{\varepsilon_x}$ increases the productivity dispersion).
Figure 7: Responses of Aggregate Variables to a Decline in Aggregate Total Factor Productivity (TFP) $z$

Note: The upper-left panel shows the movement of aggregate TFP $z$ considered in this simulation. The remaining three panels present how output $Y$, total hours worked $H$, and average labor productivity $Y/H$ respond to this decline in $z$. All the responses show the percentage deviation from the steady-state levels of the variables.
Figure 8: Responses of Individuals to a Decline in Aggregate Total Factor Productivity (TFP) $z$

Note: This figure shows individual-level responses to a drop in aggregate TFP $z$ that occurs in period 0 as shown in the upper-left panel of Figure 7. The responses presented here are those of individuals who were close to indifferent towards working before the drop in $z$. The left panel shows those with relatively low current labor productivity (25th percentile in the productivity distribution), whereas the right one is those with relatively high productivity (75th percentile). For each, the responses of employment (1: work and 0: not work), asset holding, and consumption are presented.
Figure 9: Impact of a Rise in Idiosyncratic Wage Risk $\sigma_{\varepsilon_x}$ During a Recession

Note: The upper two panels depict the movements of aggregate total factor productivity (TFP) $z$ and idiosyncratic wage risk $\sigma_{\varepsilon_x}$ considered in this simulation. The line with x shows those when both the decline in $z$ and the rise in $\sigma_{\varepsilon_x}$ occur, whereas the line with o shows those when only $z$ drops. The lower two panels show the corresponding responses of output $Y$ and total hours worked $H$. All the results shown in this figure are obtained by first taking the average of 500 independent stochastic simulations and then computing the percentage deviation from the pre-shock average (230–250 periods).
Figure 10: Total Hours Worked and Output in the U.S.

Note: This figure plots total hours worked and output of the U.S. between 1975Q1 and 2009Q3. They are logged and then detrended by the HP filter with a smoothing parameter of 1,600.
8 Appendix A: Data

8.1 A1: Individual Wage Data

The data used to determine parameters on idiosyncratic labor productivity are taken from the Family-level file of the Panel Study of Income Dynamics (PSID). Individual wages are computed, dividing annual labor income (1975: V3863–1992: V21484) by annual hours worked (1975: V3823–1992: V20344). Those individual wages are then converted real wages in terms of 1983 dollars using the CPI data. Only data of male heads of households are used. Since data on years of education are discontinuous in 1974, interview years of 1975–1992 are used.

The following observations are excluded.

- An observation if a major assignment is assigned to the labor income and/or hours.
- An observation with the wage less than one dollar (in 1983 dollars) or higher than 500 dollars (in 1983 dollars).
- The most recent Latino sample and the Survey of Economic Opportunity sample.
- An observation with less than 100 annual hours.

35 Numbers in parentheses are variable labels of the PSID.
36 More specifically, monthly “Consumer Price Index for All Urban Consumers: All Items” is taken from the FRED database at the Federal Reserve Bank of St. Louis. The annual CPI in a year is computed using the average monthly CPI in the year.
37 Interview year of 1993 is excluded because data on major assignment for labor income are not available in the year.
Top-coded observations for income.


### 8.2 A2: Individual Labor Income Data

The 1992 PSID Individual-level file provides annual labor income data for individuals, including those other than heads of households. Total labor income (ER30750) is taken to compute the Gini coefficient for annual labor income. Only individuals older than 16 are included (ER30736). Individuals are also excluded if a major assignment is assigned to their income and/or hours worked (ER30751, ER30755).

### 8.3 A3: U.S. Macroeconomic Data

Table 4 and Figure 10 are constructed using the following data between 1947Q3 and 2009Q3. Output is “Real Gross Domestic Product (billions of chained 2005 dollars)” taken from Table 1.1.6 of the Bureau of Economic Analysis (BEA). Consumption is “Personal Consumption Expenditures (PCE)” less durable goods obtained from Table 2.3.5 of the BEA. Investment is the sum of durable goods consumption in Table 2.3.5 and private fixed investment (including residential investment) in Table 5.3.5. The real values of consumption and investment are calculated using the price index for Gross Domestic Product in Table 1.1.4. Total labor hours are those constructed by Cociuba, Prescott, and Ueberfeldt (2009) and obtained from their website.\(^{39}\)

\(^{39}\)I am grateful to the authors for making the data available.
9 Appendix B: Solution Methods

9.1 B1: Steady State

1. Determine grid points for individual asset $k$ and idiosyncratic labor productivity $x$.
   Use 100 log-spaced points over $[-2, 250]$ for $k$ and 17 evenly spaced points over $[-3\bar{\sigma}_\varepsilon/\sqrt{1-\rho_z^2}, 3\bar{\sigma}_\varepsilon/\sqrt{1-\rho_z^2}]$ for $x$. Compute the transition matrix of $x$ using the method developed by Tauchen (1986).

2. Set a guess for the discount factor $\beta$.

3. Solve the individual optimization problem and obtain the beginning-of-the-period value function $V(k, x)$.

   (1) Compute the steady-state wage rate $\bar{w} = \alpha \bar{z}((1 - \alpha)\bar{z}/(\bar{r} + \delta))^{(1-\alpha)/\alpha}$ using the target steady-state rental rate of capital $\bar{r} = 0.01$ and the steady-state aggregate TFP $\bar{z} = 1.0$.

   (2) Set a guess for the beginning-of-the-period value function, $V_0(k, x)$.

   (3) Solve the consumption-saving problem for each of current employment choices:

   $$V^E_1(k, x) = \max_{k'\geq k} \{ u(w\bar{h}x + (1 + r)k - k', \bar{h}) + \beta \sum_{x'} \pi_x(x'|x)V_0(k', x') \}$$

   and

   $$V^N_1(k, x) = \max_{k'\geq k} \{ u((1 + r)k - k', 0) + \beta \sum_{x'} \pi_x(x'|x)V_0(k', x') \},$$

   where $\pi_x(x'|x)$ is the transition probability from $x$ to $x'$. Since $k'$ may take values not on its grid points, approximate the conditional expectation using univariate cubic spline interpolation in $k'$. If $V^E_1(k, x) \geq V^N_1(k, x)$, then individuals with

\footnote{Aggregate state $(z, \mu, \sigma_{\varepsilon_x})$ is constant at the steady state.}
and \( x \) choose to work. Otherwise, they do not work. Hence, set \( V_1(k, x) = \max \{ V_1^E(k, x), V_1^N(k, x) \} \).

(4) If \( V_1(k, x) \) becomes sufficiently close to \( V_0(k, x) \), then set \( V(k, x) = V_1(k, x) \) and proceed to Step 4. Otherwise, update the value function as \( V_0(k, x) = V_1(k, x) \) and return to Step 3 (3).

4. Compute the distribution of \( k \) and \( x \) across individuals at the steady state, \( \mu(k, x) \).

(1) Choose points at which the distribution \( \mu(k, x) \) is approximated. Use 2,000 log-spaced points over \([-2, 250]\) for \( k \) and the points chosen in Step 1 for \( x \).

(2) Replacing \( V_0(k, x) \) with \( V(k, x) \) obtained above, solve the individual problems shown in Step 3 (3), this time for \( 2 \times 17 \) pairs of \( (k, x) \) and find their optimal asset holding \( k'(k, x) \) and employment \( h(k, x) \).

(3) Suppose \( k_h \leq k'(k, x) \leq k_{h+1} \), where \( k_h \) and \( k_{h+1} \) are two sequential asset points. Starting from an initial guess, update \( \mu(k, x) \), so that individuals with \( (k, x) \) move to \( (k_h, x') \) with probability \( \omega \pi_x(x'|x) \) and to \( (k_{h+1}, x') \) with probability \( (1 - \omega) \pi_x(x'|x) \), where \( \omega = (k_{h+1} - k')/(k_{h+1} - k_h) \). Repeat this until \( \mu(k, x) \) converges. The converged \( \mu(k, x) \) is the steady-state distribution \( \tilde{\mu}(k, x) \).

5. Compute the steady-state aggregate capital \( \tilde{K} = \int k \tilde{\mu}([dk \times dx]) \) and aggregate efficiency-weighted labor \( \tilde{L} = \int x h(k, x) \tilde{\mu}([dk \times dx]) \). Calculate the implied steady-state rental rate of capital \( \tilde{r} = (1 - \alpha) \tilde{z} \tilde{K}^{-\alpha} \tilde{L}^\alpha - \delta \). If \( \tilde{r} \) becomes sufficiently close to the target rate (1 percent), then the steady state is found. Otherwise, set a different value to \( \beta \) and repeat Steps 3–5.

9.2 B2: Business Cycles

1. Aggregate state \((z, \mu, \sigma_{\varepsilon_z})\) varies over time. For aggregate TFP \( z \), set nine evenly spaced points over \([3\sigma_{\varepsilon_z}/\sqrt{1 - \rho_z^2}, 3\sigma_{\varepsilon_z}/\sqrt{1 - \rho_z^2}]\) and compute the transition matrix
using the method of Tauchen (1986). The distribution across individuals \( \mu(k, x) \) is a high-dimensional item, and thus approximate it with aggregate capital \( K \). Use seven evenly spaced grid points over \([0.80\bar{K}, 1.20\bar{K}]\), where \( \bar{K} (= 11.59) \) is the steady-state aggregate capital. Since \( \sigma_{\varepsilon_x} \) has only three states, use them directly.

2. For individual asset \( k \), use the 100 grid points chosen in the steady-state solution method. For idiosyncratic productivity \( x \), use 17 evenly spaced points over \([-3\bar{\sigma}_{\varepsilon_x}/\sqrt{1 - \rho_x^2}, 3\bar{\sigma}_{\varepsilon_x}/\sqrt{1 - \rho_x^2}]\) for all the risk states. The transition probabilities depend on the value of \( \sigma_{\varepsilon_x} \) and thus are different across the risk states. Compute those probabilities using the method of Tauchen (1986).

3. Specify the relationship between the approximate aggregate state \((z, K, \sigma_{\varepsilon_x})\) and the next period’s aggregate capital \( K’ \), the current equilibrium wage rate \( w \), and the current equilibrium rental rate of capital \( r \). Assume that individuals use the following forecasting rules for each risk state \((i = H, M, L)\):

\[
\ln \hat{K}' = a_{0,i} + a_{1,i} \ln K + a_{2,i} \ln z
\]

(11)

and

\[
\ln \hat{w} = b_{0,i} + b_{1,i} \ln K + b_{2,i} \ln z.
\]

(12)

Individuals compute \( \hat{r} = z(1 - \alpha)(\hat{w} / (\alpha z))^{-\alpha/(1-\alpha)} \).

4. Solve the individual optimization problem and obtain the beginning-of-the-period value function \( V(k, x; z, K, \sigma_{\varepsilon_x}) \).

(1) Set a guess for the beginning-of-the-period value function, \( V_0(k, x; z, K, \sigma_{\varepsilon_x}) \).

(2) Solve the consumption-saving problem for each of current employment choices:
\[ V^E_1(k, x; z, K, \sigma_{\epsilon_x}) = \max_{k' \geq k} \{u(\tilde{w} \bar{h} x + (1 + \hat{r})k - k', \bar{h}) \] 
\[ + \beta \sum_{x'} \sum_{z'} \sum_{\sigma'_{\epsilon_x}} \pi_x(x' | x, \sigma_{\epsilon_x}) \pi_z(z' | z) \pi_{\sigma_{\epsilon_x}}(\sigma'_{\epsilon_x} | \sigma_{\epsilon_x}) V_0(k', x', z', \hat{K}', \sigma'_{\epsilon_x}) \] 
and

\[ V^N_1(k, x; z, K, \sigma_{\epsilon_x}) = \max_{k' \geq k} \{u((1 + \hat{r})k - k', 0) \] 
\[ + \beta \sum_{x'} \sum_{z'} \sum_{\sigma'_{\epsilon_x}} \pi_x(x' | x, \sigma_{\epsilon_x}) \pi_z(z' | z) \pi_{\sigma_{\epsilon_x}}(\sigma'_{\epsilon_x} | \sigma_{\epsilon_x}) V_0(k', x', z', \hat{K}', \sigma'_{\epsilon_x}), \]

where \( \pi_x(x' | x, \sigma_{\epsilon_x}) \) is the transition probability from \( x \) to \( x' \), which depends on \( \sigma_{\epsilon_x} \), \( \pi_z(z' | z) \) is the transition probability from \( z \) to \( z' \), and \( \pi_{\sigma_{\epsilon_x}}(\sigma'_{\epsilon_x} | \sigma_{\epsilon_x}) \) is the transition probability from \( \sigma_{\epsilon_x} \) to \( \sigma'_{\epsilon_x} \). Since \( k' \) and \( \hat{K}' \) may not be on their grid points, use bivariate cubic spline interpolation in \((K, k)\) to approximate the conditional expectation. If \( V^E_1(k, x; z, K, \sigma_{\epsilon_x}) \geq V^N_1(k, x; z, K, \sigma_{\epsilon_x}) \), then individuals with \( k \) and \( x \) choose to work. Otherwise, they do not work. Hence, set
\[ V_1(k, x; z, K, \sigma_{\epsilon_x}) = \max\{V^E_1(k, x; z, K, \sigma_{\epsilon_x}), V^N_1(k, x; z, K, \sigma_{\epsilon_x})\}. \]

(3) If \( V_1(k, x; z, K, \sigma_{\epsilon_x}) \) becomes sufficiently close to \( V_0(k, x; z, K, \sigma_{\epsilon_x}) \), then set \( V(k, x; z, K, \sigma_{\epsilon_x}) = V_0(k, x; z, K, \sigma_{\epsilon_x}) \) and proceed to the next step. Otherwise, update the value function as \( V_0(k, x; z, K, \sigma_{\epsilon_x}) = V_1(k, x; z, K, \sigma_{\epsilon_x}) \) and return to Step 4 (2).

5. Generate 3,500-period model data using the beginning-of-the-period value function obtained above \( V(k, x; z, K, \sigma_{\epsilon_x}) \). Approximate the distribution across individuals \( \mu(k, x) \) at \( 2,000 \times 17 \) points set in the steady-state solution.

(1) Set initial conditions: \( z_0 = \bar{z}, \sigma_{\epsilon_x0} = \sigma_{\epsilon_xM}, \mu_0 = \bar{\mu}, \) and \( K_0 = \int k \mu_0([dk \times dx]) \).
(2) Set a guess for \(w_0, \tilde{w}_0\). Then, \(\tilde{L}_0 = (\alpha z_0/\tilde{w}_0)^{1/(1-\alpha)}K_0\) and \(\tilde{r}_0 = (1-\alpha)z_0(\tilde{w}_0/\alpha z_0)^{-\alpha/(1-\alpha)} - \delta\). Individuals’ forecast of the next-period aggregate state \(\tilde{K}_1\) is given by the forecasting rule. Replacing \(V_0(k, x; z, K, \sigma_\varepsilon)\) with \(V(k, x; z, K, \sigma_\varepsilon)\), solve the individual problems shown in Step 4 (2), this time for \(2,000 \times 17\) pairs of \((k, x)\) under \(w = \tilde{w}_0, r = \tilde{r}_0, \) and \(K' = \tilde{K}_1\). Record the optimal asset holding \(k_1(k, x)\) and employment \(h_0(k, x)\).

(3) Check whether the labor market clears; in other words, check whether
\[
\tilde{L}_0 = \int x h_0(k, x) \mu_0([dk \times dx])
\]
holds. If not, reset \(\tilde{w}_0\) and repeat Step 5 (2).\(^{41}\) Once the market-clearing wage is found, proceed to (4).

(4) Compute aggregate variables as follows:
\[
L_0 = \int x h_0(k, x) \mu_0([dk \times dx]),
K_1 = \int k_1(k, x) \mu_0([dk \times dx]),
H_0 = \int h_0(k, x) \mu_0([dk \times dx]),
Y_0 = z_0K_0^{1-\alpha}L_0^\alpha,
I_0 = K_1 - (1 - \delta)K_0, C_0 = Y_0 - I_0, \text{ and } r_0 = (1 - \alpha)z_0K_0^{1-\alpha}L_0^\alpha - \delta.
\]

(5) Obtain the next period’s distribution across individuals \(\mu_1(k, x)\) as described in Step 4 (3) of the steady-state solution.

(6) Repeat Steps 5 (2)–(5) for 3,500 periods.

6. Using the model data (disregarding the first 500 periods), update the coefficients of the forecasting rules with ordinary least squares. If those coefficients converge, then proceed to the next step. Otherwise, repeat Steps 4 and 5 using the updated forecasting rules.

7. Check whether the converged forecasting rules are sufficiently accurate. If not, assume different functional forms and repeat Steps 3–5. The forecasting rules of (11) and (12) turn out to be quite accurate as reported in Appendix C.

\(^{41}\)A similar method is used for the bond market by Krusell and Smith (1998) and Pijoan-Mas (2007), and the goods market by Khan and Thomas (2003, 2007, 2008).
## 10 Appendix C: Forecasting Rules

The table below presents the coefficients of the forecasting rules ($\ln K' = a_0 + a_1 \ln K + a_2 \ln z$ and $\ln w = b_0 + b_1 \ln K + b_2 \ln z$) and the accuracy of those rules. Two accuracy measures are reported. One is the coefficient of determination $R^2$ and the other is the standard deviation of the forecasting error $\hat{\sigma}$. A separate rule is used for each of the risk states. As shown, the forecasting rules achieve a high degree of accuracy.

<table>
<thead>
<tr>
<th></th>
<th>Constant Risk</th>
<th>Varying Risk (H / M / L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{K}'$</td>
<td>$a_0$</td>
<td>0.113</td>
</tr>
<tr>
<td></td>
<td>$a_1$</td>
<td>0.954</td>
</tr>
<tr>
<td></td>
<td>$a_2$</td>
<td>0.099</td>
</tr>
<tr>
<td></td>
<td>$R^2$</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>$\hat{\sigma}$</td>
<td>0.0072%</td>
</tr>
<tr>
<td>$\hat{w}$</td>
<td>$b_0$</td>
<td>-0.196</td>
</tr>
<tr>
<td></td>
<td>$b_1$</td>
<td>0.433</td>
</tr>
<tr>
<td></td>
<td>$b_2$</td>
<td>0.828</td>
</tr>
<tr>
<td></td>
<td>$R^2$</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>$\hat{\sigma}$</td>
<td>0.0365%</td>
</tr>
<tr>
<td></td>
<td>Countercyclical Risk (H / M / L)</td>
<td>Recalibrated Varying Risk (H / M / L)</td>
</tr>
<tr>
<td>----------------------</td>
<td>----------------------------------</td>
<td>-------------------------------------</td>
</tr>
<tr>
<td>$\ddot{K}$</td>
<td>$a_0$ 0.264 / 0.206 / 0.236</td>
<td>0.090 / 0.094 / 0.086</td>
</tr>
<tr>
<td></td>
<td>$a_1$ 0.892 / 0.916 / 0.903</td>
<td>0.964 / 0.962 / 0.965</td>
</tr>
<tr>
<td></td>
<td>$a_2$ 0.089 / 0.096 / 0.103</td>
<td>0.088 / 0.093 / 0.090</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.998 / 0.999 / 0.999</td>
<td>1.000 / 1.000 / 1.000</td>
</tr>
<tr>
<td>$\hat{\sigma}$</td>
<td>0.0524% / 0.0506% / 0.0518%</td>
<td>0.0858% / 0.0685% / 0.0856%</td>
</tr>
<tr>
<td>$\ddot{w}$</td>
<td>$b_0$ −1.083 / −0.721 / −0.883</td>
<td>−0.070 / −0.098 / −0.054</td>
</tr>
<tr>
<td></td>
<td>$b_1$ 0.795 / 0.647 / 0.715</td>
<td>0.379 / 0.393 / 0.377</td>
</tr>
<tr>
<td></td>
<td>$b_2$ 0.877 / 0.842 / 0.816</td>
<td>0.839 / 0.863 / 0.878</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.956 / 0.976 / 0.972</td>
<td>0.985 / 0.988 / 0.982</td>
</tr>
<tr>
<td>$\hat{\sigma}$</td>
<td>0.3187% / 0.2763% / 0.2700%</td>
<td>0.4508% / 0.3497% / 0.4592%</td>
</tr>
</tbody>
</table>