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# Credit Supply and Output Volatility

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**Abstract**

The link between aggregate profits and investment has been widely analysed through the impact of profits on net worth and therefore the firm's ability to borrow, in the presence of credit market imperfections. How the business cycle is affected if profits also affect investment through an impact on savings and therefore the intermediary's ability to lend, is the topic of this paper. We find that the fluctuations in the supply of credit that result from this may significantly amplify output responses to shocks in comparison to a situation where the net worth mechanism operates alone.

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# 1 Introduction

The idea that falls in current entrepreneurial profits, by decreasing borrower net worth, lead to falls in profitable investment and therefore future profits has become a mainstay of the literature on the macroeconomic effect of credit market imperfections. Following the seminal work of Bernanke and Gertler (1989), this is shown to happen when the ability to borrow for entrepreneurial investment is constrained by the net worth of the borrower/entrepreneur, This can arise theoretically for many reasons (agency costs, moral hazard, etc.) and a wide literature has provided both theoretical extensions[1] and frameworks for quantitative testing. While this link via profits and net worth has been extensively analysed, we argue that the literature has largely neglected a more traditional Keynesian link between entrepreneurial profits and investment via its effect on savings. The purpose of this paper is to explore in a simple model how these two mechanisms interact. Propagation in the model is augmented by fluctuations in the lending sector as well as in the borrowing sector.[2]

Most existing macroeconomic models of credit market imperfections assume a perfectly elastic supply of the aggregate saving. In those that do not (for example, the large model DSGE approaches to simulating the Bernanke and Gertler framework, or Matsuyama, 2007), the effect that high profits have on the supply of savings is relatively weak. Suppose at one time, in aggregate, firm projects yield higher than normal returns. Given the nature of the typical financial contract in the literature, the main consequence of this to lenders is a fall in default rates, since each contract returns the same amount provided returns are above a certain threshold. The main beneficiaries of the increase in returns are borrowers. If borrowers are short-lived, the bulk of the increase in profits is

[1] Important contributions include Kiyotaki and Moore (1997) who provide a model that is capable of incorporating a meaningful role for asset prices, and Matsuyama (2007) who shows the importance of net worth in determining the *composition* of credit rather than the quantity when agents choose from a menu of projects that have different expected returns and associated differences in size and agency cost. An important theoretical contribution that arguably lies outside a typical net worth paradigm is the model of Cooley, Marimon and Quadrini (2004) on the limited enforceability of contracts. This model demonstrates considerable amplification and persistence given the inability to exclude from the market those who have repudiated contracts. A full survey is, of course, beyond the scope of this paper.

[2] Current market conditions do not require emphasising the general importance of this feature. However, we also give the following quote from Walsh (2003, page 359): "While the bank lending channel as part of monetary policy transmission process may be not operative, it might still be the case that shifts in bank loan supply are a cause of economic fluctuations. In the United States, the 1989-1992 period generated decline in bank lending and stories, particularly from New England, of firms facing difficulty borrowing, led many to seek evidence that credit markets played an independent role in contributing in the 1990-1991 recession."

then simply consumed without leading to a correspondingly large increase in savings.

Since, typically, the incentives will be such that entrepreneurs will invest all of their collateral in their own future projects, this relatively small impact of profits on savings is also largely unaffected by the assumption of longer-lived entrepreneurs. The stronger link between profits and savings here is obtained by assuming that (at least some) entrepreneurs face a consumption decision on completion of an investment project. Very loosely, the model tries to capture the savings behavior of the owners of equity, as well as of the owners of debt.

If there is an exogenous lower bound on the real rates of return faced by savers - here it is simply the return on physical storage - then the model yields two possible regimes. If the supply of credit is so large that returns to saving are driven down to the lower bound[3] then fluctuations in savings due to cyclical changes in profits will have no effect on interest rates, leaving only the well known net worth mechanism in effect. If the supply of credit is not large enough for this, then a savings mechanism also operates; namely that in periods of high output, the increase in savings and therefore credit supply causes the interest rate to fall, leading to increased investment and higher future output. This leads to both increased persistence and amplification of shocks.

The model of financial market imperfections used is based on the static model of Repullo and Suarez (2000), which here then, by construction, becomes dynamic though any model that motivates a net worth borrowing constraint should yield similar implications. An overlapping generations framework is used for tractability. We take advantage of this to avoid distinguishing sharply between borrowers and lenders in terms of their ex-ante characteristics. Since a consumption decision is made after entrepreneurial activity, last period's borrowers are now this period's lenders. Youth is spent working, and this is followed by entrepreneurial activity. In the next phase of life, the proceeds from entrepreneurial activity are consumed or saved for consumption in retirement. This saving funds the investment of the following cohort, and so on.

It is important to note that the main purpose of the paper is to examine dynamics that might be incorporated in many macroeconomic models of credit market imperfection (yielding increased persistence), rather than the more ambitious aim of providing a

[3]During much of the great moderation, the supply of liquidity was so abundant that the real rates of return faced by savers were in fact very low. The idea is that if a permanent fall in the *level* of credit supply alters this situation, it may lead to a permanent increase in the *volatility* of output.

model of financial market imperfections that explains much of the observed persistence in the business cycle.[4] Section 2 describes the model while section 3 presents results and conclusions.

## 2 The Model

Time is infinite and discrete, and a large and constant number of agents are born each period. Agents live for three periods which we label youth, adulthood and maturity, so at any one time agents are young, adult or mature. Agents are born into youth owning an indivisible project of size 1, and agent $i$ has entrepreneurial skill $s_i$. Entrepreneurial skill could be thought to increase project returns given a required investment, but it will be slightly more convenient to assume that skill reduces the size of the required investment without affecting returns. So for a project to commence, it requires an investment $1 - s_i$. Again it is slightly more convenient to treat this as additive rather than multiplicative, but neither of these assumptions is important to the story. $s$ is i.i.d. across agents according to a time-invariant density function $f(s)$ on a support $[0, 1]$. Agents also receive a time-varying endowment $w_t$ which is constant across agents ($s$ could equivalently of course be viewed as a heterogeneous endowment across agents). $w_t$ is then a measure of aggregate net worth.

Projects are started in youth and yield returns at the beginning of adulthood. In youth, the two options open to an agent are as follows. She may use her endowment to start her project, borrowing additional funds if necessary via a set of perfectly competitive intermediaries. No consumption occurs in youth, so if she does not start a project, she stores her endowment for future consumption which yields an exogenous gross return $\bar{R}$. We assume that young agents cannot save their endowment with intermediaries, which we argue allows a cleaner interpretation of the model as discussed below. It is also a conservative assumption, in that if we allow the young to save, the results are typically

[4] Initially, attempts at quantifying the persistence produced by the financial accelerator found relatively weak effects (see for example Fuerst, 1995, and Carlstrom and Fuerst, 1997, on the Bernanke-Gertler mechanism and Kocherlakota, 2000, Cordoba and Ripoll, 2004, on Kiyotaki and Moore.) Set in a New Keynesian Model with sticky prices, Bernanke et al. (1999), however, show significantly greater amplification and persistence due to the financial accelerator. This work has been further consolidated in a new generation of dynamic stochastic general equilibrium (DSGE) models. See Christiano et al. (2003),(2007), Christensen and Dib (2008) among others.

much stronger.[5] Adult agents, after the returns from projects or savings are realised, then make savings decisions as to how to spread their consumption over adulthood and maturity. In doing so, they have the options of storing their adult wealth or lending to the intermediaries.

In Repullo and Suarez (2000) these intermediaries face an exogenous interest rate. In the dynamic framework we have described, this becomes endogenous, since intermediaries now offer savers a gross interest rate $\eta R_t$ that equates the aggregate demand for loans from intermediaries to the aggregate supply of savings from adult agents. It is clear that, in equilibrium, this cannot fall below the return from storage, so we have $\eta R_t \geqslant \bar{R}$. Intermediaries lend at a rate $R_t$, and $1 - \eta$ represents the fraction of funds that they 'gobble up' to meet their costs.

We assume that the endowment $w_t$ agents receive in youth is funded in the following simple (and stylised) way. In each period an exogenous fraction $\beta_t$ of adult agents, drawn at random, die before they are able to consume and their wealth is then distributed evenly among young agents. If $X_t^a$ is aggregate adult wealth, then

$$w_t = \beta_t X_t^a. \tag{1}$$

This provides a very simple way of capturing the procyclicality of net worth, that we moderately preferred to the standard approach[6] in order to be able interpret $\beta_t$ as a financial parameter governing the flow of funds into internal finance.

## 2.1 Consumption

The endowments of agents when young can be divided between investing in the agent's project or storage. The returns from these options are obtained in adulthood and agents receive no other income over the course of their lives. These returns, which constitute adult wealth, may be consumed in adulthood, or saved, via intermediaries or storage, for consumption in maturity. No utility is derived from consumption in youth.

[5] Contact authors for details.

[6] This is to assume that projects produce capital (that often depreciates within a period), that when combined with the labour of the young produces output. So entrepreneurs receive the rental from capital and young agents receive a wage $w_t$ that is then procyclical (see Bernanke and Gertler, 1989). This modeling approach, though making (1) non-linear, would make no significant difference to the analysis or results, particularly regarding fluctuations around the steady state. The main difference is one of interpretation - the equivalent of $\beta_t$ would then be interpreted as a production function parameter.

At the beginning of adulthood, project returns are realised, a fraction $\beta_t$ of agents die, and their wealth is redistributed to the next generation of young agents. The remaining adult agents fall into one of two types; a fraction $(1-\alpha_t)(1-\beta_t)$ being of type 1, that consume only in adulthood, and a fraction $\alpha_t(1-\beta_t)$ being of type 2, that spread consumption over adulthood and maturity.

Agent $i$ has utility $U_t^j(c_{i,t}^a, c_{i,t+1}^m)$ where $c_{i,t}^a$ and $c_{i,t+1}^m$ are consumption in adulthood and maturity respectively, $j$ is her type, and $t$ is the time at which the agent is an adult. For the fraction $1-\alpha_t$ agents of type 1, we assume that $U_t^1(c_{i,t}^a, c_{i,t+1}^m) = c_{i,t}^a$. Noting that the introduction of discounting would add very little to the analysis, for type 2 agents we assume,

$$U_t^2(c_{i,t}^a, c_{i,t}^m) = \left[\frac{\left(c_{i,t}^a\right)^\rho}{2} + \frac{\left(c_{i,t+1}^m\right)^\rho}{2}\right]^{1/\rho}. \tag{2}$$

Therefore, if $X_{i,t}^a$ is the adult wealth of an agent $i$ of type 2 and if she faces a real interest rate $\eta R_t$,

$$c_{i,t}^a = \frac{X_{i,t}^a}{1+(\eta R_t)^{\sigma-1}}.$$

where $\sigma$ is the intertemporal elasticity of substitution. Since we have assumed that young agents cannot save with intermediaries, those that have insufficient skill to invest in projects are limited to physical storage. This is a very helpful assumption in order to be to simulate the model with a reasonable savings ratio (otherwise we easily find that the supply of savings becomes too large) and it also allows a much cleaner interpretation of the parameters, since then $\alpha$ governs flows into saving. As noted above, allowing the young to save typically greatly strengthens the effect of the saving mechanism. Hence aggregate saving is given by:

$$S_t^a = \alpha_t(1-\beta_t)\frac{(\eta R_t)^{\sigma-1}}{1+(\eta R_t)^{\sigma-1}}X_t^a. \tag{3}$$

The stochastic processes that generate $\alpha_t$ and $\beta_t$ are exogenous to the model and are the source of shocks to savings and net worth respectively; all we assume is that each agent knows his type at or before the beginning of adulthood and that $\alpha_t$ and $\beta_t$ are common knowledge at the beginning of period $t$ (and uncorrelated with the success of any project). Since agents cannot consume when young, the possibility of death in adulthood will not affect the behaviour of the young. Furthermore, since indirect utility is linear in adult wealth, we have the following very obvious remark:

*Remark 1:* In youth, all agents will aim to maximise their expected wealth in adulthood.

## 2.2 Projects

We now discuss the financial market imperfection of Repullo and Suarez (henceforth RS) incorporated in the model. As a result of this, agents can only borrow to fund projects if and only if their skill level is above a threshold value $\bar{s}_t$, derived below. This and the other implications of this section are summarised in remark 2 below.

As discussed above, projects can only be begun in youth, and yield returns in adulthood. As a result of remark 1, the analysis of projects closely follows that of RS. A project succeeds with probability $p$ yielding a return of $A$, or fails with probability $1-p$ with a zero return. As in RS, while these returns are publicly observable, and $p$ is an unobservable choice of the agent made after the project begins (here in youth). The reason the agent does not always choose $p = 1$ is that, regardless of the success of the project, she also receives in adulthood a private benefit $\phi(p, \lambda)A$ which is decreasing in $p$ (and the shift parameter $\lambda$) and is also unobservable to all except to the agent. The unobservability of $p$ and $\phi(.)$ is the source of *moral hazard* in the model, since financial contracts can then only be contingent on the success of the project.[7]

Due to a limited liability assumption, the lender cannot recover anything if the project fails (since the private benefit is unobservable), and so the contract simply specifies the gross borrowing rate $B_t$ that the borrower pays in the event of project success. The lowest $B_t$ for which lenders are willing to participate is given by

$$pB_t = R_t. \tag{4}$$

As in RS, the function $\phi(.)$ is identical across agents.[8] They work with a specific

[7] Note that the type of composition effects found in Matsuyama (2007) can be incorporated if entrepreneurs choose from a menu of projects $A_j$, $\lambda_j$, with suitable relationships between $A_j$ and $\lambda_j$ across $j$.

[8] If there two possible sorts of finance, $\phi$ can also depend on the sort of finance. As RS show, this can motivate the coexistence of both forms of finance in equilibrium. We do not include this here in order to present the dynamics in as simple a setting as possible (Cantore and Satchi, 2008, provides an extension of this dynamic framework to include two forms of finance).

functional form for $\phi(.)$, and we choose a similar form here:

$$\varphi(p, \lambda) = \frac{1}{2\lambda}(k - p^2), \tag{5}$$

generalizing trivially from RS by allowing $k \neq 1$.

It is much simpler to treat the private benefit as a cash flow (that can then be saved) rather than a benefit-in-kind (that could only be naturally interpreted as being consumed during the project) and to avoid unrevealing complexity, we do so. Suppose an agent with skill $s$ borrows $1 - \theta$ to fund her project. Since she needs to borrow at least $1 - s - w_t$, we must have $\theta \leq w_t + s$. As noted in remark 1, young agents at time $t$ will choose $p$ to maximise expected wealth in adulthood. This is:

$$pA - pB_t(1 - \theta) + \phi(p, \lambda)A. \tag{6}$$

Taking the resulting first order condition and substituting in $pB_t = R_t$, we get

$$Ap^2 - \lambda Ap + \lambda R_t(1 - \theta) = 0. \tag{7}$$

Agents can only borrow if $\theta$ is such that the first order condition for $p$ (7) has a real solution. Otherwise profits are decreasing in $p$, and the agent cannot borrow since the lender knows that he will set $p = 0$. This occurs iff $\theta \geq \bar{\theta}_t$, where

$$\bar{\theta}_t = 1 - \frac{\lambda A}{4R_t}. \tag{8}$$

$1 - \bar{\theta}_t$ is then the maximum an agent can borrow to fund a project. Hence only agents with skill level $s_t \geq \bar{s}_t$ can fund a project, where

$$\bar{s}_t = \bar{\theta}_t - w_t = 1 - w_t - \frac{\lambda A}{4R_t}. \tag{9}$$

In this case (the entrepreneur prefers the largest root),

$$p(\theta) = \min\left\{\frac{\lambda}{2}\left[1 + \left(\frac{\theta - \bar{\theta}_t}{1 - \bar{\theta}_t}\right)^{\frac{1}{2}}\right], 1\right\} \tag{10}$$

and the entrepreneur has expected income $u(\theta, R_t)$:

$$u(\theta, R_t) = \frac{1}{2}\left[Ap(\theta) - R_t(1-\theta)\right] + \frac{Ak}{2\lambda}. \tag{11}$$

Note that (8)-(11) imply that

$$\frac{\partial u}{\partial \theta} = \frac{R_t}{2}\left[1 + \left(\frac{1-\bar{\theta}_t}{\theta - \bar{\theta}_t}\right)^{\frac{1}{2}}\right] > R_t \quad \text{for all } \theta \in [\bar{\theta}_t, 1). \tag{12}$$

Since young agents who cannot invest in projects, i.e. those with $s < \bar{s}$, and are limited to physical storage. Given (12), all agents with $s \geqslant \bar{s}_t$ will invest in projects to the full extent they can if $u(\bar{\theta}_t) > \bar{R}w_t$. The latter occurs iff[9]

$$\frac{\lambda A}{8} + \frac{Ak}{2\lambda} > \bar{R}w_t. \tag{13}$$

For convenience, we summarise these results in the following remark:

*Remark 2:* An agent invests in a project iff she has skill $s \geqslant \bar{s}_t = \bar{\theta}_t - w_t = 1 - \frac{\lambda A}{4R_t} - w_t$. If $s \geqslant 1 - w_t$, i.e. the endowment is sufficient to wholly fund the project, she uses it to finance the project and stores the rest. If not, she invests all of the endowment in the project, and borrows the remainder at a gross borrowing rate $B = R_t/p(\theta)$ where $p = p(\theta)$ given by (10) is the probability of success of the project and $\theta = w_t + s$.

## 2.3 Loan supply and demand

Given remark 2, the demand for bank loans is simply:

$$L_t^d(\bar{s}_t, w_t) = \int_{\bar{s}_t}^{1-w_t} (1 - s - w_t) f(s) ds. \tag{14}$$

Note that, holding interest rates constant, how loan demand is affected by the endowment $w_t$ depends on the shape of the density function $f$. For example if $f(s)$ is decreasing on $[0, 1]$ (so population density falls as skill levels increase), an increase in the endowment $w_t$ will cause loan demand to rise so long as it does not cause the threshold skill level

[9] This is slightly different from the equivalent condition given in RS, since here we have replaced their heterogenous endowment with a homogenous one, with heterogeneity coming here from the introduction of 'skill.' This condition, which is mostly important for tractability, is satisfied for all of the results presented in the paper.

to fall below zero.[10] In this case, the fact that a high endowment allows more agents to invest outweighs the fact that each has to borrow less.

The demand for loans is met through the supply of domestic savings $S_t$. Total savings constitute aggregate adult saving, given by equation (3), since the young are limited to investment or physical storage. If $Z_t$ is the portion of funds that are saved via storage, and $1-\eta$ the portion consumed by the intermediation process, then

$$\eta\alpha_t(1-\beta_t)\frac{(\eta R_t)^{\sigma-1}}{1+(\eta R_t)^{\sigma-1}}X_t^a = \eta S_t = Z_t + L_t^d(\bar{s}_t, w_t). \tag{15}$$

The domestic interest rate $R_t$ will then move to clear the domestic loan market. $R_t$ is the (gross) interest rate offered to savers by the perfectly competitive set of intermediaries, who lend these funds to agents investing in projects. Since, for savers, the alternative to saving with intermediaries is storage which yields an exogenous gross return $\bar{R}$, in order for the intermediaries to attract any funds we must have

$$\eta R_t \geqslant \bar{R} \text{ and } Z_t \geqslant 0. \tag{16}$$

Clearly, at least one of the weak inequalities in (16) must hold with equality, giving us two possible types of equilibrium. In a *liquidity-rich* equilibrium, the supply of savings is large enough to push $R_t$ down to $\eta R_t = \bar{R}$ and savings $S_t$ are allocated between investing in projects $L_t^d$ and storage $Z_t \geqslant 0$. In a *liquidity-constrained* equilibrium, $\eta R_t$ remains strictly above $\bar{R}$, so all savings are allocated to project investment with $Z_t = 0$ and $\eta S_t = L_t^d$. Fluctuations in savings will then cause $R_t$ to change.

Aggregate adult wealth $X_t^a$ and the shocks $\alpha_t$ and $\beta_t$ are all predetermined at time $t$. Hence, taking $X_t^a$, $w_t = \beta_t X_t^a$ and $\alpha_t$ as given and writing $Z_t = 0$, (15), (14) and (9) yield a solution for $R_t$ and $\bar{s}_t$. If this is a valid solution (i.e. satisfies $\eta R_t \geqslant \bar{R}$) we must be in a *liquidity-constrained* equilibrium that is described by this solution. Otherwise a *liquidity-rich* equilibrium must hold, where $\bar{s}_t$ is given by substituting $R_t = \bar{R}/\eta$ into (9).

The description of the simple dynamics is then completed by noting that $\bar{s}_t$ and $R_t$ then determine $X_{t+1}^a$, the aggregate wealth of adult agents in period $t+1$ (subject to a possible technology shock which would be straightforward to add). The realisation of

[10] From (9), $\frac{\partial L_t^d}{\partial w_t} = \int_{\bar{s}_t}^{1-w_t} f(\bar{s}_t) - f(s)ds$. If $f(s)$ is decreasing on $[0,1]$ and $\bar{s}_t \geqslant 0$ then the integrand is positive.

the shocks $\alpha_{t+1}$ and $\beta_{t+1}$ then determine $w_{t+1} = \beta X^a_{t+1}$ and $\bar{s}_{t+1}$ and $R_{t+1}$ as described above. This determines $X^a_{t+2}$, and so on. From (10) and (11), the expression for $X^a_{t+1}$ in terms of $R_t$ and $\bar{s}_t$ is:

$$\begin{aligned} X^a_{t+1} &= \bar{R}w_t F(\bar{s}_t) + \int_{\bar{s}_t}^{1-w_t} \frac{1}{2}\left(Ap(s+w_t) - R_t(1-w_t-s) + \frac{Ak}{\lambda}\right) f(s)ds \quad (17) \\ &+ \int_{1-w_t}^{1} \left(\frac{A}{2}\left[\lambda + \frac{k}{\lambda}\right] - \bar{R}\left[1-w_t-s\right]\right) f(s)ds. \end{aligned}$$

The three separate terms in the above expression represent, respectively, the wealth of those who are unable to borrow, the wealth of those who borrow from banks to fund projects, and those who can fund projects without borrowing. We note that the steady state is then obtained by solving for the values of $w$, $\bar{s}$ and $R$ that keep $S$ and $X$ unchanged while holding $\alpha_t$ and $\beta_t$ at their means $\alpha$ and $\beta$. Finally we note that while (17) is the correct expression for $X^a_{t+1}$ in the case $0 < \lambda \leq 1$, the case with $\lambda > 1$ is equally straightforward to obtain and is omitted here for brevity.

# 3 Results

We now present some numerical simulations depicting the adjustment of the model to transient single period shocks. The model clearly has limited scope from the point of view of calibration and in interpreting the parameters below, it is perhaps best to see the model as an attempt to capture the flow of profits into three possible sources; namely consumption, saving and internal finance. We then want to determine how the variation of these flows affects the volatility of output and other variables of interest. Note, importantly, that we are conducting a relative exercise; rather than looking at absolute levels of persistence and amplitude, the primary purpose of the simulations is to compare the levels of output volatility in liquidity-rich and liquidity-constrained equilibria of similar parametrisation. This identifies the importance of the savings mechanism, which only operates in the liquidity constrained equilibria, while the net worth mechanism operates in both.

The strength of these two mechanisms will clearly depend on the relative size of the flows of profits (and wealth) into savings and net worth, which are respectively determined by the steady state values of $\alpha_t$ and $\beta_t$, $\alpha$ and $\beta$. Nonetheless, even when the flow of profits into savings, roughly $\alpha(1-\beta)/2$, is relatively small compared to the flows into the net worth of future agents, $\beta$, we find that the presence of the savings mechanism can substantially increase the degree of output volatility. Subject to the fact that this result depends on the presence of significant financial frictions in the model, it is relatively insensitive to the remaining parameters in the model with the exceptions described below.[11]

Given the relatively stylized nature of the model, it is important to stress that the purpose is not to demonstrate a particular level of persistence at a baseline calibration. Nonetheless it is also useful to initially note - in order to motivate the discussion - that the model does demonstrate non-trivial persistence to endowment (and technology[12]) shocks for a broad range of parameters.

## 3.1 Parametrisation

The are relatively few parameters to set. Since a period represents the length of the production process, we take a period to represent at least a year, though since we are interested in the relative volatility of the two regimes this has no qualitative impact on the results. For all the simulations, we normalize the rate of return on physical storage $\bar{R}$ to 1, and set $A = 1$, so that an agent with zero skill who did not need to borrow would make a return on a project equivalent to that of physical storage. We also set the cost of intermediation to 0, so $\eta = 1$. As might be expected, none of these choices is important for the results.

We use a very simple form for the density function $f(s)$ for the distribution of skill. We assume that there is a mass of $1-c$ agents who have zero skill, where $0 \leqslant c \leqslant 1$, and then that $f(s) = c$ on the support is $[0, 1]$. This form has the useful property that given $\lambda, A$ and the steady state interest rate value of $R_t$, say $R$, the percentage fluctuation in loan demand caused by a movement in interest rates is independent of $c$. $c$ represents the proportion of projects that would be viable were financial markets perfect. This is clearly somewhat arbitrary to calibrate, but initially we set $c = 0.5$. Density functions with a negative (positive) slope at the steady state, so that the population density falls (rises)

[11]Contact authors for further details. We avoid a full sensitivity analysis here for brevity.

[12]While technology shocks are not formally described here, they are very straightforward to introduce. The savings mechanism significantly increases persistence in response to single-period technology shocks; with serially correlated shocks it also increases amplitude.

as skill levels increase, cause an increase (decrease) in the level of persistence generated by the model.

The remaining parameters to choose are $\lambda$, $\sigma$, $k$, $\alpha$ and $\beta$. The approach we take is as follows. For a given choice of these parameters, we find the value of $k$ in (5) that results in a steady state with the real interest rate over one time period to be 2%, i.e. so that the steady state value of interest rate $R$ is equal to 1.02.[13] Since $R > \bar{R} = 1$, this must describe a liquidity-constrained equilibrium. We then increase the savings parameter $\alpha$ by a sufficient amount in order to reduce the $R$ to 1 obtaining a liquidity-rich equilibrium, maintaining the value of the remaining parameters including $c$. We then compare the response to shocks in both cases, the difference between the two indicating the effect of the savings mechanism. Given the uncertainty in interpreting the length of a period, this steady state choice of $R$ has no significant impact on the results; the value close to 1 merely allows a switch to a liquidity-rich equilibrium with a relatively small adjustment in the average savings rate.

Our baseline choices for $\lambda, \sigma, \alpha$ and $\beta$ are as follows. We set the intertemporal elasticity of substitution $\sigma = 1$.[14] $\lambda$, being inversely related to level of private benefit, determines the strength of financial frictions in the model, taking a lower value when financial frictions are greater. Financial frictions will have some impact on every entrepreneur who needs to borrow iff $\lambda \leqslant 1$. Setting $\lambda = 1$ then minimises financial frictions subject to this assumption, and is our baseline choice.

It then remains to set $\alpha$ and $\beta$, the steady state values of $\alpha_t$ and $\beta_t$ that determine respectively the proportions of adult wealth that go towards saving and internal finance. There is no clear-cut method of calibration, and the effect of varying these parameters is discussed below. $1 - \beta$ can be thought of as the proportion of profits that is consumed or saved, which might include bonuses or profit-related pay. A very rough proxy for this could be the dividends/earning ratio. This is a very volatile number, but $\beta = 0.5$ serves as again a rough approximation to this for U.S. data in recent decades. The savings ratio

[13] We might wish for a mechanism whereby project returns evolved in the long run to allow the real interest rate to adjust relative to some rate of time preference for example. Adjusting $k$ alters the expected project return of projects in a way that is independent of the probability project success, which is convenient for the 'difference-in-difference' type exercises we conduct, as changes in $k$ per se do not then affect financial frictions or the dynamics in the model.

[14] 0.2 would be an example of a typical estimate of $\sigma$ from a microeconomic study though many macroeconomic models use higher values. Lower values of $\sigma$ in this model lead both to increased volatility, and an increased relative impact of the savings mechanism on the volatility of output.

out of dividends, (which tends to be higher than the overall savings ratio), is then $\alpha/2$. As a starting value, we then set $\alpha = 0.25$.

## 3.2 Response to Shocks

With our baseline choices for $\lambda = 1$, $\sigma = 1$, $c = 0.5$, $\alpha = 0.25$ and $\beta = 0.5$, setting $k = 0.7123$ gives a liquidity-constrained equilibrium with a steady-state value of $R_t$, $R = 1.02$. Maintianing these values for $\lambda$, $\sigma$, $\beta$ and $k$ but increasing $\alpha$ by 0.02 then results in a liquidity-rich equilibrium in which $R = 1.0$. Comparing simulation responses with these two parameter sets allows us to describe the basic results of the model.

### 3.2.1 Savings Shocks

The first sense in which a liquidity-constrained equilibrium will be more volatile than a liquidity-rich one is in response to savings shocks. Figure 1a shows the percentage deviations of output, interest rates, endowments and threshold skill from steady state in response to 1% single period negative shocks to savings. Output refers to total project output including private benefits, though the results are not sensitive to the latter.[15]

These responses are absent in a liquidity-rich equilibrium, since an increase in the savings rate has no effect on interest rates, and therefore investment. Figures 1b and 1c show the same shocks when the elasticity of substitution is decreased to $\sigma = 0.2$ and when financial frictions are increased so that $\lambda = 0.5$. In both cases the amplitude and persistence of the responses are increased.

### 3.2.2 Endowment Shocks

Perhaps the more interesting case is that of endowment shocks. We now consider a one-period 1% negative shock to the parameter $\beta_t$ that governs the flow into endowments, with the baseline parametrisation. Note that from (15), the fall in $\beta_t$ also increases savings as it essentially represents a switch from internal finance to dividends, some of which are saved. We refer to this as an 'ordinary' endowment shock, with a 'pure' endowment shock referring to the situation where $\alpha_t$ also changes in the first period to offset the initial effect on savings, so all that happens in the first period is a fall in endowments $w_t$. The

[15] The equivalent of (17) for output is
$Y^a_{t+1} = \int_{\bar{s}_t}^{1-w_t} \frac{1}{2}\left(Ap(s+w_t) + R_t(1-w_t-s) + \frac{Ak}{\lambda}\right) f(s)ds + \int_{1-w_t}^{1} \frac{A}{2}\left[\lambda + \frac{k}{\lambda}\right] f(s)ds.$

distinction between these two types of shocks is only relevant in a liquidity-constrained equilibrium, since fluctuations in the supply of savings have no effect in a liquidity-rich one.

Figures 2a and 2c show an pure endowment shock in the two equilibria, giving the percentage deviations of output, interest rates, endowments and threshold skill from steady state in response to a negative 1% single period negative shock to $\beta_t$. However, in the liquidity-constrained equilibrium, the shock results in a fall in savings in the periods following the shock. This results in a 'hump-shaped' output response that is both more persistent and of approximately 25% greater amplitude. In figure 1b, we show the effect of an 'ordinary' endowment shock. Despite the fact that the associated rise in savings has no effect in the liquidity-rich equilibrium and mitigates the output response in the liquidity-constrained equilibrium, the amplitude of the output responses is similar and persistence is significantly greater in the liquidity-constrained equilibrium.

Figure 3 shows shows how a liquidity-constrained economy responds to an pure endowment shock with various parameter changes; figure 3a, where we vary financial frictions so that $\lambda = 0.5$; figure 3b, where we reduce $\sigma$ to 0.2 and figure 3c, where we reduce the proportion of viable projects to $c = 1/3$. Figures 4a-4c show the responses in a liquidity-rich equilibrium for the same parameter changes (it is the difference between figures 3 and 4 that are important here, since the effect of a given endowment shock depends on the size of the economy.) We can see that all of these parameter changes yields an increased difference in volatility between the two equilibria when compared to the baseline (of course a combination of these parameter changes yields a greater difference still.)

The stronger the savings flow relative to the endowment flow the greater the difference in volatility between the two equilibria is. Figures 5a-5c show the equivalent of figures 2a-2c, except with $\beta$ raised to 2/3, in which the difference in output volatility is somewhat mitigated. However, any of the parameter changes discussed in the above paragraph would still yield a clear difference in volatility between the two regimes.

#### 3.2.3 The cost of intermediation

We finally note how the cost of intermediation affects the economy. A rise in the cost of intermediation, given by a fall in $\eta$, unambiguously reduces the size of the economy as would be expected. However it has ambiguous effects on the likelihood of a liquidity-constrained equilibrium. One the one hand and higher value of $R$ is needed for a liquidity-constrained equilibrium since the required condition is $\eta R > \bar{R}$. On the other hand $R$ is pushed up because the lending of intermediaries is reduced. The most important parameter in determining this is the elasticity of substituion $\sigma$. As a very rough rule of thumb, for a broad range of values for the remaining parameters, a rise in the cost of intermediation will make a liquidity constrained equilibrium more likely if the elasticity of substitution is less than 0.5.[16] Since this is plausible, it is possible then that a permanent rise in the cost of intermediation may not only shrink the size of the economy but also lead to a permanently more volatile regime.

### 3.3 Further work and Conclusions

The model is a stylized description of various financial flows. As a result, it is difficult to calibrate with precision, and the model needs to be more sophisticated to capture properly some important aspects of these flows, such as the division of flows between corporate investment and corporate saving, the latter being becoming increasingly important in many modern economies. However by setting them in a simple framework, the model allows consideration of shocks to these flows that are perhaps not often considered in much of the macroeconomic literature on credit market imperfections. The great moderation, and its possible current demise, yields an obvious context for the results.

We note some possible extensions to the above framework. Firstly, the original Repullo and Suarez paper (2000) contained two loan markets, each associated with a different level of moral hazard. An extension to this is described in Cantore and Satchi (2008). We are also yet to fully explore whether certain distributions for the skill level $s$ could lead to multiple equilibria. The simplicity of the OLG framework also allows the possibility of extending the entrepreneurial phase of life and, by tracking distributions of wealth, drawing links between the level of financial frictions and inequality. Finally, we note that it might be relatively straightforward to marry this model - by offering a menu of project choices with differing expected returns and corresponding levels of moral hazard - to the credit composition story of Matsuyama (2007) potentially generating some fairly involved dynamics.

[16]This should be viewed more as sufficient condition than a necessary one, as the actual threshold can be much higher if financial frictions are strong etc.

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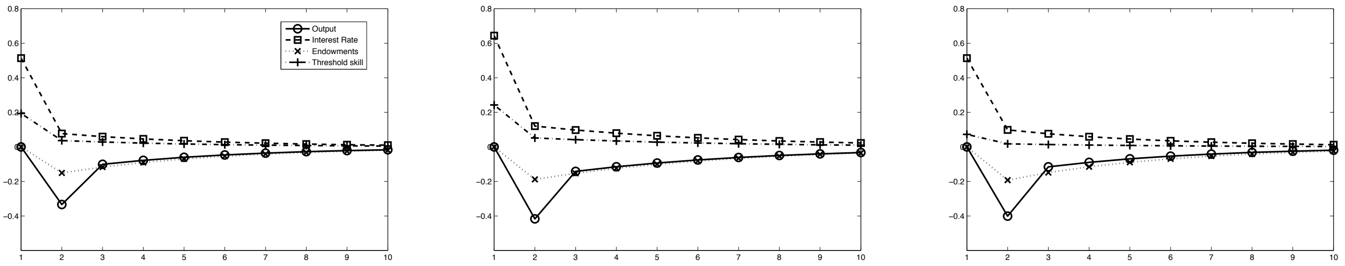
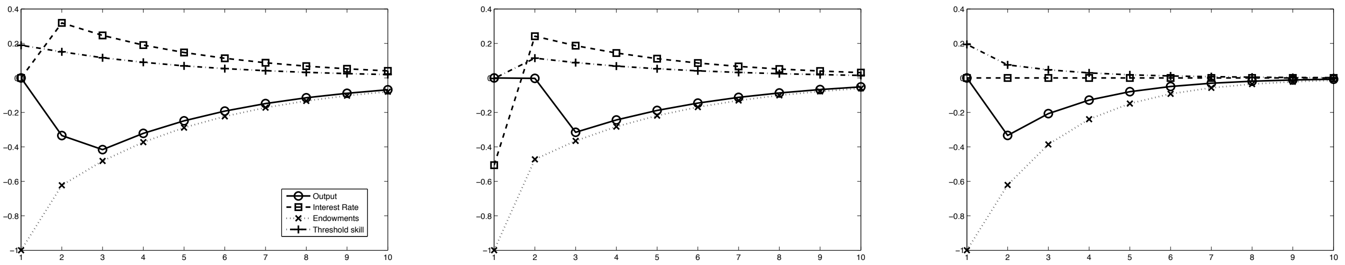


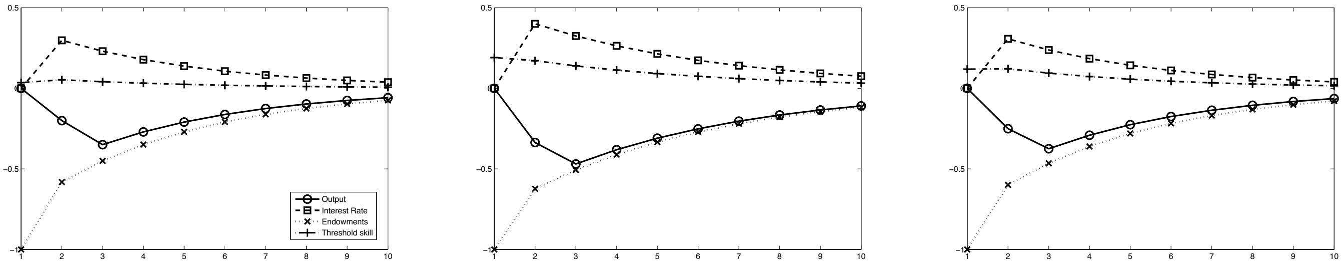

(a) baseline calibration (b) $\sigma = 0.2$ (c) $\lambda = 0.5$

Figure 1: 1% negative shock to savings



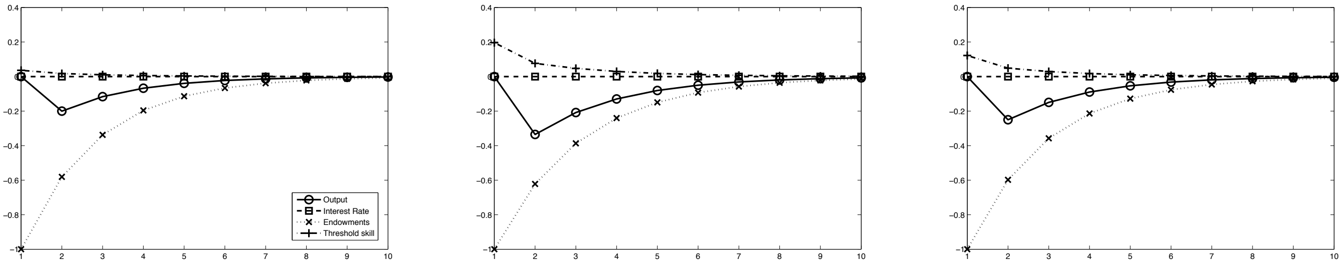

(a) 'pure' shock Liquidity-constrained eq'm (b) 'ordinary' shock Liquidity-constrained eq'm (c) 'pure' shock Liquidity-rich eq'm

Figure 2: 1% negative shock to endowments at baseline calibration



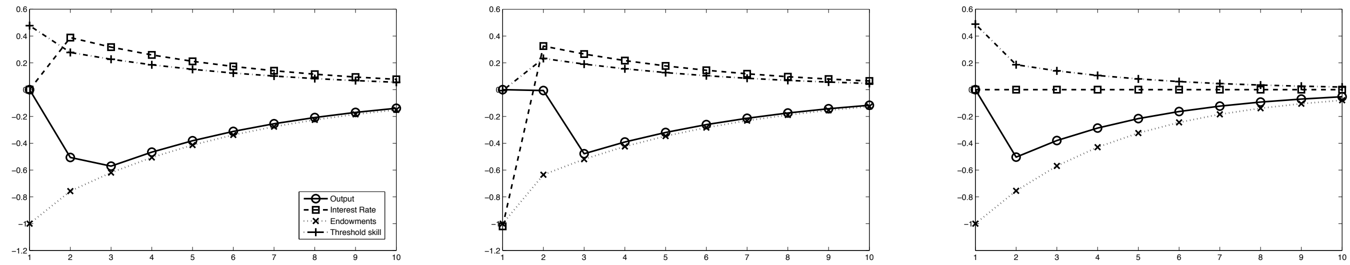

(a) $\lambda = 0.5$ (b) $\sigma = 0.2$ (c) $c = 1/3$

Figure 3: 1% 'pure' endowment shock in the Liquidity-constrained economy




(a) $\lambda = 0.5$ (b) $\sigma = 0.2$ (c) $c = 1/3$

Figure 4: 1% 'pure' endowment shock in the Liquidity-rich economy




(a) 'pure' shock Liquidity-constrained eq'm (b) 'ordinary' shock Liquidity-constrained eq'm (c) 'pure' shock Liquidity-rich eq'm

Figure 5: 1% negative shock to endowments with $\beta = 2/3$