University of Kent

Department of Economics Discussion Papers

R&D Spillovers, Concentration and Market Performance.

Anna Stepanova

February 2009

KDPE 0901

R&D Spillovers, Concentration and Market Performance.

Anna Stepanova[*†]

This version: February 2009

Abstract

In a two-stage R&D game of process innovation, we investigate the effect of exogenously changing R&D spillovers and market concentration on the equilibrium level of effective cost reduction, total output, profits and social welfare. Interpreting spillover as a measure of patent protection, we find that weaker patent protection results in less R&D. We also show that firms prefer weaker patent protection, but social welfare is maximized for higher levels of patent protection. In terms of market concentration we show that firm profits decrease with increasing numbers of firms. Social welfare is typically maximized under oligopoly with the optimal number of firms depending on the level of spillover and efficiency of R&D investment.

JEL codes: C72, L13, O31

Keywords: oligopoly R&D competition, spillover process, cost reduction, market concentration.

[*]Department of Economics, University of Kent, Keynes College, Canterbury, CT2 7NP, UK. E-mail: A.Stepanova@kent.ac.uk

[†]I would like to thank Anette Boom, Peter Norman Sørensen, Peter Sudhölter and especially Rabah Amir for helpful comments and ideas.

1 Introduction

There is a wide theoretical and empirical literature devoted to the discussion of the performance of firm's R&D activities. Among them there are papers that demonstrate the effects of R&D on the productivity of the innovating firm itself as well as on well-being of consumers and the society at large (e.g. De Bondt, Slaets and Cassiman, 1992). It is widely recognized that effects are generated not only due to the direct effect of R&D, but also due to the effects of R&D spillovers. In this paper we look at how firm's profit and social welfare depend on the amount of R&D spillovers as well as on market structure.

Our focus will be on the theoretical impact of intra-industry spillovers, i.e. spillovers between firms within an industry.[1] We use the two-stage model of Kamien, Muller and Zang (1992), henceforth, KMZ. In the first stage firms decide on the level of their R&D investment. The second stage happens according to the standard Cournot scenario. The firm's unit cost in the second stage depends on its own R&D investment and on spillovers from R&D investments of rival firms. Thus there is competition as well as spillover externalities within the same industry.

We first solve the model for the equilibrium R&D investment, effective cost reduction, per-firm and industry output and price. Our main objectives are to look at the behavior of these equilibrium characteristics as well as per-firm equilibrium profit and social welfare as (1) the level of spillover changes, and (2) the market concentration (as measured by the number of firms) changes.

Spillover externalities are captured by a spillover parameter β that can range from zero, implying no spillovers to one, implying perfect spillovers in the sense that R&D investment is a pure public good. Two possible interpretations of spillover parameter are patent enforcement and location, with a low spillover corresponding to high patent protection and relatively large distances between firms. An increase in the spillover parameter leads to a decrease in the equilibrium R&D investment and per-firm equilibrium output. Applying these results, we find that per-firm equilibrium profit is maximized under an intermediate to high degree of spillover. This is because increasing spillover offers the opportunity to free-

[1] There are two general types of spillovers: the inter-industry R&D spillovers and the intra-industry ones (e.g., see Bernstein and Nadiri, 1989, Wolf and Nadiri, 1993). Most of the theoretical papers (see, e.g., De Bondt et al. 1992, Simpson and Vonortas, 1994) focus on the impact of intra-industry R&D spillovers. There are some exceptions, e.g. Steurs (1995).

ride on the R&D investments of other firms and is associated with lower per-firm equilibrium output. The equilibrium social welfare is maximized for smaller values of the spillover. This is because the equilibrium industry output and, therefore, the equilibrium consumer surplus is larger the smaller are spillovers.

With regard to market concentration, we find that as the number of competing firms increases there is a decrease in the equilibrium R&D investment and per-firm equilibrium output, but an increase in the equilibrium industry output. Applying these results, we find that per-firm equilibrium profit decreases as the number of firms increases. With regard to social welfare we find that the type of market structure under which it reaches its maximum depends on the level of spillover as well as on the efficiency of R&D investment. If spillovers are large and R&D investment is efficient enough, then equilibrium social welfare is maximized under a monopoly. In this case, monopoly also leads to the most R&D investment. If R&D investment is relatively costly and the spillover parameter takes extreme values then perfect competition may maximize social welfare. This is because firms have no incentive to invest in R&D and we approximately have the standard static Cournot result. For most parameter values the equilibrium social welfare is inverse U-shaped in the number of competing firms and so oligopoly maximizes social welfare.

Closely related results are due to De Bondt, Slaets and Cassiman (1992). They also looked at the effect of R&D spillovers and market concentration on profit and social welfare, but using the model of d'Aspremont and Jacquemin (1988), henceforth AJ. As already mentioned, we adopt the model of KMZ and so our results differ a-priori. As discussed in detail by Amir (2000), the AJ model is distinguished from the KMZ model in the way the spillover processes are modeled and can lead to substantially different results and, therefore, policy recommendations regarding R&D cooperation.[2] In a discussion on the validity of these models Amir writes that "the AJ model appears to be of questionable validity for large values of the spillover parameter". It is, therefore, noteworthy that De Bondt, Slaets and Cassiman (1992) find that profit and social welfare are maximized for large value of the spillover parameter. This motivates our use of the KMZ model and we do indeed get different conclusions, for example, we find that equilibrium social welfare is maximized under

[2] In the AJ model firms' decision variables are the levels of reduction of their unit cost of production, and spillover effects take place additively in cost reductions, i.e. in R&D outputs. On the other hand, in the KMZ model, firms decision variables are their R&D investment levels, and spillover effects are additive in these expenditures, i.e. in R&D inputs (see also Martin, 1998).

a much lower level of spillover. We will discuss these differences in more detail below.

The rest of the paper is structured as follows. In Section 2 we present the basic theoretical framework of R&D/quantity competition and provide its equilibrium characteristics. In Section 3 we investigate the effects of the spillover level on the equilibrium characteristics of the model. This is followed by Section 4 that presents our findings on changes of the equilibrium characteristics with respect to (exogenously changed) market concentration. In Section 5 we offer some concluding remarks.

2 Model and equilibrium

Consider an industry with n identical firms facing a linear inverse demand function $P = a - bQ$, where Q is the industry output and $0 < b \leq 1$. Each firm has an initial unit cost of production c, where $0 < c < a$, and participates in the following two-stage noncooperative game. In the first stage firms simultaneously must decide how much to invest in an R&D process that will reduce their unit cost. In the second stage, upon observing the new unit costs, firms compete as in the standard quantity setting Cournot model. When each firm decides how much it would like to invest on R&D it takes into account the other's R&D expenditures as well as its resulting second stage market profit.

More precisely, in the first stage firms simultaneously decide how much to invest on R&D: $y_i \geq 0, i = \overline{1,n}$. The effective cost reduction of firm i is determined by the individual R&D investments of all firms is as follows

$$X_i = \sqrt{\frac{2}{\gamma}(y_i + \beta \sum_{j=1, j \neq i}^{n} y_j)}, \tag{1}$$

where $\gamma > 0$ is a parameter measuring the efficiency of R&D investments and a spillover parameter $\beta, \beta \in [0,1]$. The efficiency of R&D investment increases with decreasing γ. As is standard, we shall refer to γ (and subsequently, $b\gamma$) as the cost of R&D. If $\beta > 0$ then each firms unit cost is reduced by the R&D investments of other firms as well as by its own R&D investment. The larger is β then the higher is this spillover effect.

The decisions of the first stage make the unit cost of firm $i, i = \overline{1,n}$ in the second, production, stage equal to $c - X_i$. So the unit cost of each firm is determined by the R&D investments of all the firms. In the second stage firms engage in Cournot competition. Each firm's objective in the game is to maximize its individual second-stage production profit, $\pi_i, i = \overline{1,n}$, net of it's first stage R&D investment, $y_i, i = \overline{1,n}$. We restrict consideration to subgame perfect equilibria (SPEs) of the game. Firm i's second stage production profit is the Cournot equilibrium profit and has the form

$$\pi_i = \frac{1}{(n+1)^2 b}\left(a-(n+1)c_i+\sum_{j=1,j\neq i}^{n} c_j\right)^2, i,j=\overline{1,n},$$

where $c_i = c - X_i$ is the firm i's reduced unit cost and $c_j = c - X_j, j = \overline{1,n}, j \neq i$, are the reduced unit costs of its rivals. Hence, firm i's overall profit is

$$\begin{aligned}\Pi_i &= \pi_i - y_i \\ &= \frac{1}{(n+1)^2 b}\left[a-(n+1)(c-X_i)+\sum_{j=1,j\neq i}^{n}(c-X_j)\right]^2 - y_i \\ &= \frac{1}{(n+1)^2 b}\left[a-c+nX_i-\sum_{j=1,j\neq i}^{n}X_j\right]^2 - y_i.\end{aligned} \tag{2}$$

where $X_i = \sqrt{\frac{2}{\gamma}(y_i + \beta \sum_{j=1,j\neq i}^{n} y_j)}$ and $X_j = \sqrt{\frac{2}{\gamma}(y_j + \beta \sum_{k=1,k\neq j}^{n} y_k)}$, $i,j = \overline{1,n}$.

Throughout the paper we shall use the standard (Marshallian) definition of social welfare as the sum of the consumer and producer surplus:

$$\begin{aligned}SW &= \int_0^Q P(t)dt - TC \\ &= aQ - \frac{b}{2}(Q)^2 - \sum_{i=1}^{n}((c-X_i)q_i + y_i).\end{aligned} \tag{3}$$

2.1 Symmetric subgame perfect equilibrium

Assuming that all firms behave identically, we now solve for the symmetric SPE as done by KMZ. We obtain the optimal first stage R&D investment (SPE) by solving $\frac{\partial \Pi_i}{\partial y_i} = 0, i = \overline{1,n}$ with respect to y_i and setting $y_1 = ... = y_n = y^*$. Given this we can find the equilibrium characteristics of the model:

Proposition 1 *(KMZ) Assuming $b\gamma > 2$, the per-firm equilibrium R&D investment, the* *per-firm equilibrium effective cost reduction* , X^*, *per-firm equilibrium output*, q^*, *the total equilibrium output*, Q^*, *and the equilibrium price*, P^*, *are*

$$\begin{aligned}
y^* &= \frac{2(a-c)^2(n(1-\beta)+\beta)^2\gamma}{(1+(n-1)\beta)D^2} \\
X^* &= 2(a-c)\frac{(n(1-\beta)+\beta)}{D} \\
q^* &= (a-c)\frac{(n+1)\gamma}{D} \\
Q^* &= (a-c)\frac{n(n+1)\gamma}{D} \\
P^* &= a-(a-c)\frac{n(n+1)b\gamma}{D}
\end{aligned}$$

where $D = [(n+1)^2 b\gamma - 2(n(1-\beta)+\beta)]$.

The effective cost reduction X has natural boundary: $X \in [0, c]$. Given Proposition 1 to keep $c - X^* > 0$ across the feasible ranges of n and β, we shall assume $\frac{a}{c} \leq 4$. Recall that $\frac{a}{c} > 1$. The condition $b\gamma > 2$ is also necessary to avoid boundary solutions. When $b\gamma \leq 2$ for some and sometimes for most values of n (depending on $b\gamma$ and β) there is no interior solution y^* to the maximization problem. For $b\gamma \leq 2$ it would be necessary to check all possible boundary conditions. For simplification we will assume $b\gamma > 2$ throughout the remainder of the paper.

3 Effects of the level of spillovers

In the present section we shall show the impact of the spillover parameter β on the model's equilibrium characteristics. In doing so we shall see how the level of spillover affects firms' profit and social welfare and, therefore, see what level of spillover parameter maximizes them. First, we need to look at how the level of spillover affects the basic equilibrium characteristics such as effective cost reduction.

Intuitively, there is a free-rider effect: the higher the spillover parameter the more cost-reducing knowledge the firm can gain for free, the smaller should be the investment of the firm on the R&D activity. KMZ show that this intuition is correct and the equilibrium level of per-firm R&D investment, y^*, decreases as the spillover parameter increases from 0 to 1.

This means that if we look at the equilibrium effective cost reduction X^* there are two opposite effects. As we can see from equation (1) there is a positive direct effect: the larger is the spillover parameter the larger is the unit cost reduction for any level of R&D investments, $y_j, j = \overline{1,n}, j \neq i$. There is, however, the negative indirect effect discussed above: the larger the spillover parameter the smaller the per-firm equilibrium R&D investment, y^*. In Proposition 2 we demonstrate that this indirect effect is stronger than the direct one. With this it is straight forward to show the effect of β on q^*, Q^* and P^*.

Proposition 2 *An increase in the spillover level β causes:*

a) a decrease in the equilibrium effective cost reduction X^*;

b) a decrease in the per-firm equilibrium output q^*;

c) a decrease in total equilibrium output Q^*;

d) an increase in the equilibrium market price P^*.

Note that the 2-firm version of this result can be found in Amir (2000). Our result is stated for the case of n firms, but confirms the result of Amir: the equilibrium "effective levels of R&D (i.e., the sum of own and spillover levels) decrease with the spillover rate".

De Bondt, Slaets and Cassiman (1992) get different results. They showed (see their Propositions 2 and 3) that in AJ model effective R&D and firm's output are maximized for spillover equal to $\frac{1}{2}$. Our Proposition 2, in contrast, says that they are maximized at spillover equal to 0. To make sure that we are comparing like with like, let we refer to the Corollary 4.4 of Amir (2000). He defines the maximal value of spillover $\beta_{\max} = \frac{\sqrt{n}-1}{n-1}$ for which R&D processes of AJ and KMZ models are equivalent. He further concludes that for $\beta > \beta_{\max}$ the validity of the AJ model is questionable, while the KMZ model is valid for the entire range of spillovers.[3] Note that $\beta_{\max}$ is always less than $\frac{1}{2}$ and so, the result of De Bondt, Slaets and Cassiman (1992) falls into the region of questionable validity.

We now proceed to study the influence of the spillover rate β on the profitability of firms and social welfare at the equilibrium.

At first we investigate the behavior of the per-firm equilibrium profit Π_n^*. Using Proposition 1 and that Π_n^* has the form $(P^* - c + X^*)q^* - y^*$ we get

$$\Pi_n^* = \frac{(a-c)^2\gamma((n+1)^2 b\gamma(1+(n-1)\beta) - 2(\beta + n(1-\beta))^2)}{(1+(n-1)\beta)D^2},$$

[3] For spillovers above $\frac{\sqrt{n}-1}{n-1}$ joint returns to scale (in R&D expenditure and number of firms) are increasing for the AJ model, while it is non-increasing returns to scale for the KMZ over entire range of spillovers.

where $D = [(n+1)^2 b\gamma - 2(n(1-\beta)+\beta)]$.

Next, to investigate the impact of the spillover parameter on society's well-being, we analyze the behavior of the equilibrium social welfare, SW^*. From equation (3), the equilibrium social welfare can be calculated to be

$$SW^* = \frac{(a-c)^2\gamma n((n+1)^2 b\gamma(1+(n-1)\beta)(n+2) - 4(\beta+n(1-\beta))^2)}{2(1+(n-1)\beta)D^2}.$$

The following proposition details the effects of the spillover parameter on the per-firm equilibrium profit and the equilibrium social welfare for any fixed number of firms, n, and any $b\gamma$.

Proposition 3 *(a) There exists $\beta^* = argmax_\beta \Pi_n^*$ which depends on $n, b\gamma$ and always lies in the interval $(0.56, 0.74)$ such that the per-firm equilibrium profit Π_n^* is increasing for all $\beta < \beta^*$ and decreasing for $\beta > \beta^*$;*

(b) There exists $\beta^{**} = argmax_\beta SW^*$ which depends on $n, b\gamma$ and varies in the interval $(0, 0.35)$ such that the equilibrium social welfare SW^* is increasing for all $\beta < \beta^{**}$ and decreasing for $\beta > \beta^{**}$, while at β^{**} it reaches its maximum.[4]

We will discuss the two parts of Proposition 3 in turn. Part (a) of Proposition 3 shows that the maximal equilibrium profit is achieved under a surprisingly high, though moderate degree of spillover. Thus, in particular, neither zero nor full spillovers will maximize the per-firm equilibrium profit Π_n^*. Similar results were obtained by Amir and Wooders (1999) in a two-stage duopoly model with a one-way spillover structure. It can be shown by an extended computation that, as the parameters $n, b\gamma$ vary within the confines of our assumptions, the privately optimal spillover β^* always lies in the interval $(0.57, 0.73)$. This suggests that firms' preference for intermediate levels of spillovers is a feature that is robust to the type of spillover process in the industry. For example, when the number of firms is fixed at 4 and the cost of R&D, $b\gamma$, is fixed at 2.1, the maximum of equilibrium profit occurs under the spillover level $\beta^* \approx 0.62$.

To give some intuition for why β^*does not take extreme values, consider the equation

$$\frac{\partial \Pi_n^*}{\partial \beta} = \frac{2(a-c+X^*)}{(n+1)^2 b}\frac{\partial X^*}{\partial \beta} - \frac{\partial y^*}{\partial \beta}.$$

[4] *As we discuss in the appendix: β^* is the unique real solution of a qubic equation $\frac{d\Pi_n{}^*}{d\beta} = 0$ and β^{**} is the unique real solution of a qubic equation $\frac{dSW^*}{d\beta} = 0$.*

As the spillover parameter β increases there are two competing forces on the per-firm equilibrium profit: (a) A positive effect where as β increases, the firm's equilibrium R&D investment falls, resulting in decreasing fixed cost and therefore, higher per-firm equilibrium profit. (b) A negative effect where as β increases, the firm's equilibrium effective cost reduction decreases, resulting in a higher unit cost and, therefore, lower per-firm equilibrium profit. As we discuss in the Appendix when $\beta = 0$ the positive effect (a) dominates the negative effect (b) and when $\beta = 1$ the negative effect (b) dominated the positive one.

Part (*b*) of Proposition 3 states that the equilibrium social welfare is maximized under a moderate to low level of the spillover parameter.

To understand the behavior of equilibrium social welfare we look at its two components: consumer and producer surplus. The equilibrium consumer surplus is decreasing through all the valid range of the spillover parameter because total output is decreasing. Thus consumers do best if the spillover parameter is zero because then the equilibrium price is lowest. The behavior of the producer surplus as a result of spillover parameter change is determined by the behavior of the per-firm equilibrium profit (since $\frac{dPS^*}{d\beta} = n\frac{d\Pi_n^*}{d\beta}$). Hence, part (a) of Proposition 3 can be viewed as the one describing the changes in the equilibrium producer surplus. Combining consumer and producer surplus we can see why β^{**} takes values less than β^*. Also as n tends towards infinity, β^{**} tends to zero.

Adopting the model of d'Aspremont and Jacquemin (1988), De Bondt, Slaets and Cassiman (1992) found (Proposition 4) that in the case of homogeneous oligopoly, "profitability and welfare achieve a maximum for an intermediate magnitude of spillovers between $\frac{1}{2}$ and 1". They also find that the level of spillover parameter that maximizes the per-firm equilibrium profit is higher than the one at which social welfare is maximized. There are clearly some similarities between their Proposition 4 and our Proposition 3. There are, however, very important differences. De Bondt et al. find that the equilibrium social welfare attains its maximum for a relatively large level of spillover (between 0.5 and 1) while we find that the equilibrium social welfare is maximized for a moderate to low level of the spillover parameter (between 0 and 0.35). Futherrmore, the model used by De Bondt, Slaets and Cassiman (1992), as explained above, is of questionable validity for $\beta \geq \frac{1}{2}$. So even though in terms of per-firm equilibrium profit differences in the optimal spillover parameter are not so stark, it is difficult to say that out results are consistent.

4 Effects of market concentration

In this section we investigate how the market equilibrium performance responds to varying market concentration, as measured by the exogenous number of firms. The first proposition clarifies the effect of the market concentration on the equilibrium effective cost reduction, X^*,the per-firm equilibrium output, q^*, the total equilibrium output, Q^* and the equilibrium price, P^*.

Proposition 4 *As the number of competing firms increases we observe:*
(a) *a decrease in the equilibrium effective cost reduction* X^**;*
(b) *a decrease in the per-firm equilibrium output* q^**;*
(c) *an increase in the total equilibrium output* Q^**;*
(d) *a decrease in the equilibrium market price* P^**.*

Part (a) of Proposition 4 shows that the higher the number of firms on the market the lower the per-firm equilibrium effective cost reduction, X^*. This must mean that the equilibrium R&D investment, y^*, has decreased. It does so because the larger the number of firms the higher the incentive to free ride on R&D investments of other firms. The lower the equilibrium effective cost-reduction, X^*, the higher the firm's equilibrium unit cost leading to the lower per-firm equilibrium output. This effect reinforces the "standard" incentive for a firm to reduce its output when there is more competition.

Parts (c) and (d) of Proposition 4 show that, as in the case of the standard Cournot oligopoly, the model of this paper is quasi-competitive, i.e. industry output rises and price falls as number of competing firms increases. Part (b) and (c) of Proposition 4 are consistent with the standard Cournot result that as the number of firms increases (keeping unit cost the same) the per-firm equilibrium output decreases and the industry output increases (Amir and Lambson, 2000). Note, comparing parts (c) of Proposition 2 and 4, that total equilibrium industry output, Q^*, decreases with β, but increases with n. This difference merely reflects the increased number of firms.

Similar to parts (a), (c) and (d) of Proposition 4, Dasgupta and Stiglitz (1980) find that in the presence of entry barriers the per-firm R&D investment decreases, the industry output increases and the equilibrium market price decreases with the number of firms[5].

[5] In their model the market structures are endogenous.

It follows from Proposition 4 that, in the framework of this model, monopoly is the best of all market structures in terms of providing the maximum equilibrium cost-reduction and, therefore, the ability for firms to enjoy the minimum unit-cost of production. This is consistent with the Schumpeterian view that monopoly is the market structure that may lead to maximum R&D investment. (Schumpeter, 1942) The other pole of the market structures, perfect competition[6], leads to the least equilibrium cost-reduction for each firm, therefore, to the highest unit cost of production. However, in the theoretical literature there are papers that reflect an opposite view that a competitive market structure is the one that promotes more innovation than monopoly (see, e.g. Arrow, 1962).

The relationship between market concentration and R&D activity has been the subject of a large empirical literature. (see Kamien and Schwartz, 1975, Levin, Cohen and Mowery, 1985). Typically, in contrast to what we find, they observe an inverse U-shaped relationship between the market concentration and innovation. One of the possible explanations for this difference could be the endogeneity of the market concentration. We take the number of firms as exogenous. If we allow for endogeneity we may find that markets with a relatively low concentration have different characteristics (for example, different valuer of $b\gamma$ and β) that result in a positive correlation between R&D activity and market concentration.

Our next result concerns the effect of market concentration on the per-firm equilibrium profit.

Proposition 5 *The equilibrium profit per firm* Π_n^* *decreases with the number of firms* n*, regardless of the level of spillovers.*

This result is also consistent with the standard Cournot model results[7]. That it should be, however, is not something that would obviously extend to the present two-period setting with R&D investments. This is because each firm's equilibrium investment on R&D decreases when there is an increased number of rivals. This has a positive effect on the per-firm equilibrium profit. Proposition 5 shows that the decrease in profit resulting from higher competition is stronger than this positive change.

[6] Technically, it is not a perfect competition because of the presence of fixed costs. Though we choose to use this concept as it is the closest market structure that reflects a described situation in the market.

[7] In fact, investigating the effects of exogenous entry on market performance measures in standard Cournot competition with very general demand and cost functions, Amir and Lambson (2000) report that the only result that always holds is that per-firm profit decreases with the number of firms in the market.

A similar result, but in the framework of the AJ model, was found by De Bondt, Slaets and Cassiman (1992), suggesting that this result is rather robust to the way the spillover process is modeled, at least in the framework of linear demand and production costs and quadratic R&D costs.

Our final objective in this section is to describe the changes in equilibrium social welfare induced by changes in the market concentration and, therefore, find the market structure that is optimal from the point of view of society. We say that there exist a finite equilibrium social welfare maximizing number of firms n^*if $n^* = \arg\max_{n \in Z_+} SW^*$ exists. Likewise, we say that equilibrium social welfare is inverse U-shaped in n if $n^* = \arg\max_{n \in Z_+} SW^* \geq 2$, and the value of SW^* is increasing in n when $n < n^*$ and decreasing in n when $n > n^*$.[8]

The following proposition provides the results on the changes in equilibrium social welfare induced by changes in the market concentration (as measured by the number of firms).

Proposition 6 *The equilibrium social welfare SW^* is*

(a) maximized at $n^* = 1$ for β sufficiently large (at least 0.91*) and $b\gamma$ sufficiently small (at most* 2.14*)*[9]*;*

(b) increasing in n when $\beta = 0$ and $b\gamma > 2(2+\sqrt{3})$ and when $\beta = 1$ and $b\gamma > 4$;

(c) inverse U-shaped, otherwise.

Proposition 6 shows that equilibrium social welfare can be maximized under any possible market structure: monopoly, oligopoly or perfect competition. Which of these market structures would be the optimal one in terms of providing the optimum social welfare is determined by the values of the spillover parameter, β, and the cost of R&D $b\gamma$. This is illustrated in Table 1.

In order to understand Proposition 6 it is interesting to ask why monopoly or perfect competition can be the market structure that maximizes equilibrium social welfare.

In the case of monopoly (part (a) of Proposition 6): we know that monopoly has the highest equilibrium effective cost reduction (see Proposition 4). Moreover, a small $b\gamma$ means a higher efficiency of R&D investment so any cost reduction is larger and the difference between monopoly and other market structures is amplified. Less intuitive is why a large β means monopoly maximizes social welfare. Looking at Proposition 2 we see that large

[8] Note, that this does not necessarily mean that SW^* is concave in n and so, literally, inverse U-shaped.

[9] *For each β there exists λ such that for $b\gamma < \lambda$, $n^* = 1$ maximizes social welfare.*

β is associated with lower effective cost reduction, but also lower consumer surplus. This later effect means that when β is large, changes in consumer surplus are relatively small with respect to market structure. These features make monopoly the market structure that maximizes social welfare when $b\gamma$ is small and β is large as we see in Table 1. This result is consistent with the Schumpeterian hypothesis that welfare loss caused by production inefficiency of monopoly can be more than compensated by the gains from R&D activity.[10]

In the case of perfect competition (part (b) of Proposition 6): when spillover $\beta = 0$ and $b\gamma$ is large ($b\gamma \gtrsim 7.46$), meaning that R&D investment is relatively inefficient, there is a loss in social welfare due to expensive duplication of innovations. This loss is lower with growing competition. Furthermore, growing competition increases consumer surplus. This positive effect of consumer surplus outweighs negative effect of producer surplus. When the spillover parameter $\beta = 1$ and $b\gamma$ is large ($b\gamma > 4$) there is little incentive to invest in R&D as firms can free-ride and R&D investments yield small returns. Thus growing competition increases the equilibrium social welfare as in a standard model without R&D investment.

Despite the possibility that monopoly or perfect competition do maximize equilibrium social welfare, part (c) of Proposition 6 makes clear that for most parameter values oligopoly (potentially with many firms) maximizes social welfare. On the one hand this result maybe not surprising, because the equilibrium consumer surplus is increasing with n, while the equilibrium producer surplus is decreasing with n. This means that the maximum of the equilibrium social welfare is reached under oligopoly for most parameter values. On the other hand, given what know of the extreme cases, it is not so straight forward.

In fact, one of the important things that Proposition 6 emphasizes is how care is needed when using extreme values of spillover, $\beta = 0$ and $\beta = 1$. This is because conclusions can be very different with an intermediate value of β (see the column in Table 1 where $\gamma = 8.01$). This seems even more important to note given that De Bondt, Slaets and Cassiman (1992) did not find such a distinction in case of AJ model. They distinguished two tendencies in the behaviour of the equilibrium social welfare: a typical and an exceptional one. By the typical pattern they called an increase in the equilibrium social welfare with the entry of new firms up to some maximum value after which "further entry has little effect". The tendency for the equilibrium social welfare to "increase and then decrease with entry" they

[10] The Shumpeterian hypothesis, however, is based on a dynamic framework, where higher R&D investment by monopoly results from forward looking profit maximization. The model we adopt is static, but yields similar insights.

called an exceptional pattern. In our Proposition 6 we find that what De Bondt at al. call an exceptional pattern becomes more of a typical pattern.

5 Conclusion

In this paper we used the model of KMZ to analyse the effect of R&D spillovers and market concentration on the equilibrium R&D effective cost reduction, per-firm profit and social welfare. We found that equilibrium effective cost reduction is decreasing with spillover parameter and with the number of firms. Per-firm equilibrium profit is maximized under an intermediate to high degree of spillover and is decreasing with the number of firms. Finally, equilibrium social welfare is maximized for a low level of spillover and, typically, under oligopoly (but potentially under any market structure depending on the value of the parameters). We compared our results to those of De Bondt, Slaets and Cassiman (1992) and found notable differences.

One interpretation of the spillover parameter is as an inverse measure of the distance between firms with a higher spillover parameter meaning more proximity between firms.[11] With this interpretation, our results would say that as distances between firms get smaller the effective cost reduction decreases. This can be explained as the tight proximity between firms allowing a free flow of information about any process innovation that creates a "free-rider" effect. Where firms are spread apart we might think that firms tend to rely more on their own cost-reducing R&D investments, which ultimately leads to relatively high R&D. Interestingly, our results would suggest that firms tend to do best when they are in tighter proximity, while society does best when firms are relatively spread from each other.

This last point illustrates the importance of endogeneity. Clearly, firms can choose their location and, hence, potentially decide on the level of spillover. For example, Audretsch and Feldman (1996) find that the industries where research and development is more important tend to be more geographically concentrated. Ciccone and Hall (1996) showed that the localization economies (the benefits generated by the proximity of firms producing similar

[11] The theoretical work of Glaeser (1999) demonstrated that agglomeration economies can arise from knowledge spillovers. Rosenthal and Strange (2001) combines the results on this issue from the empirical and theoretical literature. Among labour market pooling, input sharing, product shipping costs, natural advantage, the proxies for knowledge spillovers are mentioned as one of the driving forces of industry agglomeration. Clusters of firms benefit from the spillovers that stimulate various forms of learning and adaptations.

goods in the same industry) are one of the determinants of spatial concentration of activity within industries. We have assumed the spillover parameter is exogenous. Theoretical work that does endogenize spillovers includes Piga and Poyago-Theotoky (2005).

This is not to say that firms have complete control over the level of spillover. For example, if we interpret the spillover parameter as the level of patent protection, then the policy maker does, in principle, have some control over it. Our results would suggest that the policy maker can maximize the social welfare by enforcing a higher level of patent protection. Firms would prefer less patent protection, but would not like either extremes of full protection or its complete absence. Of course, intuitively, firms prefer to patent their own innovations ($\beta = 0$), but to free-ride on not patented innovations of other firms ($\beta = 1$). We have assumed symmetry between firms throughout the paper, but this is something that would be nice to relax in future work.

6 Proofs

This section provides the proofs for all the results of this paper.

Proof of Proposition 1.

To find the optimal R&D investment y_i of firm i we solve the first-order condition $\frac{\partial \Pi_i}{\partial y_i} = 0, i = \overline{1,n}$, where Π_i is defined by (1). Setting $y_1 = ... = y_n = y$ we derive the following equilibrium condition for the symmetric equilibrium:

$$\frac{2}{(n+1)^2 b\gamma}\left(a - c + \sqrt{\frac{2}{\gamma}y(1+(n-1)\beta)}\right)\frac{(n(1-\beta)+\beta)}{\sqrt{\frac{2}{\gamma}y(1+(n-1)\beta)}} = 1 \tag{4}$$

We denote the solution of (4) by y^*. Solving (4) we find the equilibrium (or optimal) per-firm R&D investment:

$$y^* = \frac{2(a-c)^2(n(1-\beta)+\beta)^2\gamma}{(1+(n-1)\beta)D^2},$$

where $D = [(n+1)^2 b\gamma - 2(n(1-\beta)+\beta)]$.

For the second order condition (SOC) to be satisfied, $\frac{\partial^2 \Pi_i}{\partial (y_i)^2}|_{y_i = y^*} \geq 0, i = \overline{1,n}$,we need $b\gamma > \frac{2(n(1-\beta)+\beta)^3}{(n+1)^2(n(1-\beta^2)+\beta^2)}$. By choosing $b\gamma > 2$ we guarantee that the SOC is satisfied for any $n \geq 1$ and $\beta \in [0,1]$.

To calculate X^* we substitute y^* into the equation (1).

To find the values for q^*, Q^* and P^* we used the following relations:

$$\begin{aligned} q^* &= \frac{1}{(n+1)b}(a-(c-X^*)) \\ Q^* &= nq^* \\ P^* &= a-bQ^*. \end{aligned} \tag{5}$$

■

Proof of Proposition 2.

By differentiating X^* w.r.to β we get:

$$\frac{\partial X^*}{\partial \beta} = -\frac{2b\gamma\left(a-c\right)\left(n-1\right)\left(n+1\right)^2}{D^2}$$

where $D = [b\gamma(n+1)^2 - 2(n(1-\beta)+\beta)] > 0$. Clearly, $\frac{\partial X^*}{\partial \beta} \leq 0$ for any $a > c, n \geq 1$.

Differentiating the relations in (5) it is clear that at the equilibrium

$$sign[\frac{\partial P^*}{\partial \beta}] = -sign[\frac{\partial Q^*}{\partial \beta}] = -sign[\frac{\partial q^*}{\partial \beta}] = -sign[\frac{\partial X^*}{\partial \beta}]$$

That means $\frac{\partial q^*}{\partial \beta} \leq 0$, $\frac{\partial Q^*}{\partial \beta} \leq 0$, $\frac{\partial P^*}{\partial \beta} \geq 0$. ■

Proof of Proposition 3.

(a) Differentiating the per-firm equilibrium profit

$$\Pi_n^* = \frac{(a-c)^2\gamma(b\gamma(n+1)^2(1+(n-1)\beta) - 2(\beta+n(1-\beta))^2)}{(1+(n-1)\beta)D^2},$$

where $D = [(n+1)^2 b\gamma - 2(n(1-\beta)+\beta)] > 0$, w.r.to β we get

$$\frac{\partial \Pi_n^*}{\partial \beta} = -\frac{2(a-c)^2(n-1)\gamma A}{(1+(n-1)\beta)^2 D^3} \tag{6}$$

where $A = (2(n(1-\beta)+\beta)^3 + b\gamma(n+1)^2(3(1+(n-1)\beta)^2 - (n+1)^2))$.

As it is clear from (6), for all $\beta \in [0,1], n \geq 1$ and $b\gamma > 2$:

$$sign[\frac{\partial \Pi_n^*}{\partial \beta}] = -sign[A]$$

Solving $A = 0$ we find a unique solution $\beta^* \in [0;1]$. For $\beta \in [0;\beta^*)$ the per-firm equilibrium profit is increasing in β since $\frac{\partial \Pi_n^*}{\partial \beta} > 0$ and for $\beta \in (\beta^*; 1]$ the profit is decreasing with spillover: $\frac{\partial \Pi_n^*}{\partial \beta} < 0$.

(b) Differentiating the equilibrium social welfare

$$SW^* = \frac{(a-c)^2\gamma n(b\gamma(n+1)^2(n+2)(1+(n-1)\beta) - 4(\beta+n(1-\beta))^2)}{2(1+(n-1)\beta)D^2}$$

w.r.to β we get

$$\frac{\partial SW^*}{\partial \beta} = -\frac{(a-c)^2\gamma n(A + b\gamma(n+1)^2 n(1+(n-1)\beta)^2)}{2(1+(n-1)\beta)D^2}$$

where A is the same as in (5).

Then

$$sign[\frac{\partial SW^*}{\partial \beta}] = -sign[A + b\gamma(n+1)^2 n(1+(n-1)\beta)^2]$$

The equation $A + b\gamma(n+1)^2 n(1+(n-1)\beta)^2 = 0$ (and, therefore, $\frac{\partial SW^*}{\partial \beta} = 0$) has a unique solution β^{**} on interval $[0; 1]$. For $\beta < \beta^{**}$ the equilibrium social welfare, SW^*, is increasing in β since $\frac{\partial SW^*}{\partial \beta} > 0$ and for $\beta > \beta^{**}$: SW^* is decreasing in β since $\frac{\partial SW^*}{\partial \beta} < 0$. Hence, on the interval of $\beta \in [0; 1]$ the solution β^{**} gives a global maximum for SW^*. ■

Proof of Proposition 4.

Part (a) and (b) can be checked by differentiation of X^* and q^* (that are defined in the Proposition 1) w.r.to n:

$$\frac{dX^*}{dn} = -\frac{2b\gamma(a-c)(n+1)(n(1-\beta)+3\beta-1)}{D^2}$$
$$\frac{dq^*}{dn} = -\frac{(a-c)\gamma((n+1)^2 b\gamma - 2(1-2\beta))}{D^2}$$

where, as before, $D = [(n+1)^2 b\gamma - 2(n(1-\beta)+\beta)] > 0$. It is easy to check that for the considerable range of parameters $\beta, b\gamma$ and n we have: $\frac{dX^*}{dn} < 0$, $\frac{dq^*}{dn} < 0$. That proves that the per-firm equilibrium effective cost reduction as well as the per-firm equilibrium output is decreasing with the spillover.

(c) Differentiation of Q^* with respect to n gives

$$\frac{dQ^*}{dn} = \frac{(a-c)\gamma((n+1)^2 b\gamma - 2(n^2(1-\beta)+(2n+1)\beta))}{D^2} \tag{7}$$

It is clear from (7) that

$$sign[\frac{dQ^*}{dn}] = sign[((n+1)^2 b\gamma - 2(n^2(1-\beta) + (2n+1)\beta))].$$

For any $b\gamma \geq 2$ we have $((n+1)^2 b\gamma - 2(n^2(1-\beta)+(2n+1)\beta)) > 0$ and, therefore, $\frac{dQ^*}{dn} \geq 0$. The equilibrium level of industry output is increasing with number of firms.

(d) Since $P^* = a - bQ^*$, then

$$sign[\frac{dP^*}{dn}] = -sign[\frac{dQ^*}{dn}].$$

Hence, $\frac{dP^*}{dn} \leq 0$ and, therefore, the equilibrium market price is decreasing in n. ■

Proof of Proposition 5.

By differentiating the per-firm equilibrium profit with respect to the number of firms n and exploiting the formula $X^* = \sqrt{\frac{2}{\gamma} y^*(1+(n-1)\beta)}$, we can get

$$\begin{aligned} \frac{d\Pi_n^*}{dn} &= -\frac{1}{2}\frac{\gamma X}{(1+\beta(n-1))^2 (n(1-\beta)+\beta)^2} \times \\ &\times \Big(X[b\gamma (n+1)(1+\beta n-\beta)^2 - \beta (n(1-\beta)+\beta)^2] + \\ &\quad + 2\frac{dX}{dn}(1-2\beta)(n-1)(1+\beta(n-1))(n(1-\beta)+\beta) \Big) \end{aligned}$$

It is clear that

$$\begin{aligned} sign[\frac{d\Pi_n^*}{dn}] &= -sign[X^*[b\gamma (n+1)(1+\beta n-\beta)^2 - \beta (n(1-\beta)+\beta)^2] + \\ &\quad +2\frac{dX^*}{dn}(1-2\beta)(n-1)(1+\beta(n-1))(n(1-\beta)+\beta)] \end{aligned} \quad (8)$$

It is not difficult to show that for all feasible values of $b\gamma$ we have $b\gamma (n+1)(1+\beta(n-1))^2 - \beta (n(1-\beta)+\beta)^2 \geq 0$. That guarantees that the first term on the RHS of the expression (8) under the *sign* symbol is positive.

The positiveness of the second term on the RHS of the expression (8) depends on sign of $\frac{dX^*}{dn}$ and $(1-2\beta)$ as the rest of the multipliers in this term are positive on the considered range of parameters n and β. According to the result (a) of Proposition 4, the equilibrium cost reduction decreases with n: $\frac{dX}{dn} < 0$. Hence, we conclude that the second term on the RHS of the expression (7) is negative for $\beta \in [0; \frac{1}{2})$ and positive when $\beta \in [\frac{1}{2}; 1]$. Immediately, this tells us that$\frac{d\Pi}{dn} < 0$ for $\beta \in [\frac{1}{2}; 1]$. Hence, the per-firm equilibrium profit is decreasing on the range of spillovers from intermediate to high.

On interval $\beta \in [0; \frac{1}{2})$, we have $(1 - 2\beta) > 0$. Then,

$$X^*[b\gamma(n+1)(1+\beta n-\beta)^2 - \beta(n(1-\beta)+\beta)^2] + \\ +2\frac{dX^*}{dn}(1-2\beta)(n-1)(1+\beta(n-1))(n(1-\beta)+\beta) \gtrless 0$$

iff

$$\frac{\frac{dX^*}{dn}}{X^*} \gtrless -\frac{b\gamma(n+1)(1+\beta n-\beta)^2 - \beta(n(1-\beta)+\beta)^2}{2(1-2\beta)(n-1)(1+\beta(n-1))(n(1-\beta)+\beta)}. \tag{9}$$

Note that the RHS of (9) is decreasing in $b\gamma$.

From the result (c) of Proposition 4 we know that the equilibrium total output is increasing with n for any $b\gamma \geq 2$. By manipulations with the expression $\frac{dQ^*}{dn}$ we get

$$\frac{\frac{dX^*}{dn}}{X^*} \geq -\frac{(n^2(1-\beta)+\beta(1+2n))}{n(n+1)(n(1-\beta)+\beta)}. \tag{10}$$

Comparing the RHS of (9) with RHS of (10) we find that for any $b\gamma \geq 2$ and $\beta \in [0; \frac{1}{2})$ the following inequality holds:

$$-\frac{(n^2(1-\beta)+\beta(1+2n))}{n(n+1)(n(1-\beta)+\beta)} \geq -\frac{b\gamma(n+1)(1+\beta n-\beta)^2 - \beta(n(1-\beta)+\beta)^2}{2(1-2\beta)(n-1)(1+\beta(n-1))(n(1-\beta)+\beta)}.$$

Hence,

$$\frac{\frac{dX^*}{dn}}{X^*} \geq -\frac{b\gamma(n+1)(1+\beta n-\beta)^2 - \beta(n(1-\beta)+\beta)^2}{2(1-2\beta)(n-1)(1+\beta(n-1))(n(1-\beta)+\beta)}$$

That means $\frac{d\Pi_n^*}{dn} < 0$ for $\beta \in [0; \frac{1}{2})$ and $b\gamma > 2$ and, therefore, the per-firm equilibrium profit Π_n^* decreases with the number of firms n. ■

Proof of Proposition 6.

To investigate the behavior of the equilibrium social welfare under the model considered we solve the maximization problem $\max_n SW^*$ taking into account the constraints on the parameters of the model: $n \geq 1$, $\beta \in [0, 1], b\gamma > 2$.

To remind, equilibrium social welfare has form

$$SW^* = \frac{(a-c)^2\gamma n((n+1)^2 b\gamma(1+(n-1)\beta)(n+2) - 4(\beta+n(1-\beta))^2)}{2(1+(n-1)\beta)D^2} \tag{11}$$

According to the FOC the candidates for maximum should necessarily satisfy equality $\frac{dSW^*}{dn} = 0$.[12] In our case,

$$\frac{dSW^*}{dn} = \frac{(a-c)^2\gamma[(n+1)^3(1+(n-1)\beta)^2(b\gamma)^2 - 2(n+1)Bb\gamma + 4(n(1-\beta)-\beta)^3(1-\beta)]}{(1+(n-1)\beta)^2 D^3}$$

where $B = [(1-\beta)\beta^2 n^5 + (1+\beta)\beta^2 n^4 + \beta(4-11\beta+11\beta^2)n^3 + (4-16\beta+30\beta^2-21\beta^3)n^2 - (1-11\beta+20\beta^2-10\beta^3)n + \beta(1-\beta)]$ and $D = [(n+1)^2 b\gamma - 2(n(1-\beta)+\beta)]$.

If $n \geq 1$, $\beta \in [0,1], b\gamma > 2$ then $D > 0$ and, therefore, the denominator in $\frac{dSW^*}{dn}$ is strictly positive. We investigate the sign of the numerator in $\frac{dSW^*}{dn}$ by looking separately at three different cases: $\beta \in (0,1), \beta = 0$ and $\beta = 1$.

(1) Let $\beta \in (0,1)$. Since $a > c, b\gamma > 2$, then by solving

$$[(n+1)^3(1+(n-1)\beta)^2(b\gamma)^2 - 2(n+1)Bb\gamma + 4(n(1-\beta)-\beta)^3(1-\beta)] = 0 \qquad (12)$$

we will find the solution to $\frac{dSW^*}{dn} = 0$.

The RHS of (12) is quadratic in $b\gamma$ (in n it is a polynomial of degree 6) we solve it w.r.to $b\gamma$. There are two roots $b\gamma_1, b\gamma_2$ that are functions of n and β. For all n and β fixed within the appropriate region, root $b\gamma_2$ is the only one that can take values above 2 (it may take values below 2 depending on n and β). Therefore, we have only one interior solution (interiority can be checked numerically):

$$b\gamma_2 = \frac{(B + \sqrt{B^2 - 4(n+1)(n(1-\beta)+\beta)^3(1-\beta)(1+(n-1)\beta)^2})}{(n+1)^2(1+(n-1)\beta)^2}$$

This is the implicit form for optimal n, solution to $\frac{dSW^*}{dn} = 0$. We denote it by $\widehat{n}(b\gamma_2, \beta)$[13].

It can be shown numerically that for fixed $b\gamma, \beta$ we have

$$\frac{dSW^*}{dn} \geq 0 \text{ for } n \in [1, \widehat{n}(b\gamma_2, \beta)]$$
$$\frac{dSW^*}{dn} < 0 \text{ for } n \in (\widehat{n}(b\gamma_2, \beta), \infty).$$

To take into account a discrete nature of the optimal solution we check $\max\limits_{\{\lfloor \widehat{n} \rfloor, \lceil \widehat{n} \rceil\}} SW^*$[14] We denote the actual maximand by n^*. It can be shown that for combinations of β (at least

[12] Later we will take into account the discrete nature of n.

[13] Note that $\widehat{n}(b\gamma_2, \beta) > 1$.

[14] $\lfloor \widehat{n} \rfloor$ ($\lceil \widehat{n} \rceil$) stands for the nearest positive integer that is less (greater) than $\widehat{n}$ and greater than 1.

0.91) and $b\gamma$ (at most 2.14) $n^* = 1$.

(2) When $\beta = 0$ equation (11) becomes

$$SW^*|_{\beta=0} = \frac{(a-c)^2\gamma((n+1)^2(n+2)nb\gamma - 4n^3)}{2((n+1)^2b\gamma - 2n)^2}.$$

By differentiating this with respect to n and taking into account the relevant parameter assumptions, we find that

$$sign[\frac{dSW^*}{dn}|_{\beta=0}] = sign[(n+1)^3(b\gamma)^2 - 2(4n-1)(n+1)nb\gamma + 4n^3]. \qquad (13)$$

Analyzing the sign of the RHS of (13) we can conclude that

When $b\gamma \in (2; 2(2+\sqrt{3}))$:

$$\frac{dSW^*}{dn}|_{\beta=0} \geq 0 \quad \text{for} \quad n \in [1; \widehat{n}(b\gamma_2, \beta = 0)]$$
$$\frac{dSW^*}{dn}|_{\beta=0} < 0 \quad \text{for} \quad n \in (\widehat{n}(b\gamma_2, \beta = 0); +\infty)$$

When $b\gamma \geq 2(2+\sqrt{3})$:

$$\frac{dSW^*}{dn}|_{\beta=0} > 0 \quad \text{for} \quad n \geq 1$$

where $\widehat{n}(b\gamma, \beta = 0)$ can be presented in an implicit form $b\gamma(n+1)^2 - n(4n - 1 + \sqrt{12n^2 - 12n + 1}) = 0$. Therefore, at $\beta = 0$ the equilibrium social welfare is maximized under a market structure with a finite number of firms when $b\gamma$ is low ($b\gamma < 2(2+\sqrt{3})$) and the equilibrium social welfare is increasing in n when $b\gamma$ is high ($b\gamma \geq 2(2+\sqrt{3})$).

(3) When $\beta = 1$ the formula (11) becomes

$$SW^*|_{\beta=1} = \frac{(a-c)^2\gamma((n+1)^2(n+2)nb\gamma - 4)}{2((n+1)^2b\gamma - 2)^2}.$$

By differentiating the last formula with respect to n and taking into account the relevant parameter's assumptions, we find that

$$sign[\frac{dSW^*}{dn}|_{\beta=1}] = sign[n(n+2)(b\gamma - 4) + b\gamma + 6]. \qquad (14)$$

Analyzing the sign of the RHS of (14) we can conclude that

When $b\gamma \in (2;4)$:

$$\frac{dSW^*}{dn}|_{\beta=1} \geq 0 \quad \text{for} \quad n \in [1; \widehat{n}(b\gamma_2, \beta = 1)]$$
$$\frac{dSW^*}{dn}|_{\beta=1} < 0 \quad \text{for} \quad n \in (\widehat{n}(b\gamma_2, \beta = 1); +\infty)$$

When $b\gamma \geq 4$:

$$\frac{dSW^*}{dn}|_{\beta=1} > 0 \quad \text{for} \quad n \geq 1$$

where $\widehat{n}(b\gamma_2, \beta = 1) = \frac{b\gamma - 4 + \sqrt{10(4 - b\gamma)}}{4 - b\gamma}$.

As before (see part (1) of proof of Proposition 6), we take into account the discrete nature of the optimal solution by checking $\max_{\{\lfloor\widehat{n}\rfloor, \lceil\widehat{n}\rceil\}} SW^*$. We denote the actual maximand by n^*. We find that $n^* = 1$ when $b\gamma < 2.14$.

Therefore, when $\beta = 1$ and $b\gamma$ is high ($b\gamma \geq 4$) the equilibrium social welfare is increasing in n, for $b\gamma$ low ($2.14 < b\gamma < 4$) the equilibrium social welfare has an inverse U-shape form and for $b\gamma$ very low ($b\gamma \leq 2.14$) the equilibrium social welfare is maximized under monopoly.

■

References

[1] Amir, R., 2000. Modelling imperfectly appropriable R&D via spillovers. *International Journal of Industrial Organization* 18, 1013-1032.

[2] Amir, R. and V.E. Lambson, 2000. On the Effects of Entry in Cournot Markets. *Review of Economic Studies*, Blackwell Publishing, 67(2), 235-54.

[3] Amir, R. and J. Wooders, 1999. Effects of One-way Spillovers on Market shares, Industry Price, Welfare, and R&D Cooperation, *Journal of Economics and Management Strategy*, **8,** 223-249.

[4] Arrow, K. J., 1962. The economic implications of learning by doing. *Review of Economic Studies*, Blackwell Publishing, 29, 155-73.

[5] Audretsch, D.B. and M.P. Feldman, 1996. R&D spillovers and the geography of innovation and production. *American Economic Review* 86 (3), 630-640.

[6] d'Aspremont, C. and A. Jacquemin, 1988. Cooperative and noncooperative R&D in duopoly with spillovers. *American Economic Review* 78, 1133-1137.

[7] Bernstein, J.I. and M.I. Nadiri, 1989. Research and development and intra-industry spillovers: An empirical application of dynamic duality. *Review of Economic Studies 56, 249-269.*

[8] Ciccone, A. and R.E. Hall, 1996. Productivity and the density of economic activity. *American Economic Review* 86, 54-70.

[9] Dasgupta, P. and J. Stiglitz, 1980. Industrial Structure and the Nature of Innovative Activity. *The Economic Journal* 90 (358), 266-293.

[10] De Bondt, R., P. Slaets and B. Cassiman, 1992. The degree of spillovers and the number of rivals for maximum effective R&D. *International Journal of Industrial Organization* 10, 35-54.

[11] Glaeser, E., 1999. Learning in cities. *Journal of Urban Economics* 46, 254-277.

[12] Kamien, M.I., E. Muller and I. Zang, 1992. Research Joint Ventures and R&D Cartels. *American Economic Review* 82 (5), 1293-1306.

[13] Kamien, M.I. and N.L. Schwartz, 1975. Market Structure and Innovation: a survey. *Journal of economic literature* 13 (1), 1-37.

[14] Levin, R.C., W.M. Cohen and D.C. Mowery, 1985. R&D Appropriability, Opportunity and Market Structure: New Evidence on Some Schumpeterian Hypothesis. *American Economic Review*, *Papers and Proceedings,* 75, 20-24.

[15] Martin, S., 2002. Spillovers, Appropriability and R&D. *Journal of Economics* 75, 1-32.

[16] Piga, C. and J. Poyago-Theotoky, 2005. Endogenous R&D spillovers and locational choice. *Regional Science and Urban Economics* 35, 127-139.

[17] Rosenthal, S.S. and W.C. Strange, 2001. The determinants of agglomeration. *Journal of Urban Economics* 50, 191-229.

[18] Schumpeter, J., 1947. Capitalism, Socialism and Democracy. *London: Allen and Unwin.*

[19] Simpson, R.D. and N.S. Vonortas, 1994. Cournot Equilibrium with Imperfectly Appropriable R&D. *Journal of Industrial Economics* 42 (1), 79-92.

[20] Steurs, G., 1995. Inter-industry R&D spillovers: What difference do they make? *International Journal of Industrial Organization* 13, 249-276.

Table 1. The equilibrium social welfare, SW^*, and number of firms, n^* that maximizes it for different values of the spillover parameter β and cost of R&D, $b\gamma$. The equilibrium social welfare can be maximized under monopoly, oligopoly or perfect competition (see Proposition 6).

	$\gamma = 2.01$	$\gamma = 4.01$	$\gamma = 6.01$	$\gamma = 8.01$
$\beta = 0$	$SW^* = 0.571429$ $n^* = 2$	0.506616 $n^* = 4$	0.500285 $n^* = 11$	0.5 $n^* \to \infty$
$\beta = 0.2$	$SW^* = 0.590278$ $n^* = 2.$	0.526884 $n^* = 5$	0.514066 $n^* = 7$	0.508917 $n^* = 9$
$\beta = 0.4$	$SW^* = 0.590629$ $n^* = 2$	0.526043 $n^* = 5$	0.512925 $n^* = 7$	0.507828 $n^* = 8$
$\beta = 0.6$	$SW^* = 0.580852$ $n^* = 2$	0.519149 $n^* = 4$	0.50861 $n^* = 7$	0.504862 $n^* = 9$
$\beta = 0.8$	$SW^* = 0.565417$ $n^* = 2$	0.509794 $n^* = 5$	0.503414 $n^* = 9$	0.501705 $n^* = 14$
$\beta = 1$	$SW^* = 0.555556$ $n^* = 1$	0.5 $n^* \to \infty$	0.5 $n^* \to \infty$	0.5 $n^* \to \infty$

University of Kent

Department of Economics Discussion Papers

R&D Spillovers, Concentration and Market Performance.

Anna Stepanova

February 2009

KDPE 0901

R&D Spillovers, Concentration and Market Performance.

Anna Stepanova[*†]

This version: February 2009

Abstract

In a two-stage R&D game of process innovation, we investigate the effect of exogenously changing R&D spillovers and market concentration on the equilibrium level of effective cost reduction, total output, profits and social welfare. Interpreting spillover as a measure of patent protection, we find that weaker patent protection results in less R&D. We also show that firms prefer weaker patent protection, but social welfare is maximized for higher levels of patent protection. In terms of market concentration we show that firm profits decrease with increasing numbers of firms. Social welfare is typically maximized under oligopoly with the optimal number of firms depending on the level of spillover and efficiency of R&D investment.

JEL codes: C72, L13, O31

Keywords: oligopoly R&D competition, spillover process, cost reduction, market concentration.

[*]Department of Economics, University of Kent, Keynes College, Canterbury, CT2 7NP, UK. E-mail: A.Stepanova@kent.ac.uk

[†]I would like to thank Anette Boom, Peter Norman Sørensen, Peter Sudhölter and especially Rabah Amir for helpful comments and ideas.

1 Introduction

There is a wide theoretical and empirical literature devoted to the discussion of the performance of firm's R&D activities. Among them there are papers that demonstrate the effects of R&D on the productivity of the innovating firm itself as well as on well-being of consumers and the society at large (e.g. De Bondt, Slaets and Cassiman, 1992). It is widely recognized that effects are generated not only due to the direct effect of R&D, but also due to the effects of R&D spillovers. In this paper we look at how firm's profit and social welfare depend on the amount of R&D spillovers as well as on market structure.

Our focus will be on the theoretical impact of intra-industry spillovers, i.e. spillovers between firms within an industry.[1] We use the two-stage model of Kamien, Muller and Zang (1992), henceforth, KMZ. In the first stage firms decide on the level of their R&D investment. The second stage happens according to the standard Cournot scenario. The firm's unit cost in the second stage depends on its own R&D investment and on spillovers from R&D investments of rival firms. Thus there is competition as well as spillover externalities within the same industry.

We first solve the model for the equilibrium R&D investment, effective cost reduction, per-firm and industry output and price. Our main objectives are to look at the behavior of these equilibrium characteristics as well as per-firm equilibrium profit and social welfare as (1) the level of spillover changes, and (2) the market concentration (as measured by the number of firms) changes.

Spillover externalities are captured by a spillover parameter β that can range from zero, implying no spillovers to one, implying perfect spillovers in the sense that R&D investment is a pure public good. Two possible interpretations of spillover parameter are patent enforcement and location, with a low spillover corresponding to high patent protection and relatively large distances between firms. An increase in the spillover parameter leads to a decrease in the equilibrium R&D investment and per-firm equilibrium output. Applying these results, we find that per-firm equilibrium profit is maximized under an intermediate to high degree of spillover. This is because increasing spillover offers the opportunity to free-

[1] There are two general types of spillovers: the inter-industry R&D spillovers and the intra-industry ones (e.g., see Bernstein and Nadiri, 1989, Wolf and Nadiri, 1993). Most of the theoretical papers (see, e.g., De Bondt et al. 1992, Simpson and Vonortas, 1994) focus on the impact of intra-industry R&D spillovers. There are some exceptions, e.g. Steurs (1995).

ride on the R&D investments of other firms and is associated with lower per-firm equilibrium output. The equilibrium social welfare is maximized for smaller values of the spillover. This is because the equilibrium industry output and, therefore, the equilibrium consumer surplus is larger the smaller are spillovers.

With regard to market concentration, we find that as the number of competing firms increases there is a decrease in the equilibrium R&D investment and per-firm equilibrium output, but an increase in the equilibrium industry output. Applying these results, we find that per-firm equilibrium profit decreases as the number of firms increases. With regard to social welfare we find that the type of market structure under which it reaches its maximum depends on the level of spillover as well as on the efficiency of R&D investment. If spillovers are large and R&D investment is efficient enough, then equilibrium social welfare is maximized under a monopoly. In this case, monopoly also leads to the most R&D investment. If R&D investment is relatively costly and the spillover parameter takes extreme values then perfect competition may maximize social welfare. This is because firms have no incentive to invest in R&D and we approximately have the standard static Cournot result. For most parameter values the equilibrium social welfare is inverse U-shaped in the number of competing firms and so oligopoly maximizes social welfare.

Closely related results are due to De Bondt, Slaets and Cassiman (1992). They also looked at the effect of R&D spillovers and market concentration on profit and social welfare, but using the model of d'Aspremont and Jacquemin (1988), henceforth AJ. As already mentioned, we adopt the model of KMZ and so our results differ a-priori. As discussed in detail by Amir (2000), the AJ model is distinguished from the KMZ model in the way the spillover processes are modeled and can lead to substantially different results and, therefore, policy recommendations regarding R&D cooperation.[2] In a discussion on the validity of these models Amir writes that "the AJ model appears to be of questionable validity for large values of the spillover parameter". It is, therefore, noteworthy that De Bondt, Slaets and Cassiman (1992) find that profit and social welfare are maximized for large value of the spillover parameter. This motivates our use of the KMZ model and we do indeed get different conclusions, for example, we find that equilibrium social welfare is maximized under

[2] In the AJ model firms' decision variables are the levels of reduction of their unit cost of production, and spillover effects take place additively in cost reductions, i.e. in R&D outputs. On the other hand, in the KMZ model, firms decision variables are their R&D investment levels, and spillover effects are additive in these expenditures, i.e. in R&D inputs (see also Martin, 1998).

a much lower level of spillover. We will discuss these differences in more detail below.

The rest of the paper is structured as follows. In Section 2 we present the basic theoretical framework of R&D/quantity competition and provide its equilibrium characteristics. In Section 3 we investigate the effects of the spillover level on the equilibrium characteristics of the model. This is followed by Section 4 that presents our findings on changes of the equilibrium characteristics with respect to (exogenously changed) market concentration. In Section 5 we offer some concluding remarks.

2 Model and equilibrium

Consider an industry with n identical firms facing a linear inverse demand function $P = a - bQ$, where Q is the industry output and $0 < b \leq 1$. Each firm has an initial unit cost of production c, where $0 < c < a$, and participates in the following two-stage noncooperative game. In the first stage firms simultaneously must decide how much to invest in an R&D process that will reduce their unit cost. In the second stage, upon observing the new unit costs, firms compete as in the standard quantity setting Cournot model. When each firm decides how much it would like to invest on R&D it takes into account the other's R&D expenditures as well as its resulting second stage market profit.

More precisely, in the first stage firms simultaneously decide how much to invest on R&D: $y_i \geq 0, i = \overline{1,n}$. The effective cost reduction of firm i is determined by the individual R&D investments of all firms is as follows

$$X_i = \sqrt{\frac{2}{\gamma}(y_i + \beta \sum_{j=1, j\neq i}^{n} y_j)}, \tag{1}$$

where $\gamma > 0$ is a parameter measuring the efficiency of R&D investments and a spillover parameter $\beta, \beta \in [0,1]$. The efficiency of R&D investment increases with decreasing γ. As is standard, we shall refer to γ (and subsequently, $b\gamma$) as the cost of R&D. If $\beta > 0$ then each firms unit cost is reduced by the R&D investments of other firms as well as by its own R&D investment. The larger is β then the higher is this spillover effect.

The decisions of the first stage make the unit cost of firm $i, i = \overline{1,n}$ in the second, production, stage equal to $c - X_i$. So the unit cost of each firm is determined by the R&D investments of all the firms. In the second stage firms engage in Cournot competition. Each firm's objective in the game is to maximize its individual second-stage production profit, $\pi_i, i = \overline{1,n}$, net of it's first stage R&D investment, $y_i, i = \overline{1,n}$. We restrict consideration to subgame perfect equilibria (SPEs) of the game. Firm i's second stage production profit is the Cournot equilibrium profit and has the form

$$\pi_i = \frac{1}{(n+1)^2 b}\left(a - (n+1)c_i + \sum_{j=1, j\neq i}^{n} c_j\right)^2, i, j = \overline{1,n},$$

where $c_i = c - X_i$ is the firm i's reduced unit cost and $c_j = c - X_j, j = \overline{1,n}, j \neq i$, are the reduced unit costs of its rivals. Hence, firm i's overall profit is

$$\begin{aligned} \Pi_i &= \pi_i - y_i \\ &= \frac{1}{(n+1)^2 b}\left[a - (n+1)(c - X_i) + \sum_{j=1, j\neq i}^{n} (c - X_j)\right]^2 - y_i \\ &= \frac{1}{(n+1)^2 b}\left[a - c + nX_i - \sum_{j=1, j\neq i}^{n} X_j\right]^2 - y_i. \end{aligned} \tag{2}$$

where $X_i = \sqrt{\frac{2}{\gamma}(y_i + \beta \sum_{j=1, j\neq i}^{n} y_j)}$ and $X_j = \sqrt{\frac{2}{\gamma}(y_j + \beta \sum_{k=1, k\neq j}^{n} y_k)}$, $i, j = \overline{1,n}$.

Throughout the paper we shall use the standard (Marshallian) definition of social welfare as the sum of the consumer and producer surplus:

$$\begin{aligned} SW &= \int_0^Q P(t)dt - TC \\ &= aQ - \frac{b}{2}(Q)^2 - \sum_{i=1}^{n} \left((c - X_i)q_i + y_i\right). \end{aligned} \tag{3}$$

2.1 Symmetric subgame perfect equilibrium

Assuming that all firms behave identically, we now solve for the symmetric SPE as done by KMZ. We obtain the optimal first stage R&D investment (SPE) by solving $\frac{\partial \Pi_i}{\partial y_i} = 0, i = \overline{1,n}$ with respect to y_i and setting $y_1 = ... = y_n = y^*$. Given this we can find the equilibrium characteristics of the model:

Proposition 1 *(KMZ) Assuming $b\gamma > 2$, the per-firm equilibrium R&D investment, the* *per-firm equilibrium effective cost reduction* , X^*, *per-firm equilibrium output,* q^*, *the total equilibrium output,* Q^*, *and the equilibrium price,* P^*, *are*

$$\begin{aligned} y^* &= \frac{2(a-c)^2(n(1-\beta)+\beta)^2\gamma}{(1+(n-1)\beta)D^2} \\ X^* &= 2(a-c)\frac{(n(1-\beta)+\beta)}{D} \\ q^* &= (a-c)\frac{(n+1)\gamma}{D} \\ Q^* &= (a-c)\frac{n(n+1)\gamma}{D} \\ P^* &= a-(a-c)\frac{n(n+1)b\gamma}{D} \end{aligned}$$

where $D = [(n+1)^2 b\gamma - 2(n(1-\beta)+\beta)]$.

The effective cost reduction X has natural boundary: $X \in [0, c]$. Given Proposition 1 to keep $c - X^* > 0$ across the feasible ranges of n and β, we shall assume $\frac{a}{c} \leq 4$. Recall that $\frac{a}{c} > 1$. The condition $b\gamma > 2$ is also necessary to avoid boundary solutions. When $b\gamma \leq 2$ for some and sometimes for most values of n (depending on $b\gamma$ and β) there is no interior solution y^* to the maximization problem. For $b\gamma \leq 2$ it would be necessary to check all possible boundary conditions. For simplification we will assume $b\gamma > 2$ throughout the remainder of the paper.

3 Effects of the level of spillovers

In the present section we shall show the impact of the spillover parameter β on the model's equilibrium characteristics. In doing so we shall see how the level of spillover affects firms' profit and social welfare and, therefore, see what level of spillover parameter maximizes them. First, we need to look at how the level of spillover affects the basic equilibrium characteristics such as effective cost reduction.

Intuitively, there is a free-rider effect: the higher the spillover parameter the more cost-reducing knowledge the firm can gain for free, the smaller should be the investment of the firm on the R&D activity. KMZ show that this intuition is correct and the equilibrium level of per-firm R&D investment, y^*, decreases as the spillover parameter increases from 0 to 1.

This means that if we look at the equilibrium effective cost reduction X^* there are two opposite effects. As we can see from equation (1) there is a positive direct effect: the larger is the spillover parameter the larger is the unit cost reduction for any level of R&D investments, $y_j, j = \overline{1,n}, j \neq i$. There is, however, the negative indirect effect discussed above: the larger the spillover parameter the smaller the per-firm equilibrium R&D investment, y^*. In Proposition 2 we demonstrate that this indirect effect is stronger than the direct one. With this it is straight forward to show the effect of β on q^*, Q^* and P^*.

Proposition 2 *An increase in the spillover level β causes:*

a) a decrease in the equilibrium effective cost reduction X^*;

b) a decrease in the per-firm equilibrium output q^*;

c) a decrease in total equilibrium output Q^*;

d) an increase in the equilibrium market price P^*.

Note that the 2-firm version of this result can be found in Amir (2000). Our result is stated for the case of n firms, but confirms the result of Amir: the equilibrium "effective levels of R&D (i.e., the sum of own and spillover levels) decrease with the spillover rate".

De Bondt, Slaets and Cassiman (1992) get different results. They showed (see their Propositions 2 and 3) that in AJ model effective R&D and firm's output are maximized for spillover equal to $\frac{1}{2}$. Our Proposition 2, in contrast, says that they are maximized at spillover equal to 0. To make sure that we are comparing like with like, let we refer to the Corollary 4.4 of Amir (2000). He defines the maximal value of spillover $\beta_{\max} = \frac{\sqrt{n}-1}{n-1}$ for which R&D processes of AJ and KMZ models are equivalent. He further concludes that for $\beta > \beta_{\max}$ the validity of the AJ model is questionable, while the KMZ model is valid for the entire range of spillovers.[3] Note that $\beta_{\max}$ is always less than $\frac{1}{2}$ and so, the result of De Bondt, Slaets and Cassiman (1992) falls into the region of questionable validity.

We now proceed to study the influence of the spillover rate β on the profitability of firms and social welfare at the equilibrium.

At first we investigate the behavior of the per-firm equilibrium profit Π_n^*. Using Proposition 1 and that Π_n^* has the form $(P^* - c + X^*)q^* - y^*$ we get

$$\Pi_n^* = \frac{(a-c)^2\gamma((n+1)^2 b\gamma(1+(n-1)\beta) - 2(\beta + n(1-\beta))^2)}{(1+(n-1)\beta)D^2},$$

[3] For spillovers above $\frac{\sqrt{n}-1}{n-1}$ joint returns to scale (in R&D expenditure and number of firms) are increasing for the AJ model, while it is non-increasing returns to scale for the KMZ over entire range of spillovers.

where $D = [(n+1)^2 b\gamma - 2(n(1-\beta)+\beta)]$.

Next, to investigate the impact of the spillover parameter on society's well-being, we analyze the behavior of the equilibrium social welfare, SW^*. From equation (3), the equilibrium social welfare can be calculated to be

$$SW^* = \frac{(a-c)^2\gamma n((n+1)^2 b\gamma(1+(n-1)\beta)(n+2) - 4(\beta+n(1-\beta))^2)}{2(1+(n-1)\beta)D^2}.$$

The following proposition details the effects of the spillover parameter on the per-firm equilibrium profit and the equilibrium social welfare for any fixed number of firms, n, and any $b\gamma$.

Proposition 3 *(a) There exists* $\beta^* = argmax_\beta \Pi_n^*$ *which depends on* $n, b\gamma$ *and always lies in the interval* $(0.56, 0.74)$ *such that the per-firm equilibrium profit* Π_n^* *is increasing for all* $\beta < \beta^*$ *and decreasing for* $\beta > \beta^*$*;*

(b) There exists $\beta^{**} = argmax_\beta SW^*$ *which depends on* $n, b\gamma$ *and varies in the interval* $(0, 0.35)$ *such that the equilibrium social welfare* SW^* *is increasing for all* $\beta < \beta^{**}$ *and decreasing for* $\beta > \beta^{**}$*, while at* β^{**} *it reaches its maximum.*[4]

We will discuss the two parts of Proposition 3 in turn. Part (a) of Proposition 3 shows that the maximal equilibrium profit is achieved under a surprisingly high, though moderate degree of spillover. Thus, in particular, neither zero nor full spillovers will maximize the per-firm equilibrium profit Π_n^*. Similar results were obtained by Amir and Wooders (1999) in a two-stage duopoly model with a one-way spillover structure. It can be shown by an extended computation that, as the parameters $n, b\gamma$ vary within the confines of our assumptions, the privately optimal spillover β^* always lies in the interval $(0.57, 0.73)$. This suggests that firms' preference for intermediate levels of spillovers is a feature that is robust to the type of spillover process in the industry. For example, when the number of firms is fixed at 4 and the cost of R&D, $b\gamma$, is fixed at 2.1, the maximum of equilibrium profit occurs under the spillover level $\beta^* \approx 0.62$.

To give some intuition for why β^*does not take extreme values, consider the equation

$$\frac{\partial \Pi_n^*}{\partial \beta} = \frac{2(a-c+X^*)}{(n+1)^2 b}\frac{\partial X^*}{\partial \beta} - \frac{\partial y^*}{\partial \beta}.$$

[4] *As we discuss in the appendix:* β^* *is the unique real solution of a qubic equation* $\frac{d\Pi_n{}^*}{d\beta} = 0$ *and* β^{**} *is the unique real solution of a qubic equation* $\frac{dSW^*}{d\beta} = 0$.

As the spillover parameter β increases there are two competing forces on the per-firm equilibrium profit: (a) A positive effect where as β increases, the firm's equilibrium R&D investment falls, resulting in decreasing fixed cost and therefore, higher per-firm equilibrium profit. (b) A negative effect where as β increases, the firm's equilibrium effective cost reduction decreases, resulting in a higher unit cost and, therefore, lower per-firm equilibrium profit. As we discuss in the Appendix when $\beta = 0$ the positive effect (a) dominates the negative effect (b) and when $\beta = 1$ the negative effect (b) dominated the positive one.

Part (*b*) of Proposition 3 states that the equilibrium social welfare is maximized under a moderate to low level of the spillover parameter.

To understand the behavior of equilibrium social welfare we look at its two components: consumer and producer surplus. The equilibrium consumer surplus is decreasing through all the valid range of the spillover parameter because total output is decreasing. Thus consumers do best if the spillover parameter is zero because then the equilibrium price is lowest. The behavior of the producer surplus as a result of spillover parameter change is determined by the behavior of the per-firm equilibrium profit (since $\frac{dPS^*}{d\beta} = n\frac{d\Pi_n^*}{d\beta}$). Hence, part (a) of Proposition 3 can be viewed as the one describing the changes in the equilibrium producer surplus. Combining consumer and producer surplus we can see why β^{**} takes values less than β^*. Also as n tends towards infinity, β^{**} tends to zero.

Adopting the model of d'Aspremont and Jacquemin (1988), De Bondt, Slaets and Cassiman (1992) found (Proposition 4) that in the case of homogeneous oligopoly, "profitability and welfare achieve a maximum for an intermediate magnitude of spillovers between $\frac{1}{2}$ and 1". They also find that the level of spillover parameter that maximizes the per-firm equilibrium profit is higher than the one at which social welfare is maximized. There are clearly some similarities between their Proposition 4 and our Proposition 3. There are, however, very important differences. De Bondt et al. find that the equilibrium social welfare attains its maximum for a relatively large level of spillover (between 0.5 and 1) while we find that the equilibrium social welfare is maximized for a moderate to low level of the spillover parameter (between 0 and 0.35). Futherrmore, the model used by De Bondt, Slaets and Cassiman (1992), as explained above, is of questionable validity for $\beta \geq \frac{1}{2}$. So even though in terms of per-firm equilibrium profit differences in the optimal spillover parameter are not so stark, it is difficult to say that out results are consistent.

4 Effects of market concentration

In this section we investigate how the market equilibrium performance responds to varying market concentration, as measured by the exogenous number of firms. The first proposition clarifies the effect of the market concentration on the equilibrium effective cost reduction, X^*,the per-firm equilibrium output, q^*, the total equilibrium output, Q^* and the equilibrium price, P^*.

Proposition 4 *As the number of competing firms increases we observe:*
(a) *a decrease in the equilibrium effective cost reduction* X^**;*
(b) *a decrease in the per-firm equilibrium output* q^**;*
(c) *an increase in the total equilibrium output* Q^**;*
(d) *a decrease in the equilibrium market price* P^**.*

Part (a) of Proposition 4 shows that the higher the number of firms on the market the lower the per-firm equilibrium effective cost reduction, X^*. This must mean that the equilibrium R&D investment, y^*, has decreased. It does so because the larger the number of firms the higher the incentive to free ride on R&D investments of other firms. The lower the equilibrium effective cost-reduction, X^*, the higher the firm's equilibrium unit cost leading to the lower per-firm equilibrium output. This effect reinforces the "standard" incentive for a firm to reduce its output when there is more competition.

Parts (c) and (d) of Proposition 4 show that, as in the case of the standard Cournot oligopoly, the model of this paper is quasi-competitive, i.e. industry output rises and price falls as number of competing firms increases. Part (b) and (c) of Proposition 4 are consistent with the standard Cournot result that as the number of firms increases (keeping unit cost the same) the per-firm equilibrium output decreases and the industry output increases (Amir and Lambson, 2000). Note, comparing parts (c) of Proposition 2 and 4, that total equilibrium industry output, Q^*, decreases with β, but increases with n. This difference merely reflects the increased number of firms.

Similar to parts (a), (c) and (d) of Proposition 4, Dasgupta and Stiglitz (1980) find that in the presence of entry barriers the per-firm R&D investment decreases, the industry output increases and the equilibrium market price decreases with the number of firms[5].

[5] In their model the market structures are endogenous.

It follows from Proposition 4 that, in the framework of this model, monopoly is the best of all market structures in terms of providing the maximum equilibrium cost-reduction and, therefore, the ability for firms to enjoy the minimum unit-cost of production. This is consistent with the Schumpeterian view that monopoly is the market structure that may lead to maximum R&D investment. (Schumpeter, 1942) The other pole of the market structures, perfect competition[6], leads to the least equilibrium cost-reduction for each firm, therefore, to the highest unit cost of production. However, in the theoretical literature there are papers that reflect an opposite view that a competitive market structure is the one that promotes more innovation than monopoly (see, e.g. Arrow, 1962).

The relationship between market concentration and R&D activity has been the subject of a large empirical literature. (see Kamien and Schwartz, 1975, Levin, Cohen and Mowery, 1985). Typically, in contrast to what we find, they observe an inverse U-shaped relationship between the market concentration and innovation. One of the possible explanations for this difference could be the endogeneity of the market concentration. We take the number of firms as exogenous. If we allow for endogeneity we may find that markets with a relatively low concentration have different characteristics (for example, different valuer of $b\gamma$ and β) that result in a positive correlation between R&D activity and market concentration.

Our next result concerns the effect of market concentration on the per-firm equilibrium profit.

Proposition 5 *The equilibrium profit per firm* Π_n^* *decreases with the number of firms* n*, regardless of the level of spillovers.*

This result is also consistent with the standard Cournot model results[7]. That it should be, however, is not something that would obviously extend to the present two-period setting with R&D investments. This is because each firm's equilibrium investment on R&D decreases when there is an increased number of rivals. This has a positive effect on the per-firm equilibrium profit. Proposition 5 shows that the decrease in profit resulting from higher competition is stronger than this positive change.

[6] Technically, it is not a perfect competition because of the presence of fixed costs. Though we choose to use this concept as it is the closest market structure that reflects a described situation in the market.

[7] In fact, investigating the effects of exogenous entry on market performance measures in standard Cournot competition with very general demand and cost functions, Amir and Lambson (2000) report that the only result that always holds is that per-firm profit decreases with the number of firms in the market.

A similar result, but in the framework of the AJ model, was found by De Bondt, Slaets and Cassiman (1992), suggesting that this result is rather robust to the way the spillover process is modeled, at least in the framework of linear demand and production costs and quadratic R&D costs.

Our final objective in this section is to describe the changes in equilibrium social welfare induced by changes in the market concentration and, therefore, find the market structure that is optimal from the point of view of society. We say that there exist a finite equilibrium social welfare maximizing number of firms n^*if $n^* = \arg\max_{n \in Z_+} SW^*$ exists. Likewise, we say that equilibrium social welfare is inverse U-shaped in n if $n^* = \arg\max_{n \in Z_+} SW^* \geq 2$, and the value of SW^* is increasing in n when $n < n^*$ and decreasing in n when $n > n^*$.[8]

The following proposition provides the results on the changes in equilibrium social welfare induced by changes in the market concentration (as measured by the number of firms).

Proposition 6 *The equilibrium social welfare SW^* is*

(a) maximized at $n^* = 1$ for β sufficiently large (at least 0.91) and $b\gamma$ sufficiently small (at most 2.14)[9]*;*

(b) increasing in n when $\beta = 0$ and $b\gamma > 2(2 + \sqrt{3})$ and when $\beta = 1$ and $b\gamma > 4$;

(c) inverse U-shaped, otherwise.

Proposition 6 shows that equilibrium social welfare can be maximized under any possible market structure: monopoly, oligopoly or perfect competition. Which of these market structures would be the optimal one in terms of providing the optimum social welfare is determined by the values of the spillover parameter, β, and the cost of R&D $b\gamma$. This is illustrated in Table 1.

In order to understand Proposition 6 it is interesting to ask why monopoly or perfect competition can be the market structure that maximizes equilibrium social welfare.

In the case of monopoly (part (a) of Proposition 6): we know that monopoly has the highest equilibrium effective cost reduction (see Proposition 4). Moreover, a small $b\gamma$ means a higher efficiency of R&D investment so any cost reduction is larger and the difference between monopoly and other market structures is amplified. Less intuitive is why a large β means monopoly maximizes social welfare. Looking at Proposition 2 we see that large

[8]Note, that this does not necessarily mean that SW^* is concave in n and so, literally, inverse U-shaped.

[9]*For each β there exists λ such that for $b\gamma < \lambda$, $n^* = 1$ maximizes social welfare.*

β is associated with lower effective cost reduction, but also lower consumer surplus. This later effect means that when β is large, changes in consumer surplus are relatively small with respect to market structure. These features make monopoly the market structure that maximizes social welfare when $b\gamma$ is small and β is large as we see in Table 1. This result is consistent with the Schumpeterian hypothesis that welfare loss caused by production inefficiency of monopoly can be more than compensated by the gains from R&D activity.[10]

In the case of perfect competition (part (b) of Proposition 6): when spillover $\beta = 0$ and $b\gamma$ is large ($b\gamma \gtrsim 7.46$), meaning that R&D investment is relatively inefficient, there is a loss in social welfare due to expensive duplication of innovations. This loss is lower with growing competition. Furthermore, growing competition increases consumer surplus. This positive effect of consumer surplus outweighs negative effect of producer surplus. When the spillover parameter $\beta = 1$ and $b\gamma$ is large ($b\gamma > 4$) there is little incentive to invest in R&D as firms can free-ride and R&D investments yield small returns. Thus growing competition increases the equilibrium social welfare as in a standard model without R&D investment.

Despite the possibility that monopoly or perfect competition do maximize equilibrium social welfare, part (c) of Proposition 6 makes clear that for most parameter values oligopoly (potentially with many firms) maximizes social welfare. On the one hand this result maybe not surprising, because the equilibrium consumer surplus is increasing with n, while the equilibrium producer surplus is decreasing with n. This means that the maximum of the equilibrium social welfare is reached under oligopoly for most parameter values. On the other hand, given what know of the extreme cases, it is not so straight forward.

In fact, one of the important things that Proposition 6 emphasizes is how care is needed when using extreme values of spillover, $\beta = 0$ and $\beta = 1$. This is because conclusions can be very different with an intermediate value of β (see the column in Table 1 where $\gamma = 8.01$). This seems even more important to note given that De Bondt, Slaets and Cassiman (1992) did not find such a distinction in case of AJ model. They distinguished two tendencies in the behaviour of the equilibrium social welfare: a typical and an exceptional one. By the typical pattern they called an increase in the equilibrium social welfare with the entry of new firms up to some maximum value after which "further entry has little effect". The tendency for the equilibrium social welfare to "increase and then decrease with entry" they

[10] The Shumpeterian hypothesis, however, is based on a dynamic framework, where higher R&D investment by monopoly results from forward looking profit maximization. The model we adopt is static, but yields similar insights.

called an exceptional pattern. In our Proposition 6 we find that what De Bondt at al. call an exceptional pattern becomes more of a typical pattern.

5 Conclusion

In this paper we used the model of KMZ to analyse the effect of R&D spillovers and market concentration on the equilibrium R&D effective cost reduction, per-firm profit and social welfare. We found that equilibrium effective cost reduction is decreasing with spillover parameter and with the number of firms. Per-firm equilibrium profit is maximized under an intermediate to high degree of spillover and is decreasing with the number of firms. Finally, equilibrium social welfare is maximized for a low level of spillover and, typically, under oligopoly (but potentially under any market structure depending on the value of the parameters). We compared our results to those of De Bondt, Slaets and Cassiman (1992) and found notable differences.

One interpretation of the spillover parameter is as an inverse measure of the distance between firms with a higher spillover parameter meaning more proximity between firms.[11] With this interpretation, our results would say that as distances between firms get smaller the effective cost reduction decreases. This can be explained as the tight proximity between firms allowing a free flow of information about any process innovation that creates a "free-rider" effect. Where firms are spread apart we might think that firms tend to rely more on their own cost-reducing R&D investments, which ultimately leads to relatively high R&D. Interestingly, our results would suggest that firms tend to do best when they are in tighter proximity, while society does best when firms are relatively spread from each other.

This last point illustrates the importance of endogeneity. Clearly, firms can choose their location and, hence, potentially decide on the level of spillover. For example, Audretsch and Feldman (1996) find that the industries where research and development is more important tend to be more geographically concentrated. Ciccone and Hall (1996) showed that the localization economies (the benefits generated by the proximity of firms producing similar

[11] The theoretical work of Glaeser (1999) demonstrated that agglomeration economies can arise from knowledge spillovers. Rosenthal and Strange (2001) combines the results on this issue from the empirical and theoretical literature. Among labour market pooling, input sharing, product shipping costs, natural advantage, the proxies for knowledge spillovers are mentioned as one of the driving forces of industry agglomeration. Clusters of firms benefit from the spillovers that stimulate various forms of learning and adaptations.

goods in the same industry) are one of the determinants of spatial concentration of activity within industries. We have assumed the spillover parameter is exogenous. Theoretical work that does endogenize spillovers includes Piga and Poyago-Theotoky (2005).

This is not to say that firms have complete control over the level of spillover. For example, if we interpret the spillover parameter as the level of patent protection, then the policy maker does, in principle, have some control over it. Our results would suggest that the policy maker can maximize the social welfare by enforcing a higher level of patent protection. Firms would prefer less patent protection, but would not like either extremes of full protection or its complete absence. Of course, intuitively, firms prefer to patent their own innovations ($\beta = 0$), but to free-ride on not patented innovations of other firms ($\beta = 1$). We have assumed symmetry between firms throughout the paper, but this is something that would be nice to relax in future work.

6 Proofs

This section provides the proofs for all the results of this paper.

Proof of Proposition 1.

To find the optimal R&D investment y_i of firm i we solve the first-order condition $\frac{\partial \Pi_i}{\partial y_i} = 0, i = \overline{1,n}$, where Π_i is defined by (1). Setting $y_1 = ... = y_n = y$ we derive the following equilibrium condition for the symmetric equilibrium:

$$\frac{2}{(n+1)^2 b\gamma}\left(a - c + \sqrt{\frac{2}{\gamma} y(1+(n-1)\beta)}\right)\frac{(n(1-\beta)+\beta)}{\sqrt{\frac{2}{\gamma} y(1+(n-1)\beta)}} = 1 \tag{4}$$

We denote the solution of (4) by y^*. Solving (4) we find the equilibrium (or optimal) per-firm R&D investment:

$$y^* = \frac{2(a-c)^2(n(1-\beta)+\beta)^2\gamma}{(1+(n-1)\beta)D^2},$$

where $D = [(n+1)^2 b\gamma - 2(n(1-\beta)+\beta)]$.

For the second order condition (SOC) to be satisfied, $\frac{\partial^2 \Pi_i}{\partial (y_i)^2}|_{y_i = y^*} \geq 0, i = \overline{1,n}$,we need $b\gamma > \frac{2(n(1-\beta)+\beta)^3}{(n+1)^2(n(1-\beta^2)+\beta^2)}$. By choosing $b\gamma > 2$ we guarantee that the SOC is satisfied for any $n \geq 1$ and $\beta \in [0,1]$.

To calculate X^* we substitute y^* into the equation (1).

To find the values for q^*, Q^* and P^* we used the following relations:

$$
\begin{aligned}
q^* &= \frac{1}{(n+1)b}(a-(c-X^*)) \\
Q^* &= nq^* \\
P^* &= a-bQ^*.
\end{aligned} \tag{5}
$$

■

Proof of Proposition 2.

By differentiating X^* w.r.to β we get:

$$
\frac{\partial X^*}{\partial \beta} = -\frac{2b\gamma\,(a-c)\,(n-1)\,(n+1)^2}{D^2}
$$

where $D = [b\gamma(n+1)^2 - 2(n(1-\beta)+\beta)] > 0$. Clearly, $\frac{\partial X^*}{\partial \beta} \leq 0$ for any $a > c, n \geq 1$.

Differentiating the relations in (5) it is clear that at the equilibrium

$$
sign[\frac{\partial P^*}{\partial \beta}] = -sign[\frac{\partial Q^*}{\partial \beta}] = -sign[\frac{\partial q^*}{\partial \beta}] = -sign[\frac{\partial X^*}{\partial \beta}]
$$

That means $\frac{\partial q^*}{\partial \beta} \leq 0$, $\frac{\partial Q^*}{\partial \beta} \leq 0$, $\frac{\partial P^*}{\partial \beta} \geq 0$. ■

Proof of Proposition 3.

(a) Differentiating the per-firm equilibrium profit

$$
\Pi_n^* = \frac{(a-c)^2\gamma(b\gamma(n+1)^2(1+(n-1)\beta) - 2(\beta+n(1-\beta))^2)}{(1+(n-1)\beta)D^2},
$$

where $D = [(n+1)^2 b\gamma - 2(n(1-\beta)+\beta)] > 0$, w.r.to β we get

$$
\frac{\partial \Pi_n^*}{\partial \beta} = -\frac{2(a-c)^2(n-1)\gamma A}{(1+(n-1)\beta)^2 D^3} \tag{6}
$$

where $A = (2(n(1-\beta)+\beta)^3 + b\gamma(n+1)^2(3(1+(n-1)\beta)^2 - (n+1)^2))$.

As it is clear from (6), for all $\beta \in [0,1], n \geq 1$ and $b\gamma > 2$:

$$
sign[\frac{\partial \Pi_n^*}{\partial \beta}] = -sign[A]
$$

Solving $A = 0$ we find a unique solution $\beta^* \in [0;1]$. For $\beta \in [0;\beta^*)$ the per-firm equilibrium profit is increasing in β since $\frac{\partial \Pi_n^*}{\partial \beta} > 0$ and for $\beta \in (\beta^*; 1]$ the profit is decreasing with spillover: $\frac{\partial \Pi_n^*}{\partial \beta} < 0$.

(b) Differentiating the equilibrium social welfare

$$SW^* = \frac{(a-c)^2 \gamma n (b\gamma(n+1)^2(n+2)(1+(n-1)\beta) - 4(\beta + n(1-\beta))^2)}{2(1+(n-1)\beta)D^2}$$

w.r.to β we get

$$\frac{\partial SW^*}{\partial \beta} = -\frac{(a-c)^2 \gamma n (A + b\gamma(n+1)^2 n(1+(n-1)\beta)^2)}{2(1+(n-1)\beta)D^2}$$

where A is the same as in (5).

Then

$$sign[\frac{\partial SW^*}{\partial \beta}] = -sign[A + b\gamma(n+1)^2 n(1+(n-1)\beta)^2]$$

The equation $A + b\gamma(n+1)^2 n(1+(n-1)\beta)^2 = 0$ (and, therefore, $\frac{\partial SW^*}{\partial \beta} = 0$) has a unique solution β^{**} on interval $[0;1]$. For $\beta < \beta^{**}$ the equilibrium social welfare, SW^*, is increasing in β since $\frac{\partial SW^*}{\partial \beta} > 0$ and for $\beta > \beta^{**}$: SW^* is decreasing in β since $\frac{\partial SW^*}{\partial \beta} < 0$. Hence, on the interval of $\beta \in [0;1]$ the solution β^{**} gives a global maximum for SW^*. ■

Proof of Proposition 4.

Part (a) and (b) can be checked by differentiation of X^* and q^* (that are defined in the Proposition 1) w.r.to n:

$$\frac{dX^*}{dn} = -\frac{2b\gamma(a-c)(n+1)(n(1-\beta)+3\beta-1)}{D^2}$$
$$\frac{dq^*}{dn} = -\frac{(a-c)\gamma((n+1)^2 b\gamma - 2(1-2\beta))}{D^2}$$

where, as before, $D = [(n+1)^2 b\gamma - 2(n(1-\beta)+\beta)] > 0$. It is easy to check that for the considerable range of parameters $\beta, b\gamma$ and n we have: $\frac{dX^*}{dn} < 0$, $\frac{dq^*}{dn} < 0$. That proves that the per-firm equilibrium effective cost reduction as well as the per-firm equilibrium output is decreasing with the spillover.

(c) Differentiation of Q^* with respect to n gives

$$\frac{dQ^*}{dn} = \frac{(a-c)\gamma((n+1)^2 b\gamma - 2(n^2(1-\beta)+(2n+1)\beta))}{D^2} \tag{7}$$

It is clear from (7) that

$$sign[\frac{dQ^*}{dn}] = sign[((n+1)^2 b\gamma - 2(n^2(1-\beta) + (2n+1)\beta))].$$

For any $b\gamma \geq 2$ we have $((n+1)^2 b\gamma - 2(n^2(1-\beta) + (2n+1)\beta)) > 0$ and, therefore, $\frac{dQ^*}{dn} \geq 0$. The equilibrium level of industry output is increasing with number of firms.

(d) Since $P^* = a - bQ^*$, then

$$sign[\frac{dP^*}{dn}] = -sign[\frac{dQ^*}{dn}].$$

Hence, $\frac{dP^*}{dn} \leq 0$ and, therefore, the equilibrium market price is decreasing in n. ■

Proof of Proposition 5.

By differentiating the per-firm equilibrium profit with respect to the number of firms n and exploiting the formula $X^* = \sqrt{\frac{2}{\gamma} y^* (1 + (n-1)\beta)}$, we can get

$$\begin{aligned} \frac{d\Pi_n^*}{dn} &= -\frac{1}{2} \frac{\gamma X}{(1+\beta(n-1))^2 (n(1-\beta)+\beta)^2} \times \\ &\times \Big(X[b\gamma(n+1)(1+\beta n-\beta)^2 - \beta(n(1-\beta)+\beta)^2] + \\ &+ 2\frac{dX}{dn}(1-2\beta)(n-1)(1+\beta(n-1))(n(1-\beta)+\beta) \Big) \end{aligned}$$

It is clear that

$$\begin{aligned} sign[\frac{d\Pi_n^*}{dn}] &= -sign[X^*[b\gamma(n+1)(1+\beta n-\beta)^2 - \beta(n(1-\beta)+\beta)^2] + \\ &+2\frac{dX^*}{dn}(1-2\beta)(n-1)(1+\beta(n-1))(n(1-\beta)+\beta)] \end{aligned} \quad (8)$$

It is not difficult to show that for all feasible values of $b\gamma$ we have $b\gamma(n+1)(1+\beta(n-1))^2 - \beta(n(1-\beta)+\beta)^2 \geq 0$. That guarantees that the first term on the RHS of the expression (8) under the *sign* symbol is positive.

The positiveness of the second term on the RHS of the expression (8) depends on sign of $\frac{dX^*}{dn}$ and $(1-2\beta)$ as the rest of the multipliers in this term are positive on the considered range of parameters n and β. According to the result (a) of Proposition 4, the equilibrium cost reduction decreases with n: $\frac{dX}{dn} < 0$. Hence, we conclude that the second term on the RHS of the expression (7) is negative for $\beta \in [0; \frac{1}{2})$ and positive when $\beta \in [\frac{1}{2}; 1]$. Immediately, this tells us that $\frac{d\Pi}{dn} < 0$ for $\beta \in [\frac{1}{2}; 1]$. Hence, the per-firm equilibrium profit is decreasing on the range of spillovers from intermediate to high.

On interval $\beta \in [0; \frac{1}{2})$, we have $(1 - 2\beta) > 0$. Then,

$$X^*[b\gamma (n+1)(1+\beta n-\beta)^2 - \beta (n(1-\beta)+\beta)^2]+ \\ +2\frac{dX^*}{dn}(1-2\beta)(n-1)(1+\beta(n-1))(n(1-\beta)+\beta) \gtrless 0$$

iff

$$\frac{\frac{dX^*}{dn}}{X^*} \gtrless -\frac{b\gamma (n+1)(1+\beta n-\beta)^2 - \beta (n(1-\beta)+\beta)^2}{2(1-2\beta)(n-1)(1+\beta(n-1))(n(1-\beta)+\beta)}. \tag{9}$$

Note that the RHS of (9) is decreasing in $b\gamma$.

From the result (c) of Proposition 4 we know that the equilibrium total output is increasing with n for any $b\gamma \geq 2$. By manipulations with the expression $\frac{dQ^*}{dn}$ we get

$$\frac{\frac{dX^*}{dn}}{X^*} \geq -\frac{(n^2(1-\beta)+\beta(1+2n))}{n(n+1)(n(1-\beta)+\beta)}. \tag{10}$$

Comparing the RHS of (9) with RHS of (10) we find that for any $b\gamma \geq 2$ and $\beta \in [0; \frac{1}{2})$ the following inequality holds:

$$-\frac{(n^2(1-\beta)+\beta(1+2n))}{n(n+1)(n(1-\beta)+\beta)} \geq -\frac{b\gamma (n+1)(1+\beta n-\beta)^2 - \beta (n(1-\beta)+\beta)^2}{2(1-2\beta)(n-1)(1+\beta(n-1))(n(1-\beta)+\beta)}.$$

Hence,

$$\frac{\frac{dX^*}{dn}}{X^*} \geq -\frac{b\gamma (n+1)(1+\beta n-\beta)^2 - \beta (n(1-\beta)+\beta)^2}{2(1-2\beta)(n-1)(1+\beta(n-1))(n(1-\beta)+\beta)}$$

That means $\frac{d\Pi_n^*}{dn} < 0$ for $\beta \in [0; \frac{1}{2})$ and $b\gamma > 2$ and, therefore, the per-firm equilibrium profit Π_n^* decreases with the number of firms n. ■

Proof of Proposition 6.

To investigate the behavior of the equilibrium social welfare under the model considered we solve the maximization problem $\max_n SW^*$ taking into account the constraints on the parameters of the model: $n \geq 1$, $\beta \in [0,1], b\gamma > 2$.

To remind, equilibrium social welfare has form

$$SW^* = \frac{(a-c)^2\gamma n((n+1)^2 b\gamma(1+(n-1)\beta)(n+2) - 4(\beta+n(1-\beta))^2)}{2(1+(n-1)\beta)D^2} \tag{11}$$

According to the FOC the candidates for maximum should necessarily satisfy equality $\frac{dSW^*}{dn} = 0$.[12] In our case,

$$\frac{dSW^*}{dn} = \frac{(a-c)^2\gamma[(n+1)^3(1+(n-1)\beta)^2(b\gamma)^2 - 2(n+1)Bb\gamma + 4(n(1-\beta)-\beta)^3(1-\beta)]}{(1+(n-1)\beta)^2 D^3}$$

where $B = [(1-\beta)\beta^2 n^5 + (1+\beta)\beta^2 n^4 + \beta(4-11\beta+11\beta^2)n^3 + (4-16\beta+30\beta^2-21\beta^3)n^2 - (1-11\beta+20\beta^2-10\beta^3)n + \beta(1-\beta)]$ and $D = [(n+1)^2 b\gamma - 2(n(1-\beta)+\beta)]$.

If $n \geq 1$, $\beta \in [0,1], b\gamma > 2$ then $D > 0$ and, therefore, the denominator in $\frac{dSW^*}{dn}$ is strictly positive. We investigate the sign of the numerator in $\frac{dSW^*}{dn}$ by looking separately at three different cases: $\beta \in (0,1), \beta = 0$ and $\beta = 1$.

(1) Let $\beta \in (0,1)$. Since $a > c, b\gamma > 2$, then by solving

$$[(n+1)^3(1+(n-1)\beta)^2(b\gamma)^2 - 2(n+1)Bb\gamma + 4(n(1-\beta)-\beta)^3(1-\beta)] = 0 \tag{12}$$

we will find the solution to $\frac{dSW^*}{dn} = 0$.

The RHS of (12) is quadratic in $b\gamma$ (in n it is a polynomial of degree 6) we solve it w.r.to $b\gamma$. There are two roots $b\gamma_1, b\gamma_2$ that are functions of n and β. For all n and β fixed within the appropriate region, root $b\gamma_2$ is the only one that can take values above 2 (it may take values below 2 depending on n and β). Therefore, we have only one interior solution (interiority can be checked numerically):

$$b\gamma_2 = \frac{(B + \sqrt{B^2 - 4(n+1)(n(1-\beta)+\beta)^3(1-\beta)(1+(n-1)\beta)^2})}{(n+1)^2(1+(n-1)\beta)^2}$$

This is the implicit form for optimal n, solution to $\frac{dSW^*}{dn} = 0$. We denote it by $\widehat{n}(b\gamma_2, \beta)$[13].

It can be shown numerically that for fixed $b\gamma, \beta$ we have

$$\frac{dSW^*}{dn} \geq 0 \text{ for } n \in [1, \widehat{n}(b\gamma_2, \beta)]$$
$$\frac{dSW^*}{dn} < 0 \text{ for } n \in (\widehat{n}(b\gamma_2, \beta), \infty).$$

To take into account a discrete nature of the optimal solution we check $\max\limits_{\{\lfloor\widehat{n}\rfloor, \lceil\widehat{n}\rceil\}} SW^*$[14] We denote the actual maximand by n^*. It can be shown that for combinations of β (at least

[12] Later we will take into account the discrete nature of n.

[13] Note that $\widehat{n}(b\gamma_2, \beta) > 1$.

[14] $\lfloor\widehat{n}\rfloor$ ($\lceil\widehat{n}\rceil$) stands for the nearest positive integer that is less (greater) than $\widehat{n}$ and greater than 1.

0.91) and $b\gamma$ (at most 2.14) $n^* = 1$.

(2) When $\beta = 0$ equation (11) becomes

$$SW^*|_{\beta=0} = \frac{(a-c)^2\gamma((n+1)^2(n+2)nb\gamma - 4n^3)}{2((n+1)^2 b\gamma - 2n)^2}.$$

By differentiating this with respect to n and taking into account the relevant parameter assumptions, we find that

$$sign[\frac{dSW^*}{dn}|_{\beta=0}] = sign[(n+1)^3(b\gamma)^2 - 2(4n-1)(n+1)nb\gamma + 4n^3]. \quad (13)$$

Analyzing the sign of the RHS of (13) we can conclude that

When $b\gamma \in (2; 2(2+\sqrt{3}))$:

$$\frac{dSW^*}{dn}|_{\beta=0} \geq 0 \quad \text{for} \quad n \in [1; \widehat{n}(b\gamma_2, \beta = 0)]$$
$$\frac{dSW^*}{dn}|_{\beta=0} < 0 \quad \text{for} \quad n \in (\widehat{n}(b\gamma_2, \beta = 0); +\infty)$$

When $b\gamma \geq 2(2+\sqrt{3})$:

$$\frac{dSW^*}{dn}|_{\beta=0} > 0 \quad \text{for} \quad n \geq 1$$

where $\widehat{n}(b\gamma, \beta = 0)$ can be presented in an implicit form $b\gamma(n+1)^2 - n(4n - 1 + \sqrt{12n^2 - 12n + 1}) = 0$. Therefore, at $\beta = 0$ the equilibrium social welfare is maximized under a market structure with a finite number of firms when $b\gamma$ is low ($b\gamma < 2(2+\sqrt{3})$) and the equilibrium social welfare is increasing in n when $b\gamma$ is high ($b\gamma \geq 2(2+\sqrt{3})$).

(3) When $\beta = 1$ the formula (11) becomes

$$SW^*|_{\beta=1} = \frac{(a-c)^2\gamma((n+1)^2(n+2)nb\gamma - 4)}{2((n+1)^2 b\gamma - 2)^2}.$$

By differentiating the last formula with respect to n and taking into account the relevant parameter's assumptions, we find that

$$sign[\frac{dSW^*}{dn}|_{\beta=1}] = sign[n(n+2)(b\gamma - 4) + b\gamma + 6]. \quad (14)$$

Analyzing the sign of the RHS of (14) we can conclude that

When $b\gamma \in (2;4)$:

$$\frac{dSW^*}{dn}|_{\beta=1} \geq 0 \quad \text{for} \quad n \in [1;\widehat{n}(b\gamma_2, \beta=1)]$$
$$\frac{dSW^*}{dn}|_{\beta=1} < 0 \quad \text{for} \quad n \in (\widehat{n}(b\gamma_2, \beta=1);+\infty)$$

When $b\gamma \geq 4$:

$$\frac{dSW^*}{dn}|_{\beta=1} > 0 \quad \text{for} \quad n \geq 1$$

where $\widehat{n}(b\gamma_2, \beta=1) = \frac{b\gamma-4+\sqrt{10(4-b\gamma)}}{4-b\gamma}$.

As before (see part (1) of proof of Proposition 6), we take into account the discrete nature of the optimal solution by checking $\max_{\{\lfloor\widehat{n}\rfloor,\lceil\widehat{n}\rceil\}} SW^*$. We denote the actual maximand by n^*. We find that $n^*=1$ when $b\gamma < 2.14$.

Therefore, when $\beta = 1$ and $b\gamma$ is high ($b\gamma \geq 4$) the equilibrium social welfare is increasing in n, for $b\gamma$ low ($2.14 < b\gamma < 4$) the equilibrium social welfare has an inverse U-shape form and for $b\gamma$ very low ($b\gamma \leq 2.14$) the equilibrium social welfare is maximized under monopoly.

■

References

[1] Amir, R., 2000. Modelling imperfectly appropriable R&D via spillovers. *International Journal of Industrial Organization* 18, 1013-1032.

[2] Amir, R. and V.E. Lambson, 2000. On the Effects of Entry in Cournot Markets. *Review of Economic Studies*, Blackwell Publishing, 67(2), 235-54.

[3] Amir, R. and J. Wooders, 1999. Effects of One-way Spillovers on Market shares, Industry Price, Welfare, and R&D Cooperation, *Journal of Economics and Management Strategy*, **8,** 223-249.

[4] Arrow, K. J., 1962. The economic implications of learning by doing. *Review of Economic Studies*, Blackwell Publishing, 29, 155-73.

[5] Audretsch, D.B. and M.P. Feldman, 1996. R&D spillovers and the geography of innovation and production. *American Economic Review* 86 (3), 630-640.

[6] d'Aspremont, C. and A. Jacquemin, 1988. Cooperative and noncooperative R&D in duopoly with spillovers. *American Economic Review* 78, 1133-1137.

[7] Bernstein, J.I. and M.I. Nadiri, 1989. Research and development and intra-industry spillovers: An empirical application of dynamic duality. *Review of Economic Studies 56, 249-269.*

[8] Ciccone, A. and R.E. Hall, 1996. Productivity and the density of economic activity. *American Economic Review* 86, 54-70.

[9] Dasgupta, P. and J. Stiglitz, 1980. Industrial Structure and the Nature of Innovative Activity. *The Economic Journal* 90 (358), 266-293.

[10] De Bondt, R., P. Slaets and B. Cassiman, 1992. The degree of spillovers and the number of rivals for maximum effective R&D. *International Journal of Industrial Organization* 10, 35-54.

[11] Glaeser, E., 1999. Learning in cities. *Journal of Urban Economics* 46, 254-277.

[12] Kamien, M.I., E. Muller and I. Zang, 1992. Research Joint Ventures and R&D Cartels. *American Economic Review* 82 (5), 1293-1306.

[13] Kamien, M.I. and N.L. Schwartz, 1975. Market Structure and Innovation: a survey. *Journal of economic literature* 13 (1), 1-37.

[14] Levin, R.C., W.M. Cohen and D.C. Mowery, 1985. R&D Appropriability, Opportunity and Market Structure: New Evidence on Some Schumpeterian Hypothesis. *American Economic Review*, *Papers and Proceedings,* 75, 20-24.

[15] Martin, S., 2002. Spillovers, Appropriability and R&D. *Journal of Economics* 75, 1-32.

[16] Piga, C. and J. Poyago-Theotoky, 2005. Endogenous R&D spillovers and locational choice. *Regional Science and Urban Economics* 35, 127-139.

[17] Rosenthal, S.S. and W.C. Strange, 2001. The determinants of agglomeration. *Journal of Urban Economics* 50, 191-229.

[18] Schumpeter, J., 1947. Capitalism, Socialism and Democracy. *London: Allen and Unwin.*

[19] Simpson, R.D. and N.S. Vonortas, 1994. Cournot Equilibrium with Imperfectly Appropriable R&D. *Journal of Industrial Economics* 42 (1), 79-92.

[20] Steurs, G., 1995. Inter-industry R&D spillovers: What difference do they make? *International Journal of Industrial Organization* 13, 249-276.

Table 1. The equilibrium social welfare, SW^*, and number of firms, n^* that maximizes it for different values of the spillover parameter β and cost of R&D, $b\gamma$. The equilibrium social welfare can be maximized under monopoly, oligopoly or perfect competition (see Proposition 6).

	$\gamma = 2.01$	$\gamma = 4.01$	$\gamma = 6.01$	$\gamma = 8.01$
$\beta = 0$	$SW^* = 0.571429$ / $n^* = 2$	0.506616 / $n^* = 4$	0.500285 / $n^* = 11$	0.5 / $n^* \to \infty$
$\beta = 0.2$	$SW^* = 0.590278$ / $n^* = 2.$	0.526884 / $n^* = 5$	0.514066 / $n^* = 7$	0.508917 / $n^* = 9$
$\beta = 0.4$	$SW^* = 0.590629$ / $n^* = 2$	0.526043 / $n^* = 5$	0.512925 / $n^* = 7$	0.507828 / $n^* = 8$
$\beta = 0.6$	$SW^* = 0.580852$ / $n^* = 2$	0.519149 / $n^* = 4$	0.50861 / $n^* = 7$	0.504862 / $n^* = 9$
$\beta = 0.8$	$SW^* = 0.565417$ / $n^* = 2$	0.509794 / $n^* = 5$	0.503414 / $n^* = 9$	0.501705 / $n^* = 14$
$\beta = 1$	$SW^* = 0.555556$ / $n^* = 1$	0.5 / $n^* \to \infty$	0.5 / $n^* \to \infty$	0.5 / $n^* \to \infty$