Military Procurement, Industry Structure and Regional Conflict*

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Abstract

In this paper, we construct a model of market structure in the global arms industry linking concentration, military procurement, international trade and regional conflict. We show how concentration depends on the willingness of producers to import for their military needs and on the relative size of the external market of non-producers. We show that there can be substantial gains to producers from cooperation in the procurement process, but also small gains to non-producers involved in regional arms races. Arms export controls that limit the level of technology that can be exported to non-producers distribute these cooperative gains from producers to non-producers.

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1 Introduction

In this paper we construct a model of market structure in the global arms industry. This is interesting because there has been a large increase in concentration since the end of the Cold War, the share of the top 5 firms has increased from 22% in 1990 to 42% (SIPRI (2001)). However, the degree of global concentration is much lower in comparable high-tech industries and this is largely the result of the fact that governments, the only legitimate customer for major weapons systems, determine market structure.

Although certain aspects of our model are tailored to specific features of military procurement (e.g., demand is partly driven by arms races, arms exports are regulated) other aspects have more general applicability. There are a number of industries in which governments use procurement, subsidies or regulations to influence or determine market structure to meet objectives other than those of pure competition policy. This may be because the government is itself a major customer as for arms or for pharmaceuticals in many countries. The government may also influence market structure because it believes that there are important externalities such as press freedom or public service broadcasting in the media industry. Although demand is driven by quite different forces in arms, pharmaceutical or media, in each case governments perceive benefits from a greater variety of product, weapons, drugs, types of television channel, and tend to be concerned that competition alone cannot be relied upon to produce the optimal number of varieties or a structure of supply that meets national needs. These concerns are important in many industrialising countries where the state takes a strategic role in industrial policy. In industrial countries, however, such concerns and consequent government determination of market structure was more widespread in the past.

There has been very little formal modelling of government determination of market structure where the number of varieties of domestically produced products is a matter for policy. Sutton (1998) describes how until the 1970s government procurement rules in many countries restricted the purchase of telecommunications equipment from foreign suppliers and determined the number of firms. The easing of procurement rules that followed the liberalisation of the telecommunications market led to very rapid concentration in the world telecommunications industry. This is what we might expect to happen if governments ceased to care about market structure in the arms industry. Our model incorporates a number of elements unique to the arms industry. In particular, there is two-way international trade in a world where many of the recipients of this trade are non-producers engaged in regional arms races. For a small number of producer countries each government has a demand function for military capability, which is a function of the quantity and quality of a number of different types of weapons. The weapons can either be developed domestically, which involves R&D expenditure, or imported from other producer countries. The governments may have a bias for domestic rather than imported goods because of concerns about security of supply in conflict. Firms are price setters in the exports market. All governments regulate arms exports and choose domestic procurement prices which may be above or below the world market price. By choosing the number of domestic varieties of weapons procured the government determines the number of firms in the industry.

Using this model we study the consequences for the industry structure in the military sector, procurement subsidies and the welfare of producers and non-producers of: increased process innovation and the consequent rise in the costs of R&D[1]; increased trade between producers owing to the willingness to forgo the security benefits of domestic procurement; and a growing external market of non-producers. We then go on to explore the potential gains from cooperation between governments in producer countries in the procurement decisions.[2]

A basic feature of our model then is that governments choose the number of firms that compose the domestic procurement sector, whose existence they will guarantee.[3] They also choose the 'quality' of level of technology. High R&D costs associated with the latter in particular, imply that firms' existence depends on them being government providers and therefore, the government is actually choosing the number of domestic firms. In order to explain why governments want to keep several firms as domestic suppliers within the same sector, we use a 'taste for variety model'. This type of model has been traditionally used in the monopolistic competition literature starting with Dixit and Stiglitz (1977). Our

[1] These are aspects of the "Revolution in Military Affairs" discussed in Dunne *et al.* (2003)

[2] This is particularly relevant to the ongoing debate surrounding elements of a common defence policy within the EU.

[3] Our model builds on previous work involving three of the authors which modelled the international arms market but considered market structure to be exogenous; see Garcia-Alonso (1999), Levine and Smith (2000) and Levine *et al.* (2000).

paper however, uses a more general form of the Dixit-Stiglitz utility function, discussed in Benassy (1996), which allows for the taste for variety and the elasticity of substitution between the differentiated military goods to be independent.

The rest of the paper is organized as follows. Section 2 provides the basic set-up and the sequence of moves in the procurement game with governments and firms as players. There are three stages to the procurement and trade game. First, given their military expenditure (assumed to be exogenously fixed), producer governments choose the number of domestic suppliers to support, and the amount and quality of goods to procure from each. They also formulate a subgame perfect plan to import goods at the asking prices, which they will implement at stage 3 of the game. At stage 2 given domestic procurement decisions, firms produce and take part in a price-setting game in the export market. At stage 3 all governments make their import decisions given possibly out-of-equilibrium prices. In addition, importing government engaged in regional arms races, choose they levels of military expenditure. In a subgame perfect equilibrium of the entire game, sections 3, 4 and 5 solve for these stages starting at stage 3. Section 5 characterizes the equilibrium which in general can accommodate various asymmetries between both producers and non-producers. Section 6 provides analytical results for the symmetric form of this equilibrium. Section 7 compares this equilibrium with the cooperative arrangement. Section 8 illustrates the results using numerical solutions and section 9 provides some concluding remarks.

2 The Set-up

2.1 The Model

We model an international market for a military service good, consisting of ℓ producing and importing countries and r non-producers who only import. The total budget in each country available for military expenditure is given. Producer country 1 produces differentiated goods $j = 1, 2, \cdots, n_1$, country 2 produces goods $j = n_1+1, n_1+2, \cdots, n_1+n_2$ etc, so there are $\sum_{i=1}^{\ell} n_i = N$, say, goods in total. As is standard in this Dixit-Stiglitz framework each firm produces a single differentiated good so that the number of firms and varieties coincide.[4] Governments procure from domestic firms (if they exist) and overseas firms who enter or exit the market freely.

As well as horizontal differentiation there is vertical differentiation: each good can also be produced in an unlimited number of vertically differentiated varieties or 'qualities'. If quality increases by a proportion λ, say, then one unit of the good provides λ more services. The *maximum* quality of good j in country $i = 1, 2, \cdots, \ell$ is q_{ij} which is the quality of the domestically procured good. We assume that each firm can produce a lower quality good at the same cost and we allow for the possibility that there is an *arms export regime* in place that restricts the quality of the imported good by country i of variety j to $u_{ij} = \gamma_{ik} q_{kj}$, where q_{kj} is the quality of the domestically procured good by country k of variety $j = n_{k-1} + 1, n_{k-1} + 2, \cdots, n_{k-1} + n_k$. The parameter $\gamma_{ik} \leq 1$ captures the extent of the arms export constraint by the exporting country k on the importing country i. We take this regime to be exogenously imposed on the military authority making the procurement decisions and we do not go into details of how this regime can be sustained.[5]

It makes for a simpler presentation if we focus on decisions in producer country 1. Government 1 procures $d_{1j}, j = 1, 2, \cdots, n_1$ domestically produced military goods with quality q_{1j} and $m_{1j}, j = n_1 + 1, n_1 + 2, \cdot\cdot, N$ imported goods with quality u_{1j}. Military strength takes the form of a generalized Dixit-Stigliz CES utility function of the form

$$S_1 = [w_1 n_1 + (1 - w_1)(N - n_1)]^{\nu} \left[w_1 \sum_{j=1}^{n_1} (q_{1j} d_{1j})^{\alpha} + (1 - w_1) \sum_{j=n_1+1}^{N} (u_{1j} m_{1j})^{\alpha} \right]^{\frac{1}{\alpha}} ; \ \alpha \in [0, 1), \ \nu > 0 \qquad (1)$$

In (1) the weights w_1 and $1 - w_1$, with $w_1 \in [\frac{1}{2}, 1]$, express a possible bias for domestic rather than imported procurement in country 1.[6] If we put $\nu = 0$ and $w = \frac{1}{2}$, (1) reduces

[4]We can consider a 'variety' as a weapons system produced by a financially independent profit-maximizing branch of a large organization such as BAE. The former rather than the latter is a 'firm' in our set-up.

[5]At suitable stages of the game we impose symmetry between firms in the same country. Then each country produces the same quality of each variety and with $j = k$, the export control regime imposed by country k on country i is expressed by country i importing quality $u_{ik} = \gamma_{ik} q_{kk} = \gamma_{ik} q_k$, rewriting $q_{kk} = q_k$.

[6]Note that (1) can also be given an 'iceberg' technology interpretation by writing it as $U_1 = [w_1 n_1 + (1 - w_1)(N - n_1)]^{\nu} \left[\sum_{j=1}^{n_1} (q_{1j} d_{1j})^{\alpha} + \sum_{j=n_1+1}^{N} (T_1 u_{1j} m_{1j})^{\alpha} \right]^{\frac{1}{\alpha}}$, where $T_1 = \left(\frac{1-w_1}{w_1} \right)^{\frac{1}{\alpha}}$ is the fraction of the original good that actually arrives, the rest 'melting away' on route.

to the familiar Dixit-Stiglitz utility function used in the new trade and endogenous growth literatures. But as Benassy (1996) points out, this form of utility is restricted in that it implies an on-to-one correspondence between the taste for variety and the elasticity of substitution.

To see the significance of this generalized form of the Dixit-Stiglitz utility function, suppose there are two producer countries. Define a function $v_1(n_1, n_2)$ to represent the proportional capability gain from spreading a certain amount of quality-adjusted output $(n_1 + n_2)y$, say, between all $n_1 + n_2$ varieties rather than concentrating a proportion w_1 on one variety in country 1 and a proportion $1 - w_1$ on one imported variety; i.e.,

$$\begin{aligned} v_1(n_1, n_2) &= \frac{[w_1 n_1 + (1 - w_1) n_2]^{\nu} \left[w_1 \sum_{j=1}^{n_1} y^{\alpha} + (1 - w_1) \sum_{j=n_1+1}^{N} y^{\alpha} \right]^{\frac{1}{\alpha}}}{(n_1 + n_2) y} \\ &= \frac{[w_1 n_1 + (1 - w_1) n_2]^{\nu + \frac{1}{\alpha}}}{[w_1^{1+\alpha} + (1 - w_1)^{1+\alpha}]^{\frac{1}{\alpha}} (n_1 + n_2)} \end{aligned}$$

Suppose that the total number of varieties $N = n_1 + n_2$ increases keeping the proportion $\frac{n_1}{N} = \kappa$ fixed. Then putting $n_1 = \kappa N$ and $n_2 = (1 - \kappa)N$, $v_1 = v_1(N) = \text{constant} \times N^{(\nu + \frac{1}{\alpha} - 1)}$, we can define the *taste for variety* by the elasticity $\frac{N dv_1}{v_1 dN} = \tau$ say as

$$\tau = \frac{N dv_1}{v_1 dN} = \nu + \frac{1}{\alpha} - 1$$

The significance of the extra term in (1) is now apparent. If $\nu = 0$, then the taste for variety $\tau = \frac{1}{\alpha} - 1 = \frac{1}{\sigma - 1}$ which is determined solely by the elasticity of substitution σ. Thus this formulation establishes an arbitrary link between different characteristics: taste for variety and elasticity of substitution, the latter also determining the market power since the mark-up on marginal costs in the export market is $\frac{1}{\alpha}$. Introducing the extra term breaks this link and has important consequences for the subsequent analysis.

Governments in producer countries procure from domestic and foreign firms, possibly at different prices. Let p_{1j} be the price of the procured domestic good and P_j be the world market price of the traded good of variety j produced by firms in all producing countries $j = 1, 2, \cdot\cdot, N$. Then the budget constraint for government in producer country 1 is:

$$\sum_{j=1}^{n_1} p_{1j} d_{1j} + \sum_{j=n_1+1}^{N} P_j m_{1j} = G_1 \tag{2}$$

where G_i is total procurement expenditure in country i.

For the non-producing country $i = \ell + 1, \ell + 2, \cdots, \ell + r$ military strength is given by

$$S_i = N^{\nu} \left[\sum_{j=1}^{N} (u_{ij} m_{ij})^{\alpha} \right]^{\frac{1}{\alpha}} \tag{3}$$

where u_{ij} is the quality allowed to importing non-producing country i. Their budget constraint is:

$$\sum_{j=1}^{N} P_j m_{ij} = G_i \tag{4}$$

The model is completed by specifying the following cost structure for the firm. Firm j produces d_j units of variety j for its domestic government at a procurement price p_j and exports x_j units at a international market price P_j. The cost of producing total output $y_j = d_j + x_j$ for firm j in country i, at maximum quality q_j, is assumed to be

$$C(y_j, q_j) = F_i + f_i q_j^{\beta_i} + c_i y_j = H_i(q_j) + c_i y_i \tag{5}$$

say. The first term in (5) are fixed set-up costs, the second term is R&D expenditure into quality, and the final term constitutes variable costs. It follows that the firm's profit is

$$\pi_j = p_j d_j + P_j x_j - C_i(y_j, q_j) \tag{6}$$

and since there is free entry and exit, we must impose the participation constraint $\pi_j \geq 0$ on the procurement decision.

2.2 Sequencing of Events

We first consider the optimal decisions of a single government taking the decisions of other governments as given. The sequencing of events is as follows:

1. Domestic Procurement by Producers. Given military expenditure, the government in producer country 1 sets and procures domestic goods of quantity d_{1j} and quality q_{1j} at price p_{1j}, for $j = 1, 2, \cdots, n_1$. It also formulates a time-consistent plan to import goods m_{1j} of quality u_{1j}, for $j = n_1 + 1, n_1 + 2, \cdots, N$ at the world market equilibrium price P_j. All decisions are subject to a budget constraint and a non-negative profit participation constraint for domestic firms. The procurement price may be greater or less than the international market price. Firms already participating in the international market will always accept domestic procurement as long as the procurement price exceeds the marginal cost. In general, the world market price can depend on procurement decisions at this stage, but for large N (assumed in the analysis) we have monopolistic competition with the price (set in stage 2 below) given by $P_j = P = \frac{c}{\alpha}$ which depends only on the marginal cost c and the elasticity parameter α.

2. Monopolistic Competition between Firms. With a commitment to producing d_{1j}, in a price-setting equilibrium of this stage of the game, firms in producer country 1 set world prices P_j and export quantity x_{1j} of quality u_{ij} to countries $i = 2, \cdots, \ell + r$. Note that decisions on quality are decided at stage 1 by the procuring governments.

3. Military Spending by Non-Producers and Demand for Imports by all Countries. Given the world market price P_j and quality u_{ij}, and military expenditure, governments in both producer and non-producer countries $i = 1, 2, \cdots, \ell + r$ procure imports of good, m_{ij}, $j = 1, 2, \cdots, N$ of quality u_{ij}, where $i \neq j$ for producer countries $i = 1, 2, \cdots, \ell$. Non-producers anticipating these decisions allocate resources between consumption and military expenditure.

To solve for the equilibrium[7] we proceed by backward induction starting at stage 3.

3 Military Spending by Non-Producers and Demand for Imports

3.1 Non-Producers

Consider non-producers in regions $i = \ell + 1, \ell + 2, \cdots, \ell + r$ as pairs of risk-neutral, countries, A and B say, involved in a regional conflict. They have a given, GDP, Y_{iA}, Y_{iB} respectively, which can be devoted to military expenditure G_{iA}, G_{iB} or other forms of consumption expenditure C_{iA}, C_{iB}. Choose the price of the consumption good as the numeraire. The budget constraint is therefore

$$Y_{ik} = G_{ik} + C_{ik}\,;\ \ k = A, B \tag{7}$$

Consider now a war in region i leaving $\phi_i(C_{iA} + C_{iB})$ available for consumption. The parameter $\phi_i \in [0, 1]$ captures the destructive effect of a war in region i. Suppose that the

[7]Note that in the absence of procurement considerations and with $\nu = 0$, the trade equilibrium corresponds exactly to a standard 'new-trade' model. Then stage 1 of our model is simply the free-entry process.

prize for winning is some proportion of total consumption, $\theta_i \phi_i (C_{iA} + C_{iB})$. Then assuming the two countries maximize expected consumption the expected utility, following a war for country $k = A$ is given by

$$U_{iA}(S_{iA}, S_{iB}, G_{iA}, G_{iB}) = [p_{iA}(S_{iA}, S_{iB})\theta_i + (1 - p_{iA}(S_{iA}, S_{iB}))(1 - \theta_i)]\phi_i(C_{iA} + C_{iB}) \quad (8)$$

In (8), $p_i(S_{iA}, S_{iB})$ is a *Contest Success Function* (CSF) used extensively in the conflict literature.[8] Dropping the regional subscript i in the rest of this subsection, a general form of the CSF , discussed in Skaperdas (1996), takes the form

$$p_A = \frac{f(S_A)}{f(S_A) + f(S_B)} ; \; f' > 0 \quad (9)$$

$$p_B = \frac{f(S_B)}{f(S_A) + f(S_B)} = 1 - p_A \quad (10)$$

Two forms of the CSF, discussed in Hirshleifer (2000) at some length, are the *ratio form* and the *difference form*. These take the forms respectively

$$p_A = \frac{(b_A S_A)^m}{(b_A S_A)^m + (b_B S_B)^m} \quad (11)$$

$$p_A = \frac{\exp(k b_A S_A)}{\exp(k b_A S_A) + \exp(k b_B S_B)} = \frac{1}{1 + \exp(k(b_B S_B - b_A S_A))} \quad (12)$$

In both these forms b_i, $i = A, B$ is a measure of the effectiveness of the same military capability in the hands of country i; m in (11) or k in (12) are *decisiveness parameters* scaling the degree to which a side's greater military strength translates into enhanced battle success. As we shall see the form of CSF is quite crucial in determining the effect of variety and quality on the choice of military expenditure.

At stage 3, given the price P_j, and the number of differentiated goods, in a regional Nash equilibrium, the importing government in non-producer country A chooses both total expenditure G_A and a composition of imports $m_{Aj}, j = 1, 2, \cdots, N$ to maximize U_A given by (8) subject to its budget constraint (7), given the corresponding decision G_B of its rival. We decompose this optimization problem into two parts. First maximize the military strengths S_A, S_B that can be achieved with a given expenditures G_A, G_B. Let $S_A^*(G_A), S_B^*(G_B)$ be these maximized levels of military security. Then country A maximizes the utility $U_{iA}(S_{iA}^*(G_A), S_{iB}^*(G_B), G_{iA}, G_{iB})$ with respect to G_A given its budget constraint and G_B, and country B acts similarly.

[8]See, for example, Anderton (2000), Hirshleifer (2000), Skaperdas (1991), Garfinkel and Skaperdas (2000).

To carry out the first part of this optimization define a Lagrangian for non-producer country A

$$S_A - \lambda \left(\sum_{j=1}^{N} P_j m_{Aj} - G_A \right)$$

where $\lambda \geq 0$ is a Lagrange multiplier. Then the first-order conditions are:

$$\frac{1}{\alpha} \left[\sum_{j=1}^{N} (u_{Aj} m_{Aj})^{\alpha} \right]^{\frac{1}{\alpha}-1} \alpha u_{Aj}^{\alpha} m_{Aj}^{\alpha-1} = \lambda P_j \, ; \, j = 1, 2, \cdots N \tag{13}$$

Now divide the jth of (13) by the kth to give

$$\left(\frac{u_{Aj} m_{Aj}}{u_{Ak} m_{Ak}} \right)^{\alpha-1} = \frac{u_{Ak} P_j}{u_{Aj} P_k}$$

Substituting back into the budget constraint (3) we get

$$\sum_{k=1}^{N} \left(\frac{P_k}{u_{Ak}} \right) u_{Aj} m_{Aj} \left(\frac{u_{Aj} P_k}{u_{Ak} P_j} \right)^{-\frac{1}{1-\alpha}} = \sum_{k=1}^{N} \left(\frac{P_k}{u_{Ak}} \right)^{1-\sigma} \left(\frac{P_j}{u_{Aj}} \right)^{\sigma} u_{Aj} m_{Aj} = G_A$$

where $\sigma = \frac{1}{1-\alpha} > 1$. This results in the quality-adjusted demand by government A for good $j = 1, 2, \cdot \cdot \cdot, N$ given by

$$u_{Aj} m_{Aj} = \frac{G_A}{\left(\frac{P_j}{u_{Aj}} \right)^{\sigma} \sum_{k=1}^{N} \left(\frac{P_k}{u_{Ak}} \right)^{1-\sigma}} \tag{14}$$

To interpret and manipulate (14) it is convenient to define

$$\hat{P_A} = \left[\sum_{k=1}^{N} \left(\frac{P_k}{u_{Ak}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{15}$$

Then $\hat{P_A}$ is the quality-adjusted form of the familiar price index of imported goods, facing non-producer country A, used in the product differentiation literature (see, for example, Beath and Katsoulacos (1991), chapter 3). Now (14) and (15) can be written

$$u_{Aj} m_{Aj} = \frac{G_A}{\hat{P_A}^{1-\sigma}} \left(\frac{P_j}{u_{Aj}} \right)^{-\sigma} \tag{16}$$

The importance of (16) is that given $\hat{P}$, the elasticity of quality-adjusted demand for variety j on the world market with respect to the quality-adjusted price is constant with elasticity $-\sigma$. Substituting (16) into (3) and using $\sigma = \frac{1}{1-\alpha}$ we have

$$S_A^* = \frac{N^{\nu} G_A}{\hat{P}_A^{1-\sigma}} \left[\sum_{j=1}^{N} \left(\frac{P_j}{u_{Aj}} \right)^{-\alpha\sigma} \right]^{\frac{1}{\alpha}} = \frac{N^{\nu} G_A}{\hat{P}_A^{1-\sigma}} \hat{P}_A^{\frac{1-\sigma}{\alpha}} = \frac{N^{\nu} G_A}{\hat{P}_A} \tag{17}$$

Hence in terms of optimal security, the budget constraint of country A takes the convenient form

$$C_A + G_A = C_A + N^{-\nu}\hat{P}_A S_A^* = Y_A \tag{18}$$

Country A now maximizes

$$U_A(S_A^*, S_B^*, G_A, G_B) = [p_A(S_A^*, S_B^*)\theta + (1 - p_A(S_A^*, S_B^*))(1-\theta)]\phi(C_A + C_B) \tag{19}$$

with respect to G_A, subject to (18) and a corresponding budget constraint for country B, given G_B.

The first order conditions defining the Nash equilibrium are

$$(2\theta - 1)\frac{\partial p_A}{\partial S_A^*} = \frac{[p_A(S_A^*, S_B^*)\theta + (1 - p_A(S_A^*, S_B^*))(1-\theta)]N^{-\nu}\hat{P}_A}{Y_A + Y_B - N^{-\nu}\hat{P}_A S_A^* - N^{-\nu}\hat{P}_B S_B^*} \tag{20}$$

$$(2\theta - 1)\frac{\partial p_B}{\partial S_B^*} = \frac{[p_B(S_A^*, S_B^*)\theta + (1 - p_B(S_A^*, S_B^*))(1-\theta)]N^{-\nu}\hat{P}_B}{Y_A + Y_B - N^{-\nu}\hat{P}_A S_A^* - N^{-\nu}\hat{P}_B S_B^*} \tag{21}$$

With $S_A^* = \frac{N^\nu G_A}{\hat{P}_A}$ and $S_B^* = \frac{N^\nu G_B}{\hat{P}_B}$, (20) and (21) define two *reaction functions* in G_A and G_B. Their intersection is a Nash equilibrium; in general this is asymmetrical with asymmetries arising from differences in GDP Y_A and Y_B and differences in the prices facing each country. The latter can arise if export control regimes differ between the two countries. However for the most part we focus on symmetrical equilibria. Then putting $p_A = p_B = \frac{1}{2}$, $S_A = S_B = S$, say, $\hat{P}_A = \hat{P}_B = \hat{P}$, $Y_A = Y_B = Y^{np}$ and $G_A = G_B = G^{np}$, for all non-producers the, right-hand-side of the reaction functions becomes $\frac{N^{-\nu}\hat{P}}{4(Y^{np} - N^{-\nu}\hat{P}S)}$. The left-hand-side however depends on the form of the CSF. For the ratio form in a symmetric equilibrium $\frac{\partial p_A}{\partial S_A} = \frac{m}{4S}$; for the difference form $\frac{\partial p_A}{\partial S_A} = \frac{kb}{4}$ where we have put $b_A = b_B = b$. Hence substituting into (20) or (21) we arrive at the symmetric Nash equilibria for the two cases

$$\text{Ratio form of CSF} \;:\; G^{np} = \frac{(2\theta - 1)mY^{np}}{[1 + (2\theta - 1)m]} \tag{22}$$

$$\text{Difference form of CSF} \;:\; G^{np} = Y^{np} - \frac{N^{-\nu}\hat{P}}{(2\theta - 1)kb} \tag{23}$$

Thus an internal maximum $G > 0$ requires $\theta > \frac{1}{2}$ in both cases whilst for the difference form we must impose a further condition $\hat{P} < (2\theta - 1)bY^{np}$.

With the ratio form of the CSF we now see that military expenditure is independent of both the price index and the total number of varieties. This is a familiar property of the standard Dixit-Stigliz monopolistic competition model where the utility function is a Cobb-Douglas function of the numeraire good and the composite quantity index of differentiated goods (military strength in the context of our model). However with the difference form the optimal choice of military spending declines as the price index $\hat{P}$ increases, or the total number of varieties, N decreases. Since in a market symmetric equilibrium $\hat{P} = \frac{PN^{-\frac{1}{(\sigma-1)}}}{\gamma q}$, and the price P is constant (as we shall see), it follows from (23) that

$$\frac{\partial G^{np}}{\partial N} = \left(\nu + \frac{1}{\sigma - 1}\right) \frac{N^{-\nu-1}\hat{P}}{(2\theta - 1)kb} > 0 \quad (24)$$

$$\frac{\partial G^{np}}{\partial q} = \frac{N^{-\nu}\hat{P}}{q(2\theta - 1)kb} > 0 \quad (25)$$

Since in the symmetric Nash equilibrium, *any* military expenditure is inefficient in this set-up, this means that the welfare of the non-producers actually *increases* if quality decreases and/or the arms control regime is strengthened (then γ and therefore $\hat{P}$ falls) and/or N decreases. This striking result is a consequence of the unique character of military goods– they bring security to the purchaser but insecurity to a threatened rivals. We summarize our result as:

Proposition 1

With the ratio form of the CSF, the military expenditure of non-producers is independent of the number and quality of differentiated goods. With the difference form of the CSF, military expenditure falls and welfare increases as the number and quality decreases and/or the export regime is strengthened.

Which form of the CSF function is relevant for modelling a pair of countries involved in a regional conflict? An interesting general discussion of the choice between the ratio and difference form is to be found in Hirshleifer (2000). He suggests that "the ratio form of the CSF corresponds to clashes taking place under theoretically ideal conditions such as full information, absence of fatigue and a uniform battlefield without topographical features". Otherwise the difference form is more appropriate. He goes on to comment that "many real-world contexts might be best modelled by a CSF that combines elements of the ratio and difference form ". We follow this route and assume a *linear combination* of forms leading to an expression for the government spending-GDP ratio in non-producing countries as follows:

$$\frac{G^{np}}{Y^{np}} = \kappa \frac{(2\theta - 1)m}{[1 + (2\theta - 1)m]} + (1 - \kappa)\left[1 - \frac{N^{-\nu}\hat{P}}{(2\theta - 1)kbY^{np}}\right] \tag{26}$$

where $\kappa \in [0,1]$ and $\kappa = 0$ and $\kappa = 1$ apply in the 'pure' cases of the ratio and difference forms of CSF function respectively.

3.2 Producers

As for non-producers the import demand for any good $j = 1, 2, \cdots, N$ can similarly be written as

$$\begin{aligned} u_{ij}m_{ij} &= \frac{[G_i - \sum_{j=n_{i-1}+1}^{n_{i-1}+n_i} p_{ij}d_{ij}]}{\left(\frac{P_j}{u_{ij}}\right)^{\sigma} \sum_{k \neq [N_{i-1}, N_i]}^{N} \left(\frac{P_k}{u_{ik}}\right)^{1-\sigma}} \, ; \, j \neq N_{i-1} + 1, \, N_{i-1} + 2, \cdots, N_{i-1} + n_i \\ &= 0\, ; \qquad j = N_{i-1} + 1, \, N_{i-1} + 2, \cdots, N_{i-1} + n_i \end{aligned} \tag{27}$$

where we have defined $N_i = n_1 + n_2 + \cdots + n_i$ for $i \geq 1$ (in which case $N_1 = n_1$ and $N_\ell = N$), country $i = 1, 2, \cdots, \ell$ produces varieties $j = N_{i-1}+1, N_{i-1}+2, \cdots, N_{i-1}+n_i = N_i$ and imports m_{ij} units of variety $j = 1, 2, \cdots, N_{i-1}, N_i+1, N_i+2, \cdots, N$ (defining $N_0 = 0$). Again we can define an price index of imported goods for producer countries as

$$\hat{P}_i = \left[\sum_{k \neq [N_{i-1}, N_i]}^{N} \left(\frac{P_k}{u_{ik}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \; ; \; i = 1, 2, \cdots, \ell \tag{28}$$

4 Monopolistic Competition between Firms

Turning to stage 2 of the game, in producer country 1 firm $j = 1, 2, \cdots, n_1$ profit at stage 2 is given by

$$\pi_{1j} = (p_{1j} - c_1)d_{1j} + (P_j - c_1)x_{1j} - H_1(q_{1j})\, ; \, j = 1, 2, \cdots, n_1 \tag{29}$$

where exports to producers and non-producers are given by

$$x_{1j} = \sum_{i=2}^{\ell+r} m_{ij} = \sum_{i=2}^{\ell} \frac{[G_i - \sum_{j=n_{i-1}+1}^{n_{i-1}+n_i} p_{ij}d_{ij}]}{P_j^{\sigma}\hat{P}_i^{1-\sigma}} + \sum_{i=\ell+1}^{\ell+r} \frac{G_i u_{i1}^{\sigma-1}}{P_j^{\sigma}\hat{P}_i^{1-\sigma}} \tag{30}$$

The first term in the last element of (30) consists of exports to other producing countries and depends on the procurement decisions already taken at stage 1 and on all prices set at stage 2 of the game. The second term consists of exports to non-producing countries and depend on the all prices set by firms at stage 2 of the game.

For producers let $\Gamma_i = G_i - \sum_{j=N_{i-1}+1}^{N_{i-1}+n_i} p_{ij}d_{ij}$ be the part of the government budget devoted to imports and define $\Gamma_i = G_i$ and $\hat{P}_i = \hat{P}$ for non-producers. Then maximizing profits given by (29) with respect to P_j, gives the first-order conditions

$$(P_j - c_1)\frac{\partial x_{1j}}{\partial P_j} + x_{1j} = 0 \tag{31}$$

where from (30)

$$\frac{\partial x_{1j}}{\partial P_j} = -\frac{\sigma x_{1j}}{P_j} - \underbrace{P_j^{-\sigma}\sum_{i=2}^{\ell+r}\frac{\Gamma_i u_{i1}^{\sigma-1}}{\hat{P}_i^{2(1-\sigma)}}\frac{\partial(\hat{P}_i^{1-\sigma})}{\partial P_j}}_{\text{strategic interaction term}} \tag{32}$$

In working out the effect of a change in the price of variety firm j considers two effects: the first term takes the total price index of imports facing other countries $\hat{P}_i\,;\, i = 2, 3, \cdots, \ell+r$ as given. The second *strategic* term considers the effect on each of these price indices of the firms export price. If N is large, which we assume in this paper, then there are so many firms that we can ignore this strategic effect.[9] Then substituting (32) back into (31), the first order condition becomes

$$\left[-\frac{\sigma(P_j - c_1)}{P_j} + 1\right] x_{1j} = 0\,;\, j = 1, 2, \cdots, n_1 \tag{33}$$

Hence using (30) we obtain from (33) the *Lerner Index* for any variety $j \in [1, n_1]$ in country 1 as

$$L_1 = \frac{P_1 - c_1}{P_1} = \frac{1}{\sigma}$$

This is the familiar monopolistic competition result. The price of every good exported from country 1 is a constant mark-up on marginal cost: $P_1 = \frac{c_1}{\alpha}$. Similarly for country i, the price is given by $P_i = \frac{c_i}{\alpha}$

5 Domestic Procurement by Producers

We now complete the equilibrium by evaluating the optimal decisions of the government in country 1 at stage 1 of the game. Producer countries $i = 1, 2, \cdots, \ell$ face no single

[9] However as the number becomes small this strategic term becomes significant - see Garcia-Alonso and Levine (2004) for analysis of this case.

rival or enemy but invest in defensive and offensive capability to provide insurance in an uncertain world against a range of possible security needs. For instance the US military policy, since the end of the Cold War has been based on the capability to fight two regional conflicts at any particular time. We do not attempt to model their expenditure decisions, perceptions of threat are difficult to model, and take the military expenditure of producers as exogenous.

Each government in the producer countries then maximizes military strength for a given government spending. The government when choosing the procurement price, p_1, relaxes or tightens the firms' participation constraint and, in effect, chooses the number of domestic firms. Imposing symmetry between identical domestic firms, we have that $d_{1j} = d_1$ and $q_{1j} = q_1$ for all domestic varieties. Similarly, given the symmetry between all firms in countries $i = 2, 3, \cdots, \ell$ in the international market, government 1 will choose the same amount of imports of each variety from that country, m_{1i} say, of quality u_{1i}.[10] We examine a complete information Nash equilibrium of stage 1 of the game, and a subgame perfect equilibrium of the whole game, where for country 1, independent decision variables are d_1, q_1 and n_1.[11]

The optimization problem of the government in country 1 is to maximize military strength given by

$$S_1 = [w_1 n_1 + (1 - w_1)(N - n_1)]^{\nu} \left[w_1 n_1 (q_1 d_1)^{\alpha} + (1 - w_1) \sum_{i=2}^{\ell} n_i (u_{1i} m_{1i})^{\alpha} \right]^{\frac{1}{\alpha}} \tag{34}$$

with respect to the independent choice variables d_1, q_1, and n_1, given the world prices $P_i = P = \frac{c_i}{\alpha}$; $i = 2, 3, \cdots, \ell$ of each variety from country i, the corresponding decisions of other countries, and two sets of constraints. These are the budget constraint (BC_1) and

[10] Note that at stage 1 we have imposed symmetry between firms and have therefore defined m_{ij} and u_{ij} as the quantity and quality respectively imported by country i of *any good* from country j, $i, j = 1, 2, \cdots, \ell$. By contrast at stage 3 j referred to the *variety* available on the world market, $j = 1, 2, \cdots, N$.

[11] Any two from three possible decision variables, d_1, p_1 and n_1 can be assumed, but will lead to different Nash equilibria at stage 1. Our particular choice, d_1, and n_1 is made partly, for analytical convenience, but can be also justified by the need to *observe* decision variables in a more realistic incomplete information setting, where the process of dynamic adjustment towards the equilibrium, for example of a Cournot-type, needs to be addressed. It is plausible to assume that the domestic procurement decision, d_i, and the number of firms supported, n_i, $i = 1, 2, \cdots, \ell$ are more readily observed that the procurement price, p_i, $i = 1, 2, \cdots, \ell$, which involves a possibly hidden subsidy.

the representative domestic firm's participation constraint (PC_1) given by

$$BC_1 \ : \ p_1 n_1 d_1 + \sum_{i=2}^{\ell} P_i n_i m_{1i} = G_1$$
$$PC_1 \ : \ \pi_1 = (p_1 - c_1)d_1 + (P_1 - c_1)x_1 - F_1 - f_1 q_1^{\beta_1} \geq 0$$

and the corresponding constraints in the other countries. Clearly the PC constraint must bind so the procurement price is given by

$$p_1 = c_1 + \frac{F_1 + f_1 q_1^{\beta_1} - (P_1 - c_1)x_1}{d_1} = c_1 + \frac{H_1(q_1) - R_1(x_1)}{d_1} \tag{35}$$

where we have written export net revenue $(P_1 - c_1)x_1 = R_1(x_1)$ and we recall that total set-up and R&D costs in country 1 are denoted by $H_1(q_1)$. It is useful to note from (30) that exports x_1 of each variety from country 1 can be written in terms of decision variables as

$$\begin{aligned} x_1 &= \sum_{i=2}^{\ell+r} m_{i1} = \sum_{i=2}^{\ell} m_{i1} + \sum_{i=\ell+1}^{\ell+r} \frac{G_i u_{i1}^{\sigma-1}}{P_1^{\sigma}(n_1 P_1^{1-\sigma} u_{i1}^{\sigma-1} + n_2 P_2^{1-\sigma} u_{i2}^{\sigma-1} + \cdots + n_\ell P_\ell^{1-\sigma} u_{i\ell}^{\sigma-1})} \\ &= x_1^{p} + x_1^{np} \end{aligned} \tag{36}$$

where u_{ij} is the quality of each good produced by country j and imported by country i and we have written the total exports of each firm in country 1 as the sum of exports to other producers, x_1^p and to non-producers, x_1^{np}. At stage 1, country 1 only commits to a *total* import budget $P_i n_i m_{1i}$ (determined by the BC_1 as a residual given other decisions by all countries) and *not* to any procurement contract with any foreign firm. The import decision $m_{1j}, j = 1, 2, \cdots, \ell$ for any particular variety is decided at stage 3, given this total budget, and given out-of-equilibrium (at stage 1) prices P_j charged by firms.

Since we are assuming a Nash equilibrium in independent decision variables d_1, q_1 and n_1 for country 1, we can eliminate the procurement price, p_1, using the PC_1 constraint. The BC_1 constraint now becomes

$$BC_1 \ : \ n_1(c_1 d_1 + H_1(q_1) - R_1(x_1)) + \sum_{i=2}^{\ell} P_i n_i m_{1i} = G_1 \tag{37}$$

and the government now maximizes S_1 given by (34) with respect to d_1, q_1, and n_1, given (37), the corresponding budget constraints and independent decision variables of other governments and $P_i = P = \frac{c}{\alpha}$. We shall derive a Nash equilibrium in stage 1 of the game in which exports to other producers x_1^p is a linear combination of decision variables d_i, $i > 1$ and therefore x_1^p can be treated as given. The strategic interaction between the producer countries budget constraints through their trade with each other does not then affect the optimization problem which can be carried out by defining a Lagrangian:

$$\mathcal{L}_1 = S_1 \quad - \quad \lambda_1[n_1(c_1 d_1 + H_1(q_1) - R_1(x_1)) + P\sum_{i=2}^{\ell} n_i m_{1i} - G_1]$$

Then country 1 maximizes $\mathcal{L}_1$ with respect to independent decision variables d_1, q_1, n_1, and with respect to endogenous variables $\{m_{ij},\, \lambda_i\}$, $i, j = 1, 2, \cdots, \ell,\ j \neq i$, given the independent decision variables of the other countries $\{d_i,\, q_i,\, n_i\}$, $i = 2, 3, \cdots, \ell$.

The first-order conditions for an internal solution (where $n_1 \geq 0$ and $d_1 \geq 0$) are then

$$d_1 \quad : \quad \frac{\partial U_1}{\partial d_1} = S_1^{1-\alpha}[w_1 n_1 + (1-w_1)(N-n_1)]^{\alpha\nu} w_1 q_1^{\alpha} d_1^{\alpha-1} = \lambda c \tag{38}$$

$$n_1 \quad : \quad \frac{\partial S_1}{\partial n_1} = \frac{S_1^{1-\alpha}}{\alpha} w_1 q_1^{\alpha} d_1^{\alpha}[w_1 n_1 + (1-w_1)(N-n_1)]^{\alpha\nu} + \nu w_1 S_1[w_1 n_1 + (1-w_1)(N-n_1)]^{-1}$$

$$= \lambda(cd_1 + H_1(q_1) - R(x_1) - n_1\frac{\partial R_1}{\partial n_1}) \tag{39}$$

$$q_1 \quad : \quad \frac{\partial S_1}{\partial q_1} = S_1^{1-\alpha}[w_1 n_1 + (1-w_1)(N-n_1)]^{\alpha\nu} w_1 q_1^{\alpha-1} d_1^{\alpha} = \lambda\left(\frac{\partial H_1}{\partial q_1} - \frac{\partial R_1}{\partial q_1}\right) \tag{40}$$

$$m_1 \quad : \quad \frac{\partial S_1}{\partial m_1} = S_1^{1-\alpha}[w_1 n_1 + (1-w_1)(N-n_1)]^{\alpha\nu}(1-w_1) u_{1i}^{\alpha} m_1^{\alpha-1} d_1^{\alpha} = \lambda_1 P n_i \tag{41}$$

These four equations plus the constraint BC_1 solve for the decision variables n_1, d_1, q_1, and for endogenous variables m_{1j} and λ_1.

To complete the equilibrium we need to evaluate the responses of net export revenue to the number of varieties and quality, $\frac{\partial R_1}{\partial n_1}$ and $\frac{\partial R_1}{\partial q_1}$ respectively. First, at stage 2, from (36) write exports to non-producers as

$$x_1^{np} = \frac{1}{P_1}\sum_{i=\ell+1}^{\ell+r} \frac{G_i}{\sum_{k=1}^{\ell} n_k (\tilde{P}_{1k}^i)^{\sigma-1}} \tag{42}$$

where we have defined the quality-adjusted price of good 1 relative to k, both exported to non-producer i.

$$\tilde{P}_{1k}^i = \frac{P_1 u_{ik}}{P_k u_{i1}} \tag{43}$$

According to (42), the value of exports to non-producers by each firm in country 1, $P_1 x_1^{np}$, depends positively on the 'competitiveness' of its good, $\frac{1}{\tilde{P}_{1k}^i}$ and expenditure by non-producers, and negatively on the numbers of competitors, $n_k\,;\,k=1,2,\cdot\cdot\cdot,\ell$. Differentiating (42) we then have at stage 1 (where $P_i = P$) that

$$\begin{aligned}
\frac{\partial R_1}{\partial n_1} &= (P-c_1)\frac{\partial x_1^{np}}{\partial n_1} \\
&= -L_1\sum_{i=\ell+1}^{\ell+r}\frac{G_i}{\left[\sum_{k=1}^{\ell} n_k(\tilde{P}_{1k}^i)^{\sigma-1}\right]^2} + L_1\sum_{i=\ell+1}^{\ell+r}\frac{\frac{\partial G_i}{\partial n_1}}{\left[\sum_{k=1}^{\ell} n_k(\tilde{P}_{1k}^i)^{\sigma-1}\right]}
\end{aligned} \tag{44}$$

$$\begin{aligned}
\frac{\partial R_1}{\partial q_1} &= (P-c_1)\frac{\partial x_1^{np}}{\partial q_1} \\
&= \frac{L_1(\sigma-1)}{q_1}\sum_{i=\ell+1}^{\ell+r}\frac{G_i\sum_{k=2}^{\ell} n_k(\tilde{P}_{1k}^i)^{\sigma-1}}{\left[\sum_{k=1}^{\ell} n_k(\tilde{P}_{1k}^i)^{\sigma-1}\right]^2} + L_1\sum_{i=\ell+1}^{\ell+r}\frac{\frac{\partial G_i}{\partial q_1}}{\left[\sum_{k=1}^{\ell} n_k(\tilde{P}_{1k}^i)^{\sigma-1}\right]}
\end{aligned} \tag{45}$$

where, in (45) we have used the expression for the export control regime on quality exported by country to country j given by $u_{j1} = \gamma_{j1}q_1$. Note that at stage 1 where prices are equal the quality-adjusted relative price $\tilde{P}_{1k}^i = \frac{u_{ik}}{u_{i1}}$ becomes the relative quality exported.

Equations (44) and (45) are crucial for the results that follow in the paper. We have seen from proposition 1 that if we choose a difference form of the CSF then $\frac{\partial G_i}{\partial n_1} > 0$ and $\frac{\partial G_i}{\partial q_1} > 0$. Thus from (44) there are two opposing effects on the net revenue per firm in country 1 from increasing the number of firms, given q_1, d_1 and the corresponding decisions n_i, q_i and d_i, $i > 1$ of other countries. For a given expenditure by non-producers an increase in n_1 spreads a given demand over more competing domestic firms. Since costs include a fixed set-up and R&D component, average costs increase and revenue falls. This is the first negative term in (44). However with the difference form of the CSF, the second term is positive because an increase in n_1 increases the total variety available, N, and boosts demand from the external market. The sign of $\frac{\partial R_1}{\partial n_1}$ depends on which of these opposite forces dominates. By contrast, there is no ambiguity with respect to the effect on net revenue of a unilateral increase in quality. Given expenditure by non-producers, a unilateral increase in q_1 increases competitiveness and raises demand. It also raises expenditure and so both terms in (45) are positive.

We can now characterize a Nash equilibrium of stage 1 of the game. Using $H_1(q_1) = F_1 + f_1 q_1^{\beta}$ and dividing (39), and (40) by (38), in turn, we can eliminate the shadow price

λ to obtain

$$d_1 = \frac{\left[H(q_1) - R_1(x_1) - n_1 \frac{\partial R_1}{\partial n_1}\right]}{P\left[1 - \alpha + \frac{\alpha\nu}{[w_1 n_1 + (1-w_1)(N-n_1)]^{1+\alpha\nu}} \left(\frac{U_1}{d_1}\right)^{\alpha}\right]} \quad (46)$$

$$m_{1i} = d_1 \left(\frac{c_1(1-w_1)}{P w_1}\right)^{\sigma} \left(\frac{u_{1i}}{q_1}\right)^{\sigma-1} = \phi_1 d_1 \,;\ i > 1 \quad (47)$$

$$c_1 d_1 = \beta_1 f_1 q_1^{\beta_1} - q_1 \frac{\partial R_1}{\partial q_1} \quad (48)$$

The budget constraint BC_1, given by (37) and the expression for net revenue

$$R_1(x_1) = (P - c_1)x_1 = (P - c_1)[\sum_{i=2}^{\ell} m_{i1} + x_1^{np}] \quad (49)$$

with x_1^{np} given by (42), completes the solution for the single economy given the decisions on d_i, $\{m_{ij}\}$, q_i and n_i by the other countries $i > 1$. Combining these equations with analogous ones for the remaining $\ell - 1$ producer countries completes the Nash equilibrium at stage 1 of the game and the Subgame Perfect Nash equilibrium of the whole game. In our set-up asymmetries between producer countries can arise from differences in costs $\{c_i, F_i, f_i, \beta_i\}$, expenditures $\{G_i\}$, the domestic bias parameter $\{w_i\}$ and the nature of the arms export control regime imposed by country j on country i, $\{\gamma_{ij}\}$. For instance regarding the latter, if there are say 4 countries consisting of two alliances each with two countries, then a possible choice of γ_{ij} is the matrix Γ where

$$\Gamma = \begin{pmatrix} 1 & 1 & \gamma & \gamma \\ 1 & 1 & \gamma & \gamma \\ \gamma & \gamma & 1 & 1 \\ \gamma & \gamma & 1 & 1 \end{pmatrix}$$

where $0 \leq \gamma < 1$. Given the domination of one producer country in the world– the US – these asymmetries between producer countries are clearly of practical importance. However a non-symmetric non-cooperative equilibrium can only be solved by numerical solution. By contrast, a symmetric equilibrium is tractable and provides some valuable insights into the procurement and export arrangements in the EU which can be thought of as three approximately equally-sized size countries –the UK, Germany, and France – procuring and exporting arms. For the remainder of the paper we concentrate on the symmetric equilibrium.

6 A Symmetric Non-Cooperative Equilibrium

We solve for a symmetric non-cooperative equilibrium in which all producer countries and all non-producing countries are identical in every respect. Then $w_i = w$, $c_i = c$, $F_i = F$, $f_i = f$, $\beta_i = \beta$, $\gamma_{ij} = \gamma$, $G_i = G^p$ say, for producers ($i = 1, 2, \cdots, \ell$) and $G_i = G^{np}$ for non-producers ($i = \ell+1, \ell+2, \cdots, \ell+r$). Then $P_1 = P_2 = \cdots = P = \frac{c}{\alpha}$, $d_1 = d_2 = \cdots = d$, $n_1 = n_2 = \cdots = n$ etc, $N = \ell n$, $\phi_i = \phi = \left(\frac{\alpha(1-w)}{w}\right)^{\sigma} \gamma^{\sigma-1}$ and $\frac{U_i}{d_i} = \frac{U}{d} = n^{\nu+\frac{1}{\alpha}}[w + (1-w)(\ell-1)]^{\nu}[w + (1-w)(\ell-1)\phi^{\alpha}]^{\frac{1}{\alpha}}$ for producing countries. In addition from (44) and (45) we have

$$\frac{\partial R_1}{\partial n_1} = \frac{\partial R_2}{\partial n_2} = \cdots = -\frac{Lr}{N}\left[\frac{G^{np}}{N} - \frac{\partial G^{np}}{\partial N}\right] \tag{50}$$

$$\frac{\partial R_1}{\partial q_1} = \frac{\partial R_2}{\partial q_2} = \cdots = \frac{Lr}{N}\left[\frac{(\sigma-1)(\ell-1)G^{np}}{q\ell} + \frac{\partial G^{np}}{\partial q}\right] \tag{51}$$

The first-order condition (46) now becomes

$$d = \frac{(H(q) - R(x) + \Theta_1)}{P(1 - \alpha + \Theta_2)} \tag{52}$$

where we have defined

$$\Theta_1 \equiv \frac{Lr}{\ell}\left[\frac{G^{np}}{N} - \frac{\partial G^{np}}{\partial N}\right]$$

$$\Theta_2 \equiv \frac{\alpha\nu[w + (1-w)(\ell-1)\phi^{\alpha}]}{[w + (1-w)(\ell-1)]}$$

and we have used (24). Substituting for $H(q) - R(x)$ from (52) into (35) we arrive at the procurement price in the non-cooperative symmetric equilibrium

$$p = P(1 + \Theta_2) - \frac{\Theta_1}{d} \tag{53}$$

Hence for a 'traditional' Dixit-Stigliz utility function where $\nu = \Theta_2 = 0$ and in the limit as the external market becomes small (but still of sufficient size to determine the world market price), $\Theta_1 \to 0$ and we have that $p = P$; i.e., the procurement price equals the market price. Generally however either $p > P$, in which case the procurement process involves a subsidy, or $p < P$ implying that the government taxes away part of the monopolistic profits. A high taste for variety ν encourages the former and whilst the effect of large external market depends on the sign of Θ_1. It is clear from (53) that $\Theta_1 > 0$ iff the elasticity $\frac{N \partial G^{np}}{G^{np} \partial N} < 1$. If the CSFs are of the ratio form, this elasticity is zero and $\Theta_1 > 0$

unambiguously. For the difference form of the CSFs Θ_1 can be negative if the elasticity is sufficiently high. As we shall see for our calibration the elasticity with the difference form of the CSF *is* extremely high so $\Theta_1 < 1$ and the procurement price unambiguously involves a subsidy. The intuition behind this external effect is that increasing the number of differentiated goods, each produced by a single firm, reduces the net export revenue to the external market per firm and tightens the participation constraint. In a non-cooperative equilibrium each government takes into account only their own contribution to the world supply of differentiated goods and, through reducing the procurement price, lowers its optimal number of domestic firms as the external market becomes more important, provided that the effect on demand is not large. For the ratio form of the CSF this demand effect is zero but for the ratio effect it may be extremely large.We summarize this result as:

Proposition 2: The Procurement Price

In a symmetric, non-cooperative equilibrium without strategic pricing by firms, the procurement price may be above or below the world market price. A high taste for variety leads to the former and provided the elasticity $\frac{N\partial G^{np}}{G^{np}\partial N} < 1$, which is always true if CSFs are of the ratio form, a large external market leads to the latter.

In general the full solution to the symmetric non-cooperative equilibrium requires numerical solutions which are provided later in the paper. However we can derive explicit expressions for the total number of firms in the case where CSFs are of ratio form and $\frac{\partial G^{np}}{\partial N} = 0$. To do this, first put $R(x) = (P - c)x = (P - c)((\ell - 1)\phi d + \frac{rG^{np}}{NP})$. Then (52) becomes

$$d = \frac{\left[H(q) - \frac{rG^{np}}{\sigma \ell n}\left(1 - \frac{1}{\ell}\right)\right]}{P\left[(1-\alpha)(1 + (\ell - 1)\phi) + \Theta_2\right]} \tag{54}$$

Writing the symmetric budget constraint as

$$nd(p + P(\ell - 1)\phi) = G^p \tag{55}$$

and using (53) some algebra leads to

$$d = \frac{G^p + \frac{rG^{np}}{\sigma \ell^2}}{nP\left(1 + (\ell - 1)\phi + \Theta_2\right)} \tag{56}$$

Equation (54) says that given quality, the producer countries respond to increases in external demand per variety, $\frac{rG^{np}}{\ell n}$, by reducing the size of the firm. Thus the right-hand-side of (54) is upward-sloping in n. Equation (56) reflects the trade-off between size and number arising from the budget constraint and the right-hand-side of is downward-sloping in n. Hence given quality, we can solve for the equilibrium number of differentiated goods (equals the number of firms), n, and hence the total world number $N = \ell n$.

To complete the non-cooperative equilibrium we need to determine quality, using (48). Using (51), (48) becomes

$$cd = \beta f q^{\beta} - \frac{Lr(\sigma-1)(\ell-1)G^{np}}{\ell N} \tag{57}$$

Hence combining (56) and (57) and substituting $\Theta_1 = \frac{LrG^{np}}{\ell N}$ and $L = \frac{1}{\sigma}$, quality can be expressed in terms of the firm number per country, n, as:

$$\beta f q^{\beta} = \frac{1}{n}\left[(\sigma-1)(\ell-1)\frac{rG^{np}}{\sigma\ell^2} + \frac{c\left(G^p + \frac{rG^{np}}{\sigma\ell^2}\right)}{P\left(1+(\ell-1)\phi+\Theta_2\right)}\right] \tag{58}$$

Thus there is a *trade-off* between firm number and quality. According to (59), for a given quality, firm number is decreasing with that quality. According to (58), for a given number, quality is decreasing with that firm number. The equilibrium levels of n and q are determined by the intersection of these two downward-sloping curves, given government expenditures and the other parameters of the model.

We express the following result for N in terms of the total world expenditure $G = \ell G^p + rG^{np}$ and the relative size of the external market of non-producers $\frac{rG^{np}}{G}$:

$$N = \frac{G}{\beta F}\left[\theta - \frac{rG^{np}}{G}\left(\theta\left(1-\frac{1}{\sigma\ell}\right) - \frac{1}{\sigma\ell}(\ell-1)(\beta-\sigma+1)\right)\right] \tag{59}$$

where we have defined

$$\theta = \frac{\beta(1-\alpha)(1+(\ell-1)\phi)+\beta\Theta_2-\alpha}{1+(\ell-1)\phi+\Theta_2} \in ((1-\alpha),1)$$

Again we can examine the special case of a 'traditional' Dixit-Stigliz utility function where $\nu = \Theta_2 = 0$, there is no investment in quality ($\beta \to \infty$) and the limit as the external market becomes small. Then $\theta = \beta(1-\alpha)$ and we have that $N = \frac{G(1-\alpha)}{F}$, a familiar result for a closed economy monopolistic competition model.

Unambiguous results for the effect on N of changes in ν, w and the relative size of the external market, $\frac{rG^{np}}{G}$ can be obtained if we confine ourselves to the case where $\beta \to \infty$

and investment into R&D disappears. Then

$$\theta \rightarrow \frac{\beta[(1-\alpha)(1+(\ell-1)\phi)+\Theta_2]}{1+(\ell-1)\phi)+\Theta_2} \equiv \beta\Lambda \quad (60)$$

$$N \rightarrow \frac{G}{F}\left[\Lambda - \frac{rG^{np}}{G}\left(\Lambda\left(1-\frac{1}{\sigma\ell}\right) - \frac{1}{\sigma\ell}(\ell-1)\right)\right] \quad (61)$$

From (59) and the definition of Θ_1 given after (52) we can now examine the effect on the world number of firms of changes in the taste for variety parameter ν, the bias for domestic supply parameter $w \in [\frac{1}{2}, 1]$ and the relative size of the external market $\frac{rG^{np}}{G}$. First note that $\Lambda \in [1-\alpha, 1]$ (where (60) defines *Lambda*) as ν increases from 0 to ∞. Furthermore, from (59), N is increasing in Λ if $1 > \frac{rG^{np}}{G}\left(1-\frac{1}{\sigma\ell}\right)$. Since $\frac{rG^{np}}{G} < 1$, $\sigma > 1$ and $\ell \geq 1$ this condition is satisfied. Hence it follows that N is an increasing function of ν and we arrive at the expected result that an increase in the taste for variety in producer countries increases the number of differentiated goods.

Next consider an increase in the domestic procurement bias parameter, w. In the range $w \in [\frac{1}{2}, 1]$, ϕ falls from α^{σ} to 0 and Θ_2 goes from $\frac{\alpha\nu[1+(\ell-1)\alpha^{\sigma\alpha}]}{\ell}$ to $\alpha\nu$. Since $\alpha^{\alpha\sigma} < 1$, $\frac{1+(\ell-1)\alpha^{\sigma\alpha}}{\ell} < 1$ and therefore this represents an increase in Θ_2 and therefore Λ. A strengthening of the arms control regime between producers[12], modelled as a decrease in $\gamma \in [0, 1]$ has exactly the same effect as an increase in w. We have already shown that N is an increasing function of θ. It follows that as producer countries become less concerned with domestic supply and/or relax the arms control regime, Θ_2 falls and therefore the equilibrium number of firms, N, falls.

Finally from (59), N decreases with the relative size of the external market, $\frac{rG^{np}}{G}$, if the following condition is satisfied:

$$\Lambda > \frac{1}{\Lambda\ell}(\theta + \ell - 1) \quad (62)$$

Since $\theta < 1$, the right-hand side of (62) is an increasing function of ℓ and at $\ell = \infty$ equals $\frac{1}{\sigma}$. But $\theta > 1 - \alpha$. Hence (62) holds.

A willingness to procure from abroad, export more arms and the growing relative size of the international market are three features one may plausibly associate with *military globalization.* In that sense we may conclude that globalization is associated with a *decrease* in the number of firms in the world market. Summarizing our results:

[12] As opposed to export controls imposed on non-producers which only effect their military expenditure with the difference form of CSF (see proposition 1 and numerical results in section 8.6.

Proposition 3: The Number of Firms

In the case of the ratio form of the CSFs and high β, in a symmetric, non-cooperative equilibrium, the number of firms increases as the taste for variety by producer countries increases. More openness of economies in the form of a reduction in the bias of producer countries for domestic supply, a relaxation of arms controls between producers and an increase in the relative size of the external market results in a decrease in the number of firms.

7 Cooperation Between Producers

Returning to $\beta < \infty$ and staying with the case of the ratio form of the CSFs, we now examine a symmetric cooperative agreement at stage 1 where there are no export controls between producers and a common export control regime with respect to non-producers. Then $u_{ij} = q_j$ for producers $i = 1, 2, \cdots, \ell$, and $u_{ij} = \gamma q_j$ for non-producers $i = \ell+1, \ell+2, \cdots, \ell+r$. ℓ identical producers would then choose $d_1 = d_2 = \cdots = d_\ell = d$, $n_1 = n_2 = \cdots = n_\ell = n$, $q_1 = q_2 = \cdots = q_\ell = n$ and commit, at stage 1, to $m_1 = m_2 = \cdots = m_\ell = m$ to maximize $U_1 = U_2 = \cdots = U_\ell = U$ where

$$U = [w + (1-w)(\ell-1)]n^{\nu+\frac{1}{\alpha}}q[wd^\alpha + (1-w)(\ell-1)m^\alpha]^{\frac{1}{\alpha}} \quad (63)$$

subject to budget constraints $BC_1 = BC_2 = \cdots = BC$ and participation constraints $PC_1 = PC_2 = \cdots = PC$ where

$$\begin{aligned} BC &: \; n[pd + P(\ell-1)m] = G^p \\ PC &: \; \pi = (p-c)d + R(x) - H(q) = 0 \end{aligned}$$

In PC the net revenue is given by

$$R(x) = (P-c)x = (P-c)(x^p + x^{np}) = (P-c)\left[(\ell-1)m + \frac{rG^{np}}{\ell nP}\right] \quad (64)$$

Using (64) we can consolidate the BC and PC constraints as

$$n[c(d + (\ell-1)m) + H(q)] = L\frac{rG^{np}}{\ell} + G^p \quad (65)$$

Hence the optimal procurement decision for the producers together is found by maximizing $n^{\nu+\frac{1}{\alpha}}q[wd^{\alpha} + (1-w)(\ell-1)m^{\alpha}]^{\frac{1}{\alpha}}$ with respect to n, d and m subject to the consolidated constraint (65).

To carry out this optimization define a Langrangian

$$n^{\nu+\frac{1}{\alpha}}[wd^{\alpha} + (1-w)(\ell-1)m^{\alpha}]^{\frac{1}{\alpha}}q - \lambda\left[n[c(d+(\ell-1)m) + H(q)] - L\frac{rG^{np}}{\ell} - G^{p}\right]$$

where $\lambda \geq 0$ is a Lagrangian multiplier. The first-order conditions are:

$$\begin{aligned}
n &: (\nu + \frac{1}{\alpha})n^{(\nu+\frac{1}{\alpha}-1)}[wd^{\alpha} + (1-w)(\ell-1)m^{\alpha}]^{\frac{1}{\alpha}}q = \lambda\left[c(d+(\ell-1)m) + H(q) - Lr\frac{\partial G^{np}}{\partial N}\right] \\
d &: n^{(\nu+\frac{1}{\alpha})}[wd^{\alpha} + (1-w)(\ell-1)m^{\alpha}]^{\frac{1}{\alpha}-1}wd^{\alpha-1}q = \lambda nc \\
m &: n^{(\nu+\frac{1}{\alpha})}[wd^{\alpha} + (1-w)(\ell-1)m^{\alpha}]^{\frac{1}{\alpha}-1}(1-w)(\ell-1)m^{\alpha-1}q = \lambda nc(\ell-1) \\
q &: n^{(\nu+\frac{1}{\alpha})}[wd^{\alpha} + (1-w)(\ell-1)m^{\alpha}]^{\frac{1}{\alpha}} = \lambda\left[nH'(q) - \frac{Lr}{\ell}\frac{\partial G^{np}}{\partial q}\right]
\end{aligned}$$

Dividing the first, the third and the fourth first-order condition by the second we arrive at:

$$m = \left(\frac{1-w}{w}\right)^{\sigma} d \quad = \quad \bar{\phi}d \text{ say,} \tag{66}$$

$$cd\left[\left(\nu+\frac{1}{\alpha}\right)(w + (1-w)(\ell-1)\bar{\phi}^{\alpha} - w(1+(\ell-1)\bar{\phi})\right] \quad = \quad w\left[H(q) - \frac{Lr}{\ell}\frac{\partial G^{np}}{\partial N}\right] \tag{67}$$

$$cd = q\left[\beta f q^{\beta-1} - \frac{Lr}{N}\frac{\partial G^{np}}{\partial q}\right]\frac{wd^{\alpha}}{wd^{\alpha} + (1-w)(\ell-1)m^{\alpha}} \tag{68}$$

Equations (66), (67), (67) together with the constraint (65) characterize the optimal co-operative procurement agreement. Equations (67) and (68) can be simplified somewhat by noting that $[w+(1-w)(\ell-1)\bar{\phi}^{\alpha}] = w\left[1+\left(\frac{1-w}{w}\right)(\ell-1)\bar{\phi}^{\alpha}\right] = w\left[1+\left(\frac{1-w}{w}\right)^{\frac{1}{1-\alpha}}(\ell-1)\right] = w[1+\bar{\phi}(\ell-1)]$. Then putting $\frac{\partial G^{np}}{\partial N} = 0$ for the case of the ratio form of the CSFs and substituting into (67), a little algebra results in

$$N = \frac{(1-\alpha+\alpha\nu)G\left[1 - \frac{rG^{np}}{G}\left(1-\frac{1}{\sigma\ell}\right)\right]}{H(q)(1+\alpha\nu)} \tag{69}$$

whilst (66) and (68) now give

$$cd = \frac{\beta f q^{\beta}}{(1+\bar{\phi}(\ell-1))} \tag{70}$$

We can compare this result with the corresponding non-cooperative equilibrium given by (59). Putting $\ell = 1$ in the latter expression we find, as expected, that the non-cooperative equilibrium and the cooperative arrangement are the same if there is only one country. An analytical comparison can be made between the number of firms in the cooperative and non-cooperative equilibria if as before we confine ourselves to the large β case. Then $H(q) = F$ and a straightforward comparison can be made between (69) and (61). Let N^C and N^{NC} be firm numbers given by (69) (with $H(q) = F$) and (61) respectively. In the absence of an external market ($\frac{rG^{np}}{G} = 0$) it is straightforward to show that $N^C > N^{NC}$ iff $\nu \geq 0$. The intuition is that under cooperation if there is a taste for variety independent of the elasticity of substitution, then the number of varieties involves a positive externality which producers internalize on cooperating. Suppose we make the opposite assumption that the external market dominates and $\frac{rG^{np}}{G} \to 1$. Then it is equally straightforward to show that $N^C < N^{NC}$ if $\alpha \in (0, 1)$ which we have assumed throughout. The intuition here is that export revenue from exports rises as the total number of firms falls. This occurs because firms then compete less intensively and can spread their fixed set-up and R&D costs over a larger market share. As the external market increases, governments then choose to support *less* firms. Under non-cooperation, however, this reduction in firm number is too little compared with the optimum because governments acting independently only care about competition between domestic firms.

Another interesting result follows from (69). The right-hand-side is *independent of the domestic production bias parameter,* w. Since imports $m = \bar{\phi}d$ where $\bar{\phi} = \left(\frac{1-w}{w}\right)^{\sigma}$, an increase in w has no effect on the total number of firms (varieties) in the cooperative arrangement and only affects the trade between producers.[13] Note that this contrasts with the non-cooperative arrangement where a decrease in w leads to a decrease in the total number of firms (see proposition 2). As with the non-cooperative equilibrium, however, since $L < 1$, from (69) we can see that an increase in the relative size of the external market leads to a lower total number of firms under cooperation, and comparing (69) with (59), cooperation enhances this 'external effect' on the total firm number. To summarize:

[13]Compare the trade equation in the non-cooperative equilibrium, where $m = \phi d$ and $\phi = \left(\frac{c(1-w)}{Pw}\right)^{\sigma}$. With cooperation, trade is valued not at the world market price, but at the marginal cost, resulting in *more* trade.

Proposition 4: Optimal Cooperative Procurement

In the case of the ratio form of the CSFs, in the optimal cooperative procurement arrangement, the total number of firms is independent of the bias of producer countries for domestic supply. As with the non-cooperative equilibrium, given quality an increase in the relative size of the external market leads to a lower total number of firms under cooperation. Comparing the cooperative and non-cooperative outcomes in the large β case where there is no investment in quality, if the relative size of the external market is sufficiently small (large) then the number of firms is more (less) under cooperation.

The comparison between the cooperative and non-cooperative cases applies when there are no R&D investment considerations. For low β investment into quality occurs and this alters the comparison somewhat. The provision of quality has a beggar-thy-neighbour character: in a non-cooperative equilibrium when governments raise quality unilaterally this improves the competitiveness of exports and increases market share. In a symmetric equilibrium however the benefit to competitiveness disappears and countries are left with too much quality compared with that chosen cooperatively. The trade-off between quality and number then means that less investment in quality under cooperation raises the number of firms. Thus the comparison between N^C and N^{NC} must consider *three effects*: the independent taste for variety effect through the parameter ν and quality considerations lead us to expect $N^C > N^{NC}$ with less quality under cooperation. The existence of a relatively large external market suggests that $N^C < N^{NC}$. The net outcome depends on the interplay between these effects. This must be studied numerically to which we now turn.

8 Numerical Results

We now turn to numerical solutions of the non-cooperative and cooperative outcomes.[14] We address two sets of issues. First, we investigate the determinants of the number of

[14] We calibrate the model to reproduce stylized facts on 'firms' (weapons systems), R&D as a % of output in the firm and the level of subsidy. Details are provided in an Appendix of the accompanying working paper available on http://carecon.org.uk/Armsproduction/.

weapons systems procured and thus industrial structure and attempt to explain the post-Cold War increase in concentration. We analyse a symmetric equilibrium of equal firms and producers; this can be thought of as the EU producers trading in isolation from the US and other major producers. As discussed in Dunne *et al.* (2002), the number of firms has fallen far more than the decrease in world military expenditure, so in the model three mechanisms that can increase concentration are investigated. First, a rise in R&D costs as a proportion of output (a fall in β), second, a decrease in the bias for domestic production (a fall in w) and third, an increase in the relative size of the external market of non-producers (a rise in Φ). This last mechanism could reflect, for instance, a shift in the military expenditure of non-producers from low-tech, possibly domestically produced weapons, to modern major weapons system. The second issue investigated is the effect of this increase in concentration and the rising share of R&D on the security and military expenditure of arms importing countries. The third issue involves the gains from the coordination of procurement decisions between producers.

8.1 Changes in the Taste Parameter ν

Our first simulation allows ν to vary keeping $\Phi = \frac{rG^{np}}{G}$ and β at their baseline values. We begin by assuming that the CSF is of the ratio form ($\kappa = 1$ in (26)), so that G^{np} is constant. Figures 1 and 2 show output from the model for this particular choice of parameters. Figure 1 shows the number of firms per country growing as variety per se is valued more. R&D expenditure as a % of output remains constant. Figure 2 shows the procurement price increasing from a level below the world market price at $\nu = 0$ (implying a tax) to the point where at $\nu = 1$, there is a modest subsidy with a procurement price around 7% above the world market price paid by importers, a figure corresponding to our choice of subsidy in the calibration.

With ν set at $\nu = 1$, we now examine the non-cooperative and cooperative outcomes as three parameters change in turn from baseline values: β the R&D investment parameter, w the domestic bias parameter and $\Phi = \frac{rG^{np}}{G}$, the proportion of world demand for military goods coming from the external market of non-producers.

8.2 R&D Investment Costs

In our next experiment we examine the effect of R&D investment into quality which can be thought of as investing in vertical differentiation. In our model this effect is captured by a fall in the parameter β. In figure 3 we see the *trade-off between quality and variety.* As β increases, R&D investment falls as a % of output and the number of firms=the number of varieties increases. Our baseline calibration for β at $\beta = 1.5$ for the non-cooperative equilibrium reproduces data on R&D as a proportion of output reported in Dunne *et al.* (2002).

Under cooperation there is a switch from quality to variety. The beggar-thy-neighbour aspect of quality in the external market drives this result. When countries order high-tech, high quality specifications for domestic procurement, acting independently in the non-cooperative equilibrium they improve the competitiveness of their exports to the external market. This involves a significant subsidy in that the procurement price exceeds the world price as figure 4 shows. In a Nash equilibrium however these gains are wiped out: R&D expenditure is high but there is no improvement in competitiveness. Under cooperation the procurement price drops below the world price (but still exceeds the marginal cost $c = 1$). Figure 5 for the non-cooperative equilibrium shows another trade-off between firm size (measured as output per firm) and firm numbers and that large firms spend proportionally more on R&D. Finally figure 6 shows that the gains from cooperation between producer countries (to those countries) rises substantially as the incentive to invest in R&D rises.[15]

8.3 Changes in Domestic Procurement Bias

In our third experiment we allow the domestic procurement bias parameter, w to increase from $w = 0.5$ to $w = 1$ at which point producing countries are self-sufficient, and only exporting to non-producers. In figure 7 in the non-cooperative equilibrium the % expenditure on R&D rises as countries become more self-sufficient. For higher values of w R&D is fairly flat and countries also choose to raise variety. The reason for this is they the benefits of both quality and variety are internalized and therefore countries spend more on both

[15] Let U^C and U^{NC} be the utilities under cooperation and non-cooperation, respectively. The percentage gain from coordination is then is then defined as $\frac{U^C - U^{NC}}{U^{NC}} \times 100$.

through a higher procurement price as figure 8 shows. For lower values of w % R&D expenditures increases more sharply and the quality-variety trade-off sees firm numbers dropping as w increases. The net effect is the $\bigcup$-curve for firm numbers.

Under cooperation firm number is independent of w as predicted by proposition 4. The same is true for % R&D and the procurement price (figure 8) which for our chosen parameter values happens to coincide with the world price. As before, because firms are not competing in quality in the export market, countries choose a variety-quality trade-off that favours more variety in the cooperative case as compared with non-cooperation.

Figure 9 shows total output per firm in the non-cooperative equilibrium broken down into exports to non-producers and producers and domestic procurement. As w increases exports to producers fall and initially output is diverted to domestic procurement. With the increase in the number of firms, the total size of each firm falls and all three components eventually fall for higher values of w. As figure 10 shows, the % gain to producers from cooperation increases in the region where % R&D sharply increases under non-cooperation and the external market effect on quality competition dominates, but thereafter decreases as countries become more self-sufficient and internalize the variety effect entering through the ν parameter.

8.4 Changes in the Composition of World Demand

In our fourth experiment we allow the proportion of world demand from non-producers, $\Phi = \frac{rG^n}{G}$ increase from $\Phi = 0.5$ to $\Phi = 0.6$. Figure 11 shows that the the subsequent fall in the firm number under both non-cooperation and cooperation as Φ rises; as before there are too few firms and too much quality in the absence of cooperation. From figure 12 these changes in industry structure are brought about by initially a subsidy under non-cooperative giving way to a procurement price below the world market price but above the marginal cost at higher values of Φ. The optimal (cooperative) procurement price for the producers, by contrast, involves a procurement price falling further below the world market price as Φ increases. All these results are consistent with the results of propositions 2 to 4.

A falling number of firms as Φ rises is associated with a rise in the size of each firm. Figure 13 shows this happening in the non-cooperative equilibrium with a switch of output from domestic procurement and internal trade to the external market. Figure 14 shows that the gains to cooperation between producers rising as the external market becomes more important. This is largely the result of excessive investment into quality in the non-cooperative case, as figure 11 shows.

8.5 Changes in the Composition of World Demand with a Difference Form of CSF

All the results up to now have assumed a ratio form of CSF where the military expenditure of non-producers is fixed and independent of variety and quality. We now allow a small role for a difference form of CSF by choosing $\kappa = 0.95$ in (26). Now an increase in variety or quality results in an increase in non-producers' military expenditure and there is a role for export controls in the form of a restriction on the quality that producers export to non-producers. As proposition 1 has shown, this has the effect that non-producer military expenditure (G^{np}) falls if either the total number of varieties N falls and/or quality falls. As Φ increases N actually falls and quality rises. The former dominates in the effect on G^{np} and non-producer military expenditure falls in both the cooperative and non-cooperative cases as figure 15 shows. Still without export controls, in figure 16 we can see a gain from cooperation between producers to both the latter (this is substantial rising from a % gain of around 37% at $\Phi = 0.5$ to about 55% at $\Phi = 0.6$.) *and also (albeit slightly) to non-producers.*

Now consider cooperation between producers in the context of cooperative arrangement that also constrains producers to an arms export control regime allowing the quality exported to be γq, where q is the quality of domestic procurement and $\gamma < 1$. From (15) the quality-adjusted price $\hat{P}$ now rises which causes a fall in the optimal military expenditure of non-producers as figure 15 shows. Producers respond by choosing a different mix of quality and varieties that takes into account the endogenous response of non-producers. In fact with chosen parameter values, given the marginal effects of quality and variety on producers' expenditure given by (24) and (25) they increase domestic quality and reduce variety. The net effect is to increase the quality-adjusted price $\hat{P}$ as we have seen. With $\gamma = 0.75$ figure 16 shows that non-producers benefit from cooperation coupled with export controls and there is a region of Φ in which producers also benefit as compared with non-cooperation without export controls. To summarize, we have shown that a regime of procurement cooperation between producers coupled with arms export controls that limit the quality exported *can benefit both producers and non-producers.*

9 Conclusions

This paper has constructed a partial equilibrium model of military procurement with two-way international trade in a world where many of the recipients of this trade are non-producers engaged in regional arms races. The model captures many of the important features of the present day post Cold War international industry and trade, abstracting from the complex political realities and so allowing a focus on the fundamentals driving changes in market structure. In particular the model enables us to investigate the effect of changes in the variety and R&D induced quality of weapons systems on the non-producing countries, the impact of procurement pricing policy on the numbers of firms and hence concentration, and the possible gains available from producer cooperation in the procurement decision. The main results of our analysis are as follows:

1. **The Effect of Variety and Quality on Non-Producers.**
   These depend critically on the form of the CSF used to model conflict between importers. With the ratio form of CSF, military expenditure (as a proportion of GDP) is fixed whereas if there is a difference component it increases with the total number of varieties available and their quality. This reduces welfare because non-producers are engaged in arms races where the military expenditure of a rival cancels out the security provided by its own military expenditure.

2. **Procurement Pricing Policy and Industry Concentration.**
   The domestic procurement price chosen determines the number of domestic firms and, in the world as a whole, the total number of firms in the market (i.e., industry concentration). The procurement price can be above or below the world market price, but must be above the marginal cost of production. There is a trade-off between quality and firm number. Keeping quality fixed, we show that firm number decreases as producers become more open to trade with other producers and as the relative size of the external market rises. However openness also reduces the

incentive to provide quality and this tends to increase firm number. The net effect on firm number is ambiguous as we have seen in figure 7. What we can predict unambiguously from our model is that *if quality and R&D expenditure are held fixed or indeed rise, then openness and an increased relative size of the external market can help to explain the observed increase in industry concentration in excess of that expected from the reduction in world expenditure.* This is an empirically relevant condition because as Dunne *et al.* (2002) report, R&D expenditure as a % of output *has* actually risen in the post-war period.

3. **Gains from Producer Cooperation in the Procurement Decision.** Cooperative gains from a joint decision on firm number and domestic procurement originate from the external benefits of variety and the beggar-thy-neighbour aspect of competition in the external market. Cooperation lowers quality produced, but can increase the number of firms compared with non-cooperation. The former reduces military expenditure by non-producers and the latter has the opposite effect. The net effect depends on parameter values; for our calibration the lower quality effect dominates and the military expenditure of non-producers rises resulting in an increase in their welfare. Arms export controls taking the form of a restriction on the quality that can be exported has the effect of reducing non-producers' military expenditure and distributing the cooperative gains from the producers to non-producers.

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A (Not for Publication) Calibration of Parameter Values

By choice of units we can put $c = f = 1$. We exclude arms export regimes for now and put $\gamma = 1$. We examine a symmetric equilibrium of three countries (say, the UK, Germany and France in a EU setting) so $\ell = 3$. We can calibrate the parameter α as follows. From the binding participation constraint we have that revenue equals total costs, $P(d + x) = Py$ where we recall that d =domestic procurement, x =exports and $y = d + x$=output, all per firm. In equilibrium the procurement price equals the international market price $P = \frac{c}{\alpha}$ where c =marginal cost (equals average production cost given our assumption of constant returns to scale). Thus we have

$$Py = \frac{c}{\alpha}y = \text{Total Costs(TC)} = F + fq^{\beta} + cy \tag{A.1}$$

where q is is quality. In (A.1) let us associate the second quality component of total costs with R&D, the third with variable cost leaving F as fixed set-up costs. Denote the shares of fixed, R&D and variable cost in total cost as γ_F, γ_R and γ_V respectively. Thus

$$\frac{cy}{\frac{cy}{\alpha}} = \frac{\text{variable costs}}{\text{total cost}} = \alpha = \gamma_V \tag{A.2}$$

From Dunne et al (2002), a reasonable value for $\gamma_V = 0.5$ for Europe which is therefore our chosen value for α.

In the rest of the calibration we restrict ourselves to the case of where the CSF of the non-producers is of the ratio form. Then G^{np} is constant. Define $\Phi = \frac{rG^{np}}{\ell G^p}$ to be the ratio of military expenditure in producer and non-producer countries. A reasonable value for this parameter is $\Phi = 0.5$. In our baseline calibration we assume no home bias in military procurement so $w = 0.5$.

The remaining parameters to be calibrated are $[F,\ \nu, \beta] = \Xi$, say. Given Ξ we can compute the non-cooperative equilibrium. Suppose that we have data for three outputs: firm number per country $n = \hat{n}$, R&D expenditure by firms as a proportion of output , $\hat{RD}$ and the subsidy as a proportion of the world price $s = \frac{p-P}{P} = \hat{s}$. In our baseline calibration we choose $\hat{n} = 35$,[16] $\hat{RD} = 22\%$ and $\hat{s} = 7\%$. From the non-cooperative equilibrium we

[16] Dunne *et al.* (2002) report an inverse Herfindahl index of 27 in 1990 for non-US firms in the global market. This is a good measure of the symmetric equilibrium equivalent number of firms. Given our interpretation of a 'firm' as a independent branch producing a single weapons system within a larger organisation, by choosing around 100 firms we are assuming each produces about 4 such varieties.

have a solution $n = n(\Xi)$, $RD = RD(\Xi)$ and $s = s(\Xi)$. Then Ξ can be calibrated as the solution to:

$$\begin{aligned} \hat{n} &= n(\Xi) \\ \hat{RD} &= RD(\Xi) \\ \hat{s} &= s(\Xi) \end{aligned}$$

The result of this exercise is a model calibrated in a non-cooperative equilibrium to be consistent with stylized facts regarding firm number, R&D expenditure and the level of subsidies given to the defence industry. Clearly this procedure can be extended to other parameters such as α if we had more stylized facts. Our baseline calibration is summarized in the following table

Parameter	Value	Method and Source
$c = f$	1	normalization
γ	1	assumption (no arms exports)
α	0.5	calibration (data from Dunne *et al.* (2002))
ℓ	3	assumption (EU context)
w	0.5	assumption (no home bias)
Φ	0.5	data from Dunne *et al.* (2002)
$\hat{n}$	30-35	imposed
$\hat{RD}$	22%	data from Dunne *et al.* (2002)
$\hat{s}$	7%	data from Dunne *et al.* (2002)
F	0.00027	calibrated using non-cooperative equilibrium
β	1.5	calibrated using non-cooperative equilibrium
ν	1	calibrated using non-cooperative equilibrium

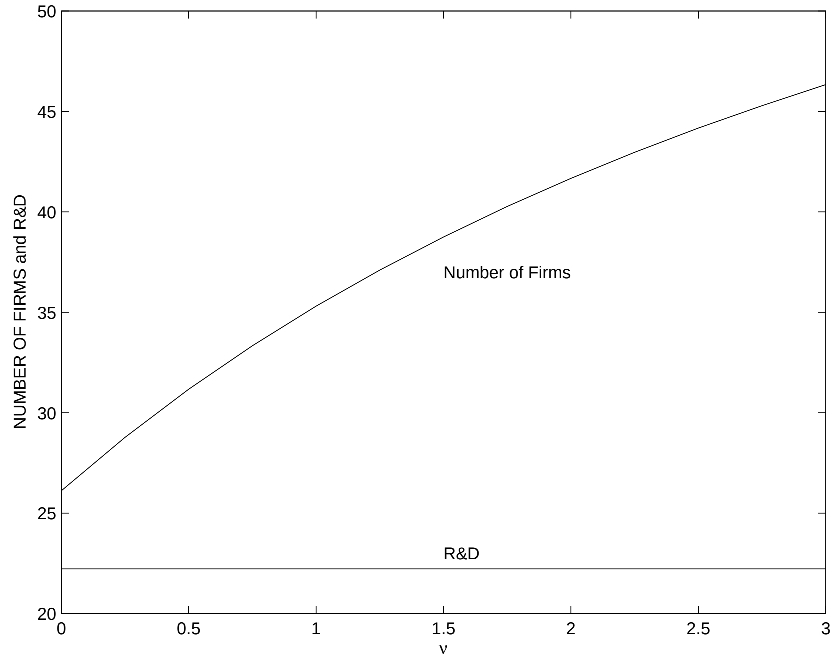

Figure 1: **Number of Firms per Country and R&D Expenditure as % of Output as ν increases.**

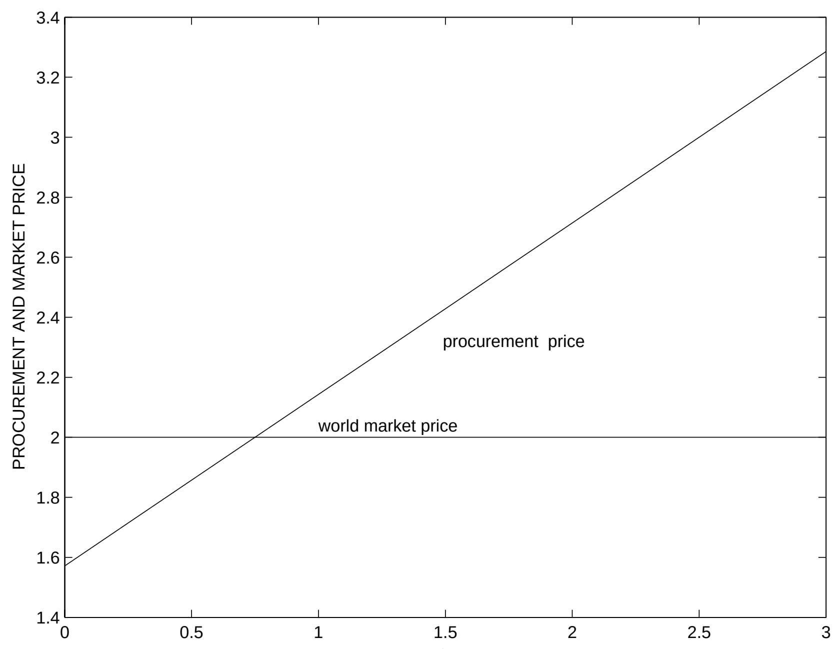

Figure 2: **Procurement and World Market Prices as ν increases.**

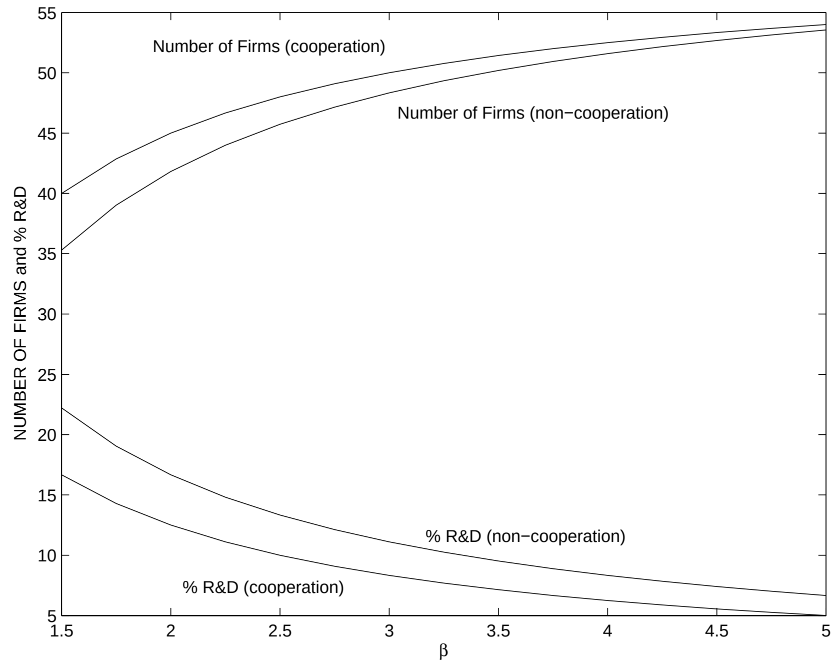

Figure 3: **Number of Firms per Country as β increases: Non-Cooperation compared with Cooperation.**

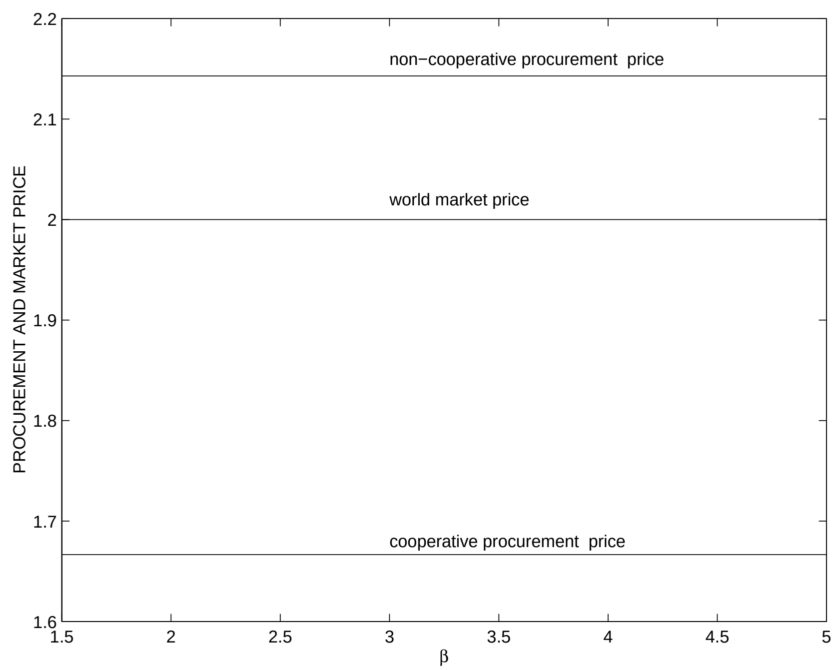

Figure 4: **Non-Cooperative and Cooperative Procurement and World Market Prices as β increases.**

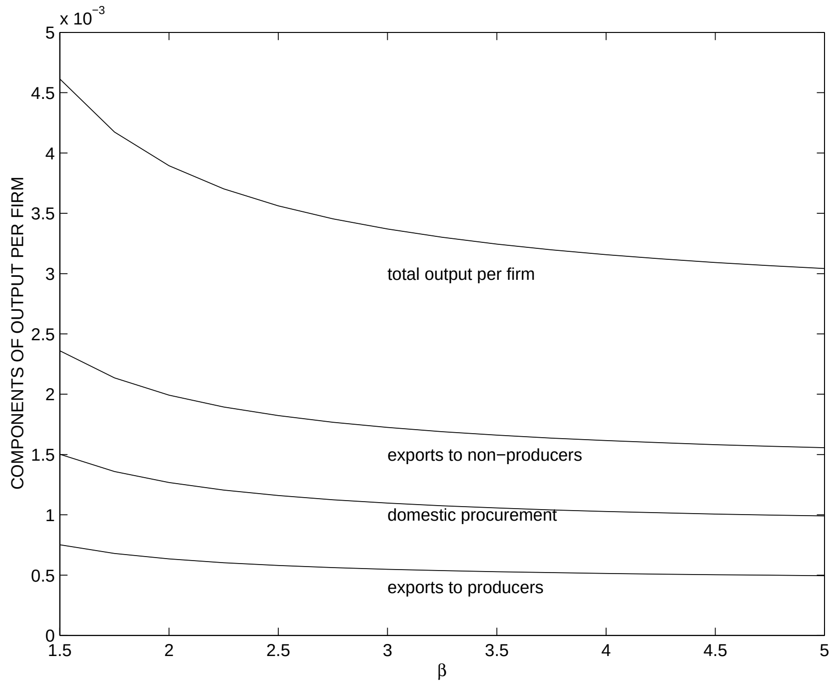

Figure 5: **R&D Expenditure as a Proportion of Output as β increases.**

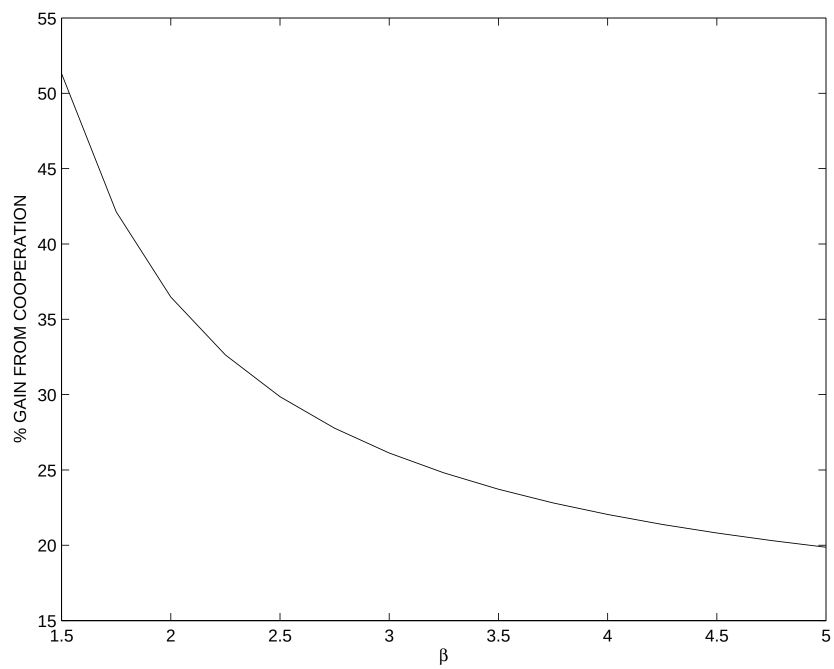

Figure 6: **Gain in Military Strength from Procurement Cooperation between Producers as β increases.**

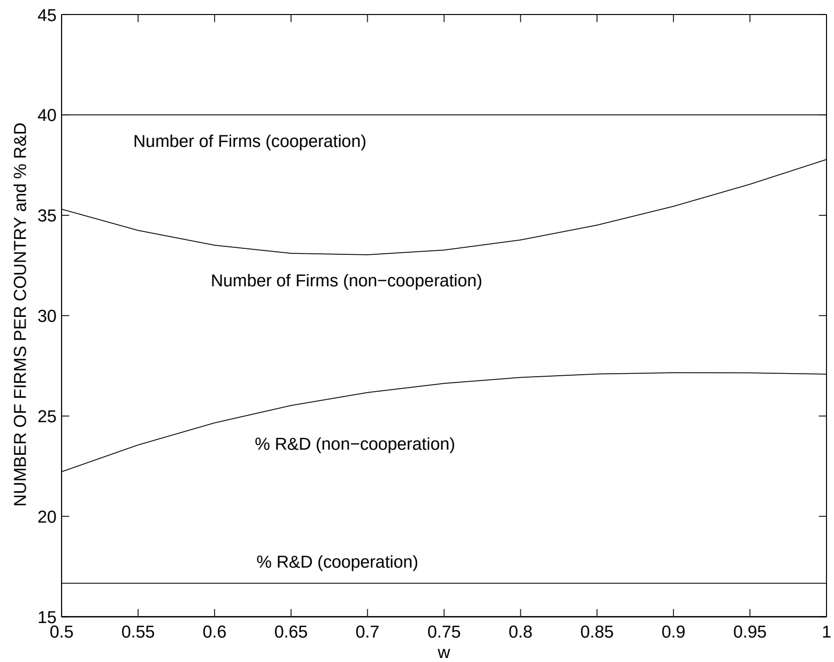

Figure 7: **Number of Firms per Country as w increases: Non-Cooperation compared with Cooperation.**

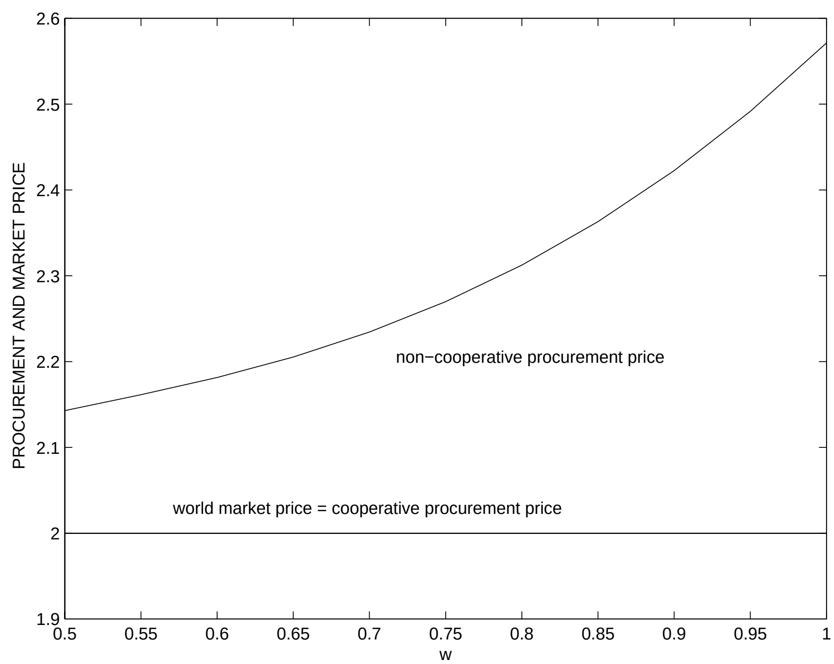

Figure 8: **Non-Cooperative and Cooperative Procurement and World Market Prices as w increases.**

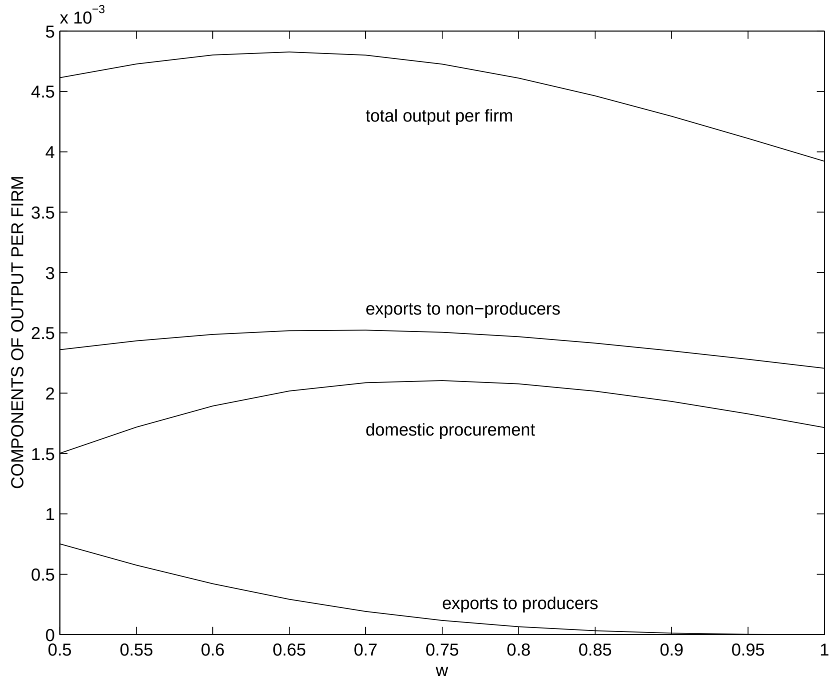

Figure 9: **Components of Output per Firm in the Non-Cooperative Equilibrium as w increases.**

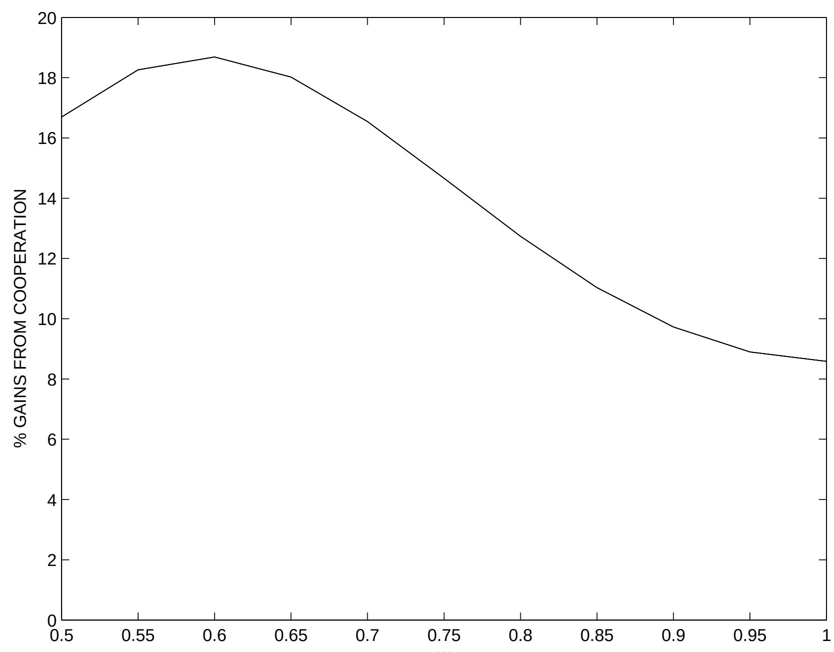

Figure 10: **Gain in Military Strength from Procurement Cooperation between Producers as w increases.**

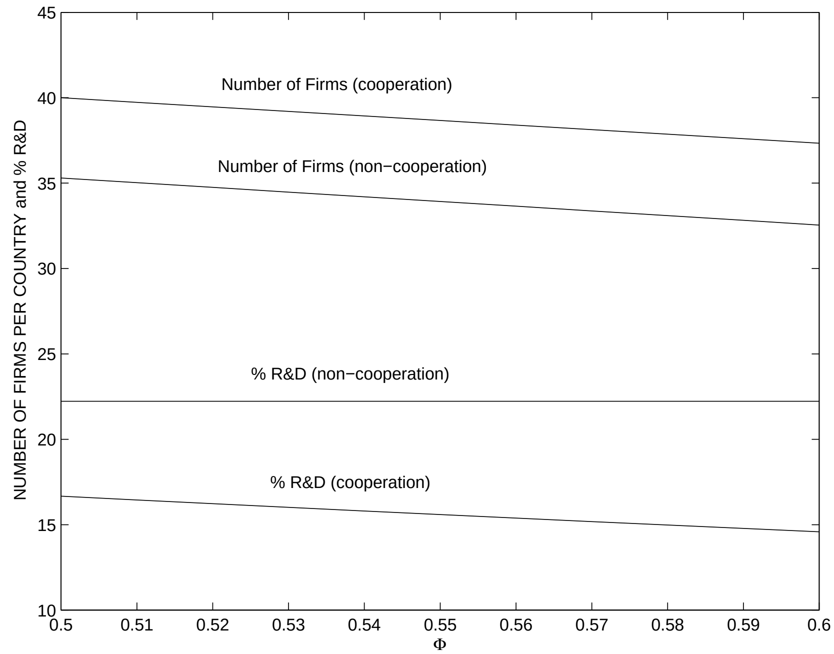

Figure 11: **Number of Firms per Country as Φ increases: Non-Cooperation compared with Cooperation.**

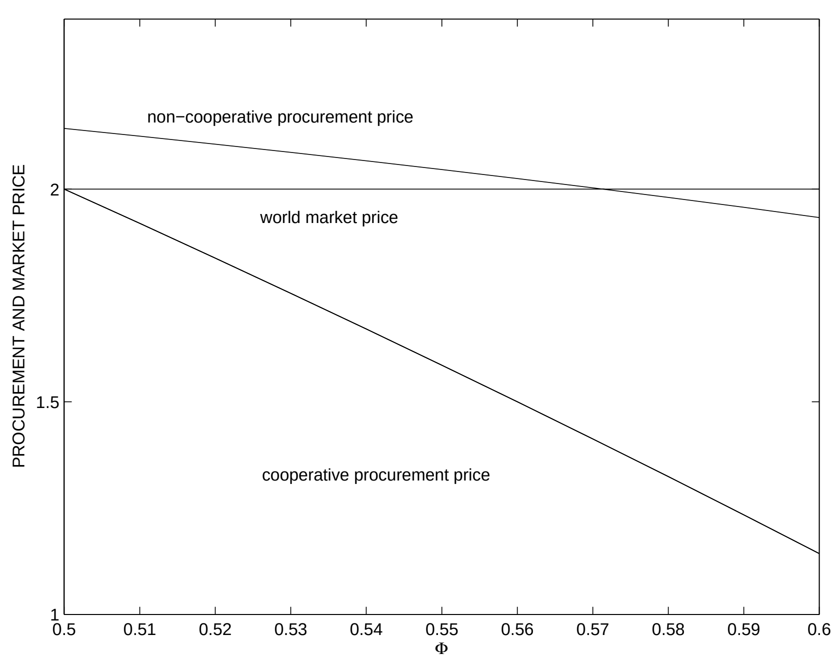

Figure 12: **Non-Cooperative and Cooperative Procurement and World Market Prices as Φ increases.**

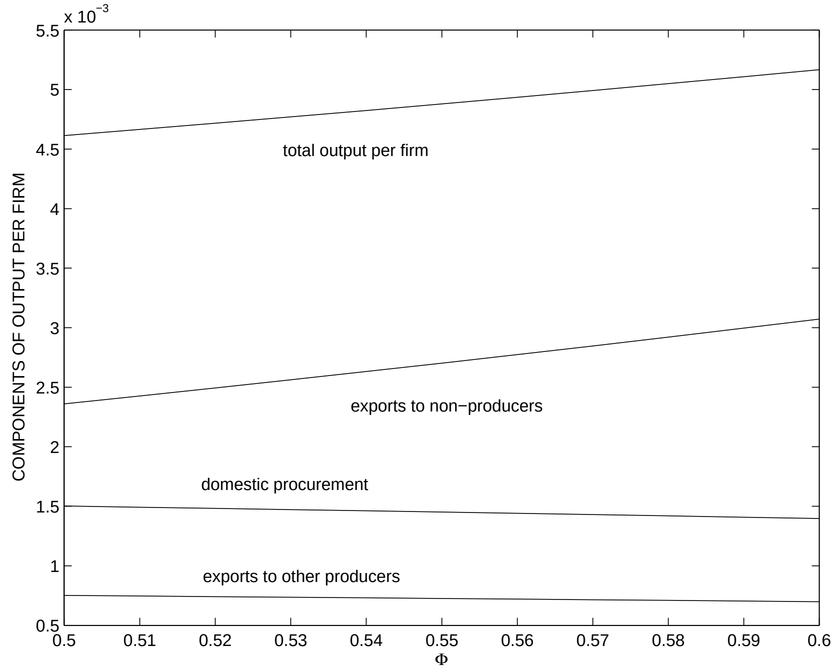

Figure 13: **Components of Output per Firm in the Non-Cooperative Equilibrium as Φ increases.**

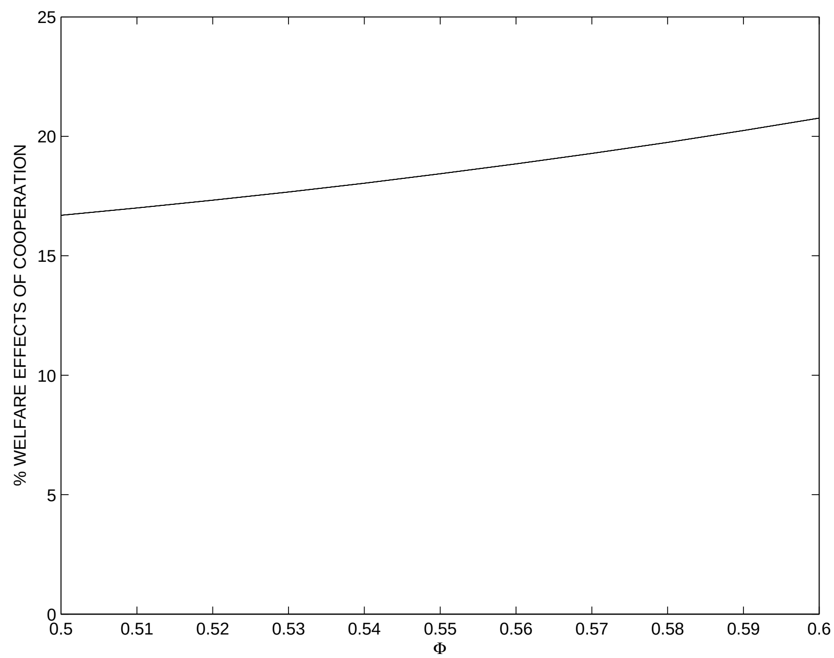

Figure 14: **Gain in Military Strength from Procurement Cooperation between Producers as Φ increases.**

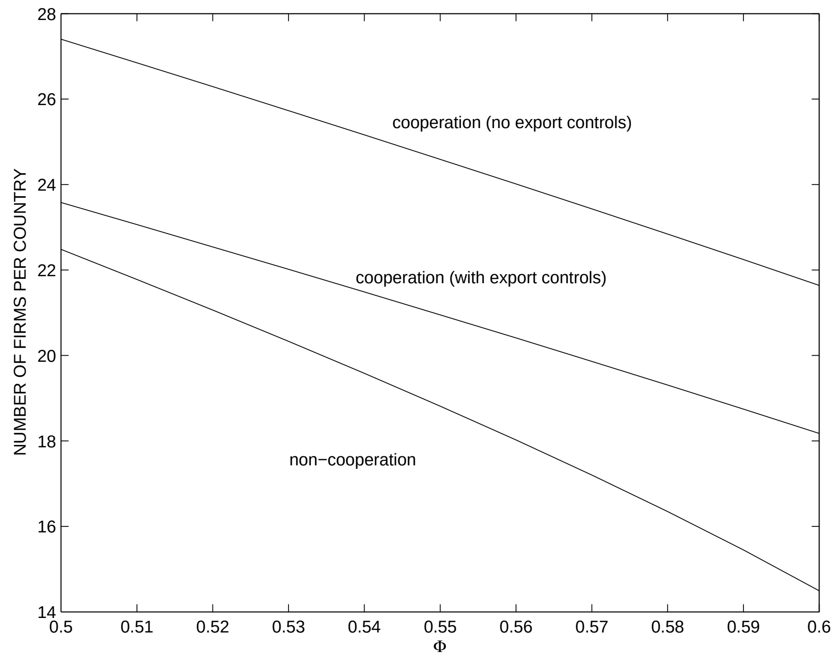

Figure 15: **Difference Form of CSF: Number of Firms per Country as Φ increases: Non-Cooperation compared with Cooperation.**

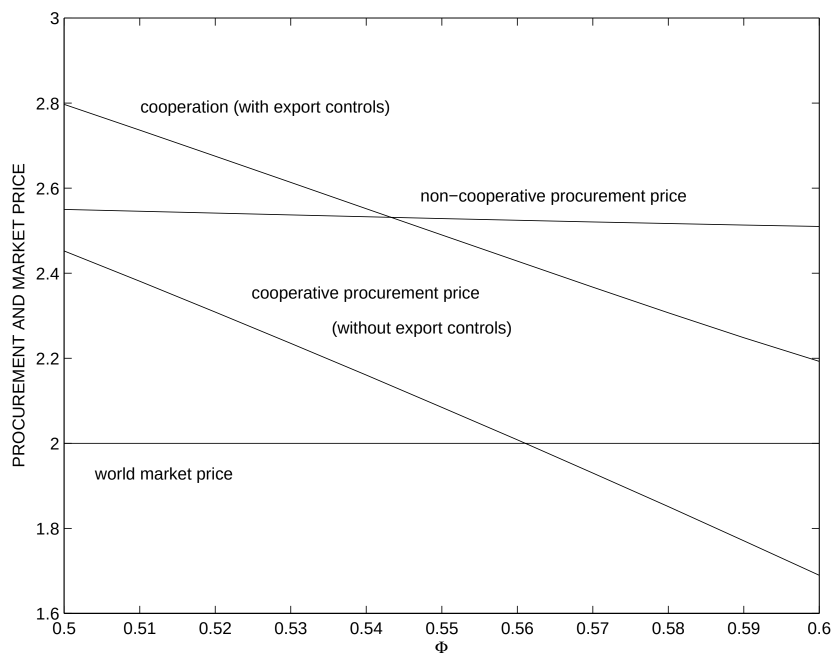

Figure 16: **Difference Form of CSF: Non-Cooperative and Cooperative Procurement and World Market Prices as Φ increases.**

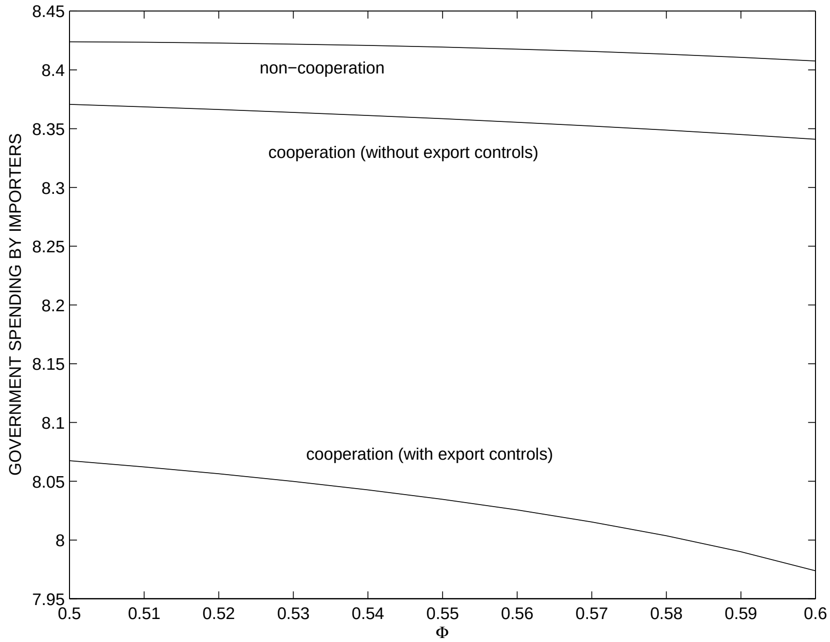

Figure 17: **Difference Form of CSF: Military Expenditure of Non-Producers as Φ increases.**

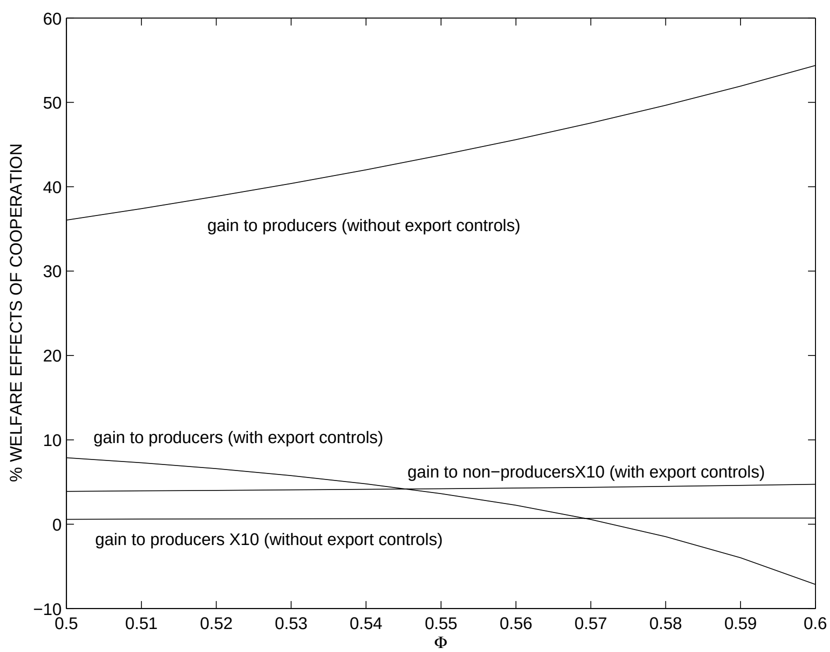

Figure 18: **Difference Form of CSF: Gain in Military Strength from Procurement Cooperation between Producers as Φ increases.**