1. **Title of the module**

MAST6005 (MA692) - Operators and Matrices

MAST7005 (MA792) - Operators and Matrices

1. **School or partner institution which will be responsible for management of the module**

School of Mathematics, Statistics and Actuarial Science

1. **The level of the module (Level 4, Level 5, Level 6 or Level 7)**

Level 6 (MAST6005), Level 7 (MAST7005)

1. **The number of credits and the ECTS value which the module represents**

15 credits (7.5 ECTS)

1. **Which term(s) the module is to be taught in (or other teaching pattern)**

Autumn or Spring

1. **Prerequisite and co-requisite modules**

Level 6:

Pre-requisite: MAST5013 (Real Analysis 2)

Co-requisite: None

Level 7:

Pre-requisite: Students are expected to have studied material equivalent to that covered in the modules above.

Co-requisite: None

1. **The programmes of study to which the module contributes**

For the level 6 module, BSc Mathematics (including programme with a Year in Industry), BSc Mathematics with a Foundation Year, MMath Mathematics, Graduate Diploma in Mathematics, International MSc in Mathematics and its Applications, MSc in Mathematics and its Applications (including programmes with an Industrial Placement)..

For the level 7 module, MMath Mathematics, International MSc in Mathematics and its Applications, MSc in Mathematics and its Applications (including programmes with an Industrial Placement)..

1. **The intended subject specific learning outcomes.  
   On successfully completing the level 6 module students will be able to:**

8.1 demonstrate systematic understanding of key aspects of matrix and operator theory;

8.2 demonstrate the capability to deploy established approaches accurately to analyse and solve problems using a reasonable level of skill in calculation and manipulation of the material in the following areas: Hermitian matrices and their spectral properties, Hilbert spaces, linear operators and functionals, compact operators, spectral theory;

8.3 apply key aspects of operator theory in well-defined contexts, showing judgement in the selection and application of tools and techniques.

**On successfully completing the level 7 module students will be able to:**

8.4 demonstrate systematic understanding of the theory of linear operators;

8.5 demonstrate the capability to solve complex problems using a very good level of skill in calculation and manipulation of the material in the following areas: Hermitian matrices and their spectral properties, Hilbert spaces, linear operators and functionals, compact operators, spectral theory;

8.6 apply a range of concepts and principles in Hilbert space theory and operator theory in loosely defined contexts, showing good judgment in the selection and application of tools and techniques.

1. **The intended generic learning outcomes.  
   On successfully completing the level 6 module students will be able to:**

9.1 manage their own learning and make use of appropriate resources;

9.2 understand logical arguments, identifying the assumptions made and the conclusions drawn;

9.3 communicate straightforward arguments and conclusions reasonably accurately and clearly;

9.4 manage their time and use their organisational skills to plan and implement efficient and effective modes of working;

9.5 solve problems relating to qualitative and quantitative information;

9.6 communicate technical material competently;

9.7 demonstrate an increased level of skill in numeracy and computation;

9.8 demonstrate the acquisition of the study skills needed for continuing professional development.

**On successfully completing the level 7 module students will be able to:**

9.9 work competently and independently, be aware of their own strengths and understand when help is needed;

9.10 demonstrate a high level of capability in developing and evaluating logical arguments;

9.11 communicate arguments confidently with the effective and accurate conveyance of conclusions;

9.12 manage their time and use their organisational skills to plan and implement efficient and effective modes of working;

9.13 solve problems relating to qualitative and quantitative information;

9.14 communicate technical material effectively;

9.15 demonstrate an increased level of skill in numeracy and computation;

9.16 demonstrate the acquisition of the study skills needed for continuing professional development.

1. **A synopsis of the curriculum**

Matrix theory: Hermitian and symmetric matrices, spaces of these matrices and the associated inner product, diagonalization, orthonormal basis of eigenvectors, spectral properties, positive definite matrices and their roots

Hilbert space theory: inner product spaces and Hilbert spaces, L^2 and l^2 spaces, orthogonality, bases, Gram-Schmidt procedure, dual space, Riesz representation theorem

Linear operators: the space of bounded linear operators with the operator norm, inverse and adjoint operators, Hermitian operators, infinite matrices, spectrum, compact operators, Hilbert-Schmidt operators, the spectral theorem for compact Hermitian operators.

Additional topics, may include:

- the Rayleigh quotient and variational characterisations of eigenvalues,

- the functional calculus,

- applications to Sturm-Liouville systems.

At level 7, topics will be studied and assessed to greater depth.

1. **Reading list (Indicative list, current at time of publication. Reading lists will be published annually)**

J.R. Giles: Introduction to the Analysis of Normed Linear Spaces. Cambridge University Press (2000).

V.L. Hansen: Functional Analysis – Entering Hilbert Space. World Scientific (2006).

R. Horn , C. Johnson: Matrix Analysis. Cambridge University Press (1985).

C.D. Meyer: Matrix Analysis and Applied Linear Algebra. SIAM (2000).

B. Rynne, M. Youngson: Linear Functional Analysis. Springer (2008).

G. Strang: Linear Algebra and its Applications, 3rd edition. Saunders (1988).

N. Young: An Introduction to Hilbert space. Cambridge University Press (1988).

F. Zhang: Matrix Theory – Basic Results and Techniques. Springer (2011).

Additional reading for level 7:

G. Teschl: Topics in Real and Functional Analysis. Lecture notes available at http://www.mat.univie.ac.at/~gerald/ftp/book-fa/index.html

1. **Learning and teaching methods**

Total contact hours: 42

Private study hours: 108

Total study hours: 150

1. **Assessment methods**
   1. Main assessment methods

**Level 6**

Assessment 1 Exercises, requiring on average between 10 and 15 hours to complete 20%

Assessment 2 Exercises, requiring on average between 10 and 15 hours to complete 20%

Examination 2 hours 60%

The coursework mark alone will not be sufficient to demonstrate the student’s level of achievement on the module.

**Level 7**

Assessment 1 Exercises, requiring on average between 10 and 15 hours to complete 20%

Assessment 2 Exercises, requiring on average between 10 and 15 hours to complete 20%

Examination 2 hours 60%

The coursework mark alone will not be sufficient to demonstrate the student’s level of achievement on the module.

13.2 Reassessment methods

Like-for-like

1. **Map of module learning outcomes (sections 8 & 9) to learning and teaching methods (section12) and methods of assessment (section 13)**

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Level 6 Module learning outcome** | 8.1 | 8.2 | 8.3 | 9.1 | 9.2 | 9.3 | 9.4 | 9.5 | 9.6 | 9.7 | 9.8 |
| **Learning/ teaching method** |  |  |  |  |  |  |  |  |  |  |  |
| Private Study and Assessment | x | x | x | x | x | x | x | x | X | x | x |
| Lectures/exercise classes | x | x | x | x | x | x |  | x | X | x |  |
| Revision classes | x | x | x |  | x | x |  | x | X | x |  |
| **Assessment method** |  |  |  |  |  |  |  |  |  |  |  |
| Examination | x | x | x | x | x | x | x | x | X | x | x |
| Coursework | x | x | x | x | x | x | x | x | X | x | x |

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Level 7 Module learning outcome** | 8.4 | 8.5 | 8.6 | 9.9 | 9.10 | 9.11 | 9.12 | 9.13 | 9.14 | 9.15 | 9.16 |
| **Learning/ teaching method** |  |  |  |  |  |  |  |  |  |  |  |
| Private Study and Assessment | x | x | x | x | x | x | x | x | x | x | x |
| Lectures/exercise classes | x | x | x | x | x | x |  | x | x | x |  |
| Revision classes | x | x | x |  | x | x |  | x | x | x |  |
| **Assessment method** |  |  |  |  |  |  |  |  |  |  |  |
| Examination | x | x | x | x | x | x | x | x | x | x | x |
| Coursework | x | x | x | x | x | x | x | x | x | x | x |

1. **Inclusive module design**

The School recognises and has embedded the expectations of current equality legislation, by ensuring that the module is as accessible as possible by design. Additional alternative arrangements for students with Inclusive Learning Plans (ILPs)/declared disabilities will be made on an individual basis, in consultation with the relevant policies and support services.

The inclusive practices in the guidance (see Annex B Appendix A) have been considered in order to support all students in the following areas:

a) Accessible resources and curriculum

b) Learning, teaching and assessment methods

1. **Campus(es) or centre(s) where module will be delivered**

Canterbury

1. **Internationalisation**

Mathematics is an international language with techniques developed and refined by mathematicians across the globe. Mastery of the subject-specific learning outcomes, 8.1 to 8.6, will equip students to apply the theories and techniques of this module in a wide range of international contexts. The module team is drawn from the School of Mathematics, Statistics and Actuarial Science, which includes many members of staff with international experience of teaching and research collaboration.

In compiling the reading list, consideration has been given to the range of texts that are available internationally and a selection of texts has been identified to complement the delivery of the material.

The support SMSAS provides to its students is also internationally attuned given our international student body.

**FACULTIES SUPPORT OFFICE USE ONLY**

**Revision record – all revisions must be recorded in the grid and full details of the change retained in the appropriate committee records.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Date approved | Major/minor revision | Start date of the delivery of revised version | Section revised | Impacts PLOs (Q6&7 cover sheet) |
| July 2023 | Minor | September 2023 | 6, 13 | No |
|  |  |  |  |  |