1. **Title of the module**

MAST5680 (MA568) - Orthogonal Polynomials and Special Functions

MAST7026 (MA7526) - Orthogonal Polynomials and Special Functions

1. **School or partner institution which will be responsible for management of the module**

School of Mathematics, Statistics and Actuarial Science

1. **The level of the module (Level 4, Level 5, Level 6 or Level 7)**

Level 6 (MAST5680), Level 7 (MAST7026)

1. **The number of credits and the ECTS value which the module represents**

15 credits (7.5 ECTS)

1. **Which term(s) the module is to be taught in (or other teaching pattern)**

Autumn or Spring

1. **Prerequisite and co-requisite modules**

**Level 6:**

For delivery to students completing Stage 1 before September 2016:

Pre-requisite: MA321 (Calculus and Mathematical Modelling), MA322 (Proofs and Numbers), MA323 (Matrices and Probability), MA552 (Analysis), MA588 (Mathematical Techniques and Differential Equations).

Co-requisite: None

For delivery to students completing Stage 1 after September 2016:

Pre-requisite: MAST4004 (Linear Algebra); MAST4010 (Real Analysis 1); MAST5013 (Real Analysis 2); MAST5012 (Ordinary differential equations).

Co-requisite: None

**Level 7:**

Pre-requisite: Students are expected to have studied material equivalent to that covered in the modules above.

Co-requisite: None

1. **The programmes of study to which the module contributes**

For the level 6 module, BSc Mathematics, BSc Mathematics and Statistics (including programmes with a Year in Industry), BSc Mathematics with a Foundation Year, MMath Mathematics, MMathStat Mathematics and Statistics, Graduate Diploma in Mathematics, International MSc in Mathematics and its Applications (including programme with an Industrial Placement).

For the level 7 module, MMath Mathematics, International MSc in Mathematics and its Applications, MSc in Mathematics and its Applications (including programmes with an Industrial Placement).

1. **The intended subject specific learning outcomes.  
   On successfully completing the level 6 module students will be able to:**

8.1 demonstrate systematic understanding of key aspects of orthogonal polynomial sequences and in particular classical polynomials, special functions and their properties;

8.2 demonstrate the capability to deploy established approaches accurately to analyse and solve problems using a reasonable level of skill in calculation and manipulation of the material in the following areas: analysis of solutions to linear differential equations with polynomial coefficients which includes their asymptotic behaviour; approximation theory; numerical analysis techniques; mathematical physics problems; probability theory;

8.3 apply key aspects of orthogonal polynomials and special functions in well-defined contexts, showing judgement in the selection and application of tools and techniques;

8.4 show judgement in the selection and application of Maple as appropriate.

**On successfully completing the level 7 module students will be able to:**

8.5 demonstrate systematic understanding of: orthogonal polynomial sequences and in particular classical polynomials, special functions and their properties, including elliptic functions;

8.6 demonstrate the capability to solve complex problems using a very good level of skill in calculation and manipulation of the material in the following areas: analysis of solutions to differential equations with polynomial coefficients which includes their asymptotic behaviour; approximation theory; numerical analysis techniques; mathematical physics problems; probability theory;

8.7 apply a range of concepts and principles in orthogonal polynomials and special functions in loosely defined contexts, showing good judgment in the selection and application of tools and techniques;

8.8 make effective and well-considered use of Maple as a symbolic computational language.

1. **The intended generic learning outcomes.  
   On successfully completing the level 6 module students will be able to:**

9.1 manage their own learning and make use of appropriate resources;

9.2 understand logical arguments, identifying the assumptions made and the conclusions drawn;

9.3 communicate straightforward arguments and conclusions reasonably accurately and clearly;

9.4 manage their time and use their organisational skills to plan and implement efficient and effective modes of working;

9.5 solve problems relating to qualitative and quantitative information;

9.6 make competent use of information technology skills such online resources (Moodle), internet communication);

9.7 communicate technical material competently;

9.8 demonstrate an increased level of skill in numeracy and computation;

9.9 demonstrate the acquisition of the study skills needed for continuing professional development.

**On successfully completing the level 7 module students will be able to:**

9.10 work competently and independently, be aware of their own strengths and understand when help is needed;

9.11 demonstrate a high level of capability in developing and evaluating logical arguments;

9.12 communicate arguments confidently with the effective and accurate conveyance of conclusions;

9.13 manage their time and use their organisational skills to plan and implement efficient and effective modes of working;

9.14 solve problems relating to qualitative and quantitative information;

9.15 make effective use of information technology skills such as online resources (Moodle), internet communication;

9.16 communicate technical material effectively;

9.17 demonstrate an increased level of skill in numeracy and computation;

9.18 demonstrate the acquisition of the study skills needed for continuing professional development.

1. **A synopsis of the curriculum**

This module provides an introduction to the study of orthogonal polynomials and special functions. They are essentially useful mathematical functions with remarkable properties and applications in mathematical physics and other branches of mathematics. Closely related to many branches of analysis, orthogonal polynomials and special functions are related to important problems in approximation theory of functions, the theory of differential, difference and integral equations, whilst having important applications to recent problems in quantum mechanics, mathematical statistics, combinatorics and number theory. The emphasis will be on developing an understanding of the structural, analytical and geometrical properties of orthogonal polynomials and special functions. The module will utilise physical, combinatorial and number theory problems to illustrate the theory and give an insight into a plank of applications, whilst including some recent developments in this field. The development will bring aspects of mathematics as well as computation through the use of MAPLE. The topics covered will include: The hypergeometric functions, the parabolic cylinder functions, the confluent hypergeometric functions (Kummer and Whittaker) explored from their series expansions, analytical and geometrical properties, functional and differential equations; sequences of orthogonal polynomials and their weight functions; study of the classical polynomials and their applications as well as other hypergeometric type polynomials. At level 7, topics will be studied and assessed to greater depth.

1. **Reading list (Indicative list, current at time of publication. Reading lists will be published annually)**

The module does not follow a specific text. However, the following texts cover the material.

R. Askey, Orthogonal Polynomials and Special Functions, Society for Industrial and Applied Mathematics, Philadelphia, PA, 1975

R. Beals and R. Wong, Special Functions – A Graduate Text, Cambridge University Press, Cambridge, 2010

T.S. Chihara, An Introduction to Orthogonal Polynomials, Dover Publ., Mineola, N.Y., 2011

M. Ismail, Classical and Quantum Orthogonal Polynomials in One Variable, Cambridge University Press, Cambridge, 2005

F.W.J. Olver, D.W. Lozier, C.W. Clark,R.F. Boisvert, Digital Library of Mathematical Functions, National Institute of Standards and Technology, Gaithersburg, U.S.A., 2010 (http://dlmf.nist.gov)

I.N. Sneddon, Special Functions of Mathematical Physics and Chemistry, 3rd Edition, Longman, London, 1980

G. Szego, Orthogonal Polynomials, 4th Ed., American Mathematical Society, Providence, RI, 1975

1. **Learning and teaching methods**

Total contact hours: 42

Private study hours: 108

Total study hours: 150

1. **Assessment methods**
   1. Main assessment methods

**Level 6**

Assessment 1 Exercises, requiring on average between 10 and 15 hours to complete 10%

Assessment 2 Exercises, requiring on average between 10 and 15 hours to complete 10%

Examination 3 hours 80%

The coursework mark alone will not be sufficient to demonstrate the student’s level of achievement on the module.

**Level 7**

Assessment 1 Exercises, requiring on average between 10 and 15 hours to complete 10%

Assessment 2 Exercises, requiring on average between 10 and 15 hours to complete 10%Examination 3 Hours 80%

The coursework mark alone will not be sufficient to demonstrate the student’s level of achievement on the module.

13.2 Reassessment methods

Like-for-like

1. **Map of module learning outcomes (sections 8 & 9) to learning and teaching methods (section12) and methods of assessment (section 13)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Level 6 Module learning outcome** | 8.1 | 8.2 | 8.3 | 8.4 | 9.1 | 9.2 | 9.3 | 9.4 | 9.5 | 9.6 | 9.7 | 9.8 | 9.9 |
| **Learning/ teaching method** |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Private Study and Assessment | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** |
| Lectures/Exercise classes | **X** | **X** | **X** | **X** |  | **X** | **X** |  | **X** |  | **X** | **X** |  |
| Revision classes | **X** | **X** | **X** |  |  | **X** | **X** |  | **X** |  | **X** | **X** |  |
| **Assessment method** |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Examination | **X** | **X** | **X** |  | **X** | **X** | **X** | **X** | **X** |  | **X** | **X** | **X** |
| Coursework | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Level 7 Module learning outcome** | 8.5 | 8.6 | 8.7 | 8.8 | 9. 10 | 9.  11 | 9.  12 | 9.  13 | 9.  14 | 9.  15 | 9.  16 | 9.  17 | 9. 18 |
| **Learning/ teaching method** |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Private Study and Assessment | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** |
| Lectures/Exercise classes | **X** | **X** | **X** | **X** |  | **X** | **X** |  | **X** |  | **X** | **X** |  |
| Revision classes | **X** | **X** | **X** |  |  | **X** | **X** |  | **X** |  | **X** | **X** |  |
| **Assessment method** |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Examination | **X** | **X** | **X** |  | **X** | **X** | **X** | **X** | **X** |  | **X** | **X** | **X** |
| Coursework | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** |

1. **Inclusive module design**

The School recognises and has embedded the expectations of current equality legislation, by ensuring that the module is as accessible as possible by design. Additional alternative arrangements for students with Inclusive Learning Plans (ILPs)/declared disabilities will be made on an individual basis, in consultation with the relevant policies and support services.

The inclusive practices in the guidance (see Annex B Appendix A) have been considered in order to support all students in the following areas:

a) Accessible resources and curriculum

b) Learning, teaching and assessment methods

1. **Campus(es) or centre(s) where module will be delivered**

Canterbury

1. **Internationalisation**

Mathematics is an international language with techniques developed and refined by mathematicians across the globe. Mastery of the subject-specific learning outcomes, 8.1 to 8.8, will equip students to apply the theories and techniques of this module in a wide range of international contexts. The module team is drawn from the School of Mathematics, Statistics and Actuarial Science, which includes many members of staff with international experience of teaching and research collaboration.

In compiling the reading list, consideration has been given to the range of texts that are available internationally and a selection of texts has been identified to complement the delivery of the material.

The support SMSAS provides to its students is also internationally attuned given our international student body.

**DIVISIONAL OFFICE USE ONLY**

**Revision record – all revisions must be recorded in the grid and full details of the change retained in the appropriate committee records.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Date approved | Major/minor revision | Start date of the delivery of revised version | Section revised | Impacts PLOs (Q6&7 cover sheet) |
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