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Positivity in Dynamics and Analysis

The classical Perron-Frobenius theory concerns the spectral properties of nonnegative matrices, and is considered one of the most beautiful topics in matrix analysis. Since its inception in the early nineteen-hundreds it has had a profound influence on many parts of pure and applied mathematics; particularly in probability theory, dynamical systems theory, and discrete mathematics.

Nonlinear Perron-Frobenius theory extends this classical linear theory to nonlinear positive operators. It deals with questions like: When does a nonlinear positive operator have an eigenvector in the cone corresponding to the cone spectral radius? When does the eigenvector lie in the interior of the cone? What are the dynamical properties of the iterates of such operators?

In the nineteen-fifties Birkhoff discovered that one can use Hilbert’s metric to analyse nonlinear positive operators. Hilbert’s metric spaces are a natural generalisation of hyperbolic space and play a role in the solution of Hilbert’s 4th problem. The synergy between nonlinear Perron-Frobenius theory and metric geometry has only recently started to fully crystallise, and provides a rich source of ideas to investigate nonlinear positive operators and their applications.

A range of research topics is available related to nonlinear Perron-Frobenius theory, e.g., nonlinear generalisations of the Krein-Rutman theorem, the cone spectral radius and measures of non-compactness, Denjoy-Wolff type theorems for nonlinear positive operators, geometry of Hilbert’s metric spaces, the behaviour of monotone dynamical systems, and order-isomorphisms between partially ordered vector spaces.

References


