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The use of ranked set sampling in spray deposit assessment

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Summary

Ranked set sampling is a modification of simple random sampling that is intended to give improved precision. The method involves selecting small sets of samples and ranking them, for example visually. A single sample from each set is then measured. The performance of the method in estimating spray deposits on the leaves of apple trees was assessed. Leaves sprayed with a fluorescent tracer dye were ranked visually under ultra-violet light. Despite errors in ranking, ranked set sampling improved the precision of estimating the mean percentage of the upper leaf surface covered with deposit, and the total deposit on the upper surface of the leaf. However, when the additional overhead associated with ranked set sampling was taken into account, the method was more efficient than simple random sampling only for estimating total deposit.

Key words: Apple, leaf deposits, random sampling, ranking, sampling precision

Introduction

The procedure of sampling a population by selecting individuals completely at random is sometimes called simple random sampling, to distinguish it from other forms of random sampling. Textbooks on sampling theory discuss many generalisations of simple random sampling, but in this article, we describe a modification of simple random sampling, termed ranked set sampling, which seldom appears in textbooks, although it was proposed almost 50 years ago.

Ranked set sampling, as its name suggests, involves ranking small sets of samples, by some method that does not involve direct measurement. Samples at particular ranks are then selected for measurement. The objective is to obtain a more precise estimate of the quantity of interest (*e.g.* the population mean) than would be got from a simple random sample of the same size. Ranking imposes an additional overhead on the sampling procedure, and ranked set sampling is therefore most effective when the ranking procedure is cheap in comparison with the cost of taking a sample measurement. In most applications, ranking is based on visual assessment. Although visual assessment may sometimes be unreliable, ranked set sampling can be effective even when errors occur in the ranking process.

In this paper, we discuss the possible use of ranked set sampling for assessing spray deposits on the leaves of apple trees. Foliar spraying in orchards is done usually with an axial fan airblast sprayer. The work was prompted by a MAFF-funded programme studying the effect of different sprayer settings on the distribution of deposit on the leaves. Studies of this type involve an extensive sampling effort, and even modest gains in efficiency can be worthwhile. If trees are sprayed with a fluorescent tracer, leaf deposits can be viewed subsequently under ultraviolet light, and visual ranking is possible.

The paper is organised as follows. First, we provide an overview of ranked set sampling, emphasising the essential ideas and highlighting some of the practical difficulties that may arise. Then we describe a small pilot study, intended to assess whether ranked set sampling might be useful for assessing spray deposits on leaves. Finally, we use some of the data from this study to assess the performance of ranked set sampling in practice.

Overview of Ranked Set Sampling

The method of ranked set sampling was proposed originally by McIntyre (1952), in connection with the estimation of pasture yields. It was re-discovered by Takahasi & Wakimoto (1968), who developed the underlying theory of the method. Ranked set sampling has been used for various sampling problems in agricultural and forestry research (*e.g.* Halls & Dell, 1966, Cobby, Ridout, Bassett & Large, 1985). Recently there has been interest in the method for environmental sampling problems (*e.g.* Kaur, Patil, Shirk & Taillie, 1996). The annotated bibliography of Kaur, Patil, Sinha & Taillie (1995) provides an excellent introduction to the literature on ranked set sampling.

Ranked set sampling is intended to improve the precision with which the mean of a population is estimated. It is most effective in situations where the measurement of sample values is costly, but small sets of samples can be ranked cheaply and with reasonable accuracy, for example based on visual assessment. The number of samples in each set that is ranked, termed the *set size*, will be denoted by m. Because accurate ranking usually becomes increasingly difficult as m increases, m is generally chosen to be 3, 4 or 5.

The sampling procedure is as follows. A random sample of size *m* is chosen. These *m* samples are ranked, the sample ranked smallest is measured accurately and the rest are discarded. A second random sample of size *m* is then chosen and ranked. This time the sample ranked second smallest is measured. This is repeated, measuring samples with increasingly high rank, until *m* sets have been selected and ranked, and one sample of each rank has been measured. The whole process is repeated *r* times, so that in all m^2r samples are examined, but only *mr* of these are measured. The *mr* measured samples comprise *r* of each rank.

The population mean is estimated in the usual way, by taking the arithmetic mean of the *mr* measured values. This is an *unbiased* estimator of the population mean, even when there are errors in ranking (Dell & Clutter, 1972). The variance of the ranked set sampling (*RSS*) estimator is

$$\mathrm{var}(RSS) = \left(\frac{1}{m} \sum_{k=1}^{m} \sigma^2_{[k:m]} \right) \Big/ mr$$

where $\sigma^2_{[k:m]}$ is the variance of the sample judged to be k^{th} smallest in a random sample of size *m*. The variance of the sample mean from a simple random sample (*SRS*) of size *mr* is $\mathrm{var}(SRS) = \sigma^2 / mr$, where σ^2 is the population variance. To compare the two sampling methods we can therefore calculate the *relative precision* (*RP*) which is defined as

$$RP = \frac{\mathrm{var}(SRS)}{\mathrm{var}(RSS)} = \sigma^2 \Big/ \frac{1}{m} \sum_{k=1}^{r} \sigma^2_{[k:m]}$$

Takahasi & Wakimoto (1968) showed that, when ranking is always done correctly, $1 \le RP \le (m+1)/2$, and Dell & Clutter (1972) showed that this inequality continues to apply when ranking errors can occur. The lower bound of unity indicates that ranked set sampling is never less precise than simple random sampling, for estimating the population mean. The upper bound of $(m+1)/2$ occurs only when the variable of interest has a uniform distribution over the population. Other distributions give lower values of *RP*, but for many distributions, the values of *RP* remain high. Table 1 shows values of *RP* for *m*=2,3,4,5 when the population distribution is uniform, normal or exponential and ranking is perfect. Dell & Clutter (1972) give a similar table that includes many more distributions. The exponential distribution was amongst the worst performing of the distributions considered by Dell & Clutter (1972); nonetheless, with *m*=5 ranked set sampling is more than twice as precise as simple random sampling.

Table 1. *Relative efficiency of ranked set sampling for various population distributions, assuming perfect ranking*

		Set size, *m*		
Distribution	2	3	4	5
Uniform	1.50	2.00	2.50	3.00
Normal	1.47	1.91	2.35	2.77
Exponential	1.33	1.64	1.92	2.19

In practice, two factors can reduce *RP* (Ridout & Cobby, 1987). The first is that practical constraints may make it impossible to use proper random sampling in forming sets. A common situation is when sets must be ranked visually in the field, and therefore the samples need to be close together and cannot be chosen at random. Often this means that samples within a set are less variable than they would be under random sampling, and this will reduce *RP*.

The second factor that can reduce *RP* is ranking errors. Dell & Clutter (1972) and David & Levine (1972) proposed a simple and useful model for ranking errors. They assumed that the true sample value is X, but that ranking is done on the basis of a different variable Y, which can be thought of as the ranker's 'perception' of X. Under some additional assumptions, the reduction in *RP* depends in a simple way on the correlation, ρ, between X and Y. Table 2 shows results for a normal distribution. The decline in *RP* as ρ decreases from one (corresponding to perfect ranking) is quite rapid.

Table 2. *Relative precision of ranked set sampling when the population distribution is Normal, and ranking is on the basis of a covariate, whose correlation with the variable of interest is* ρ.

		Set size, *m*		
ρ	2	3	4	5
1	1.47	1.91	2.35	2.77
0.9	1.35	1.63	1.87	2.07
0.75	1.22	1.37	1.48	1.56
0.5	1.09	1.14	1.17	1.19
0.25	1.02	1.03	1.04	1.04

It is also possible to estimate the population variance, σ^2, from a ranked set sample (Stokes, 1980). However, the relative precision of ranked set sampling to simple random sampling for estimating the population variance is seldom much greater than one. The relative precision also depends on r. For example, when sampling from a normal distribution with m=5 and no ranking errors, relative precision increases from 1.03 for r=1 (an unrealistically small value of r) with the limiting value 1.27 as r increases indefinitely. When the sample size mr is very small, ranked set sampling may even give a less precise estimate than simple random sampling (Stokes, 1980), though this is unlikely to occur in practice.

To summarise, ranked set sampling can potentially give a more precise estimate of the population mean than simple random sampling, but there is little difference in the precision with which the population variance is estimated. In practice of course the improvement in precision for estimating the sample mean must be considered in relation to the additional costs incurred by ranked set sampling. Relative precision compares the variances when the two methods use the same sample size. *Relative efficiency* (*RE*) compares the variances when the sampling costs of the two methods are the same. The relationship between relative efficiency and relative precision is

$$RE = \left(\frac{C}{C+R} \right) RP$$

where C is the per sample cost of simple random sampling, and R is the *additional* per sample cost of ranked set sampling (Dell & Clutter, 1972). R is the cost of selecting m-1 additional samples, ranking the full set of m, and selecting the sample thought to have the desired rank.

Materials and Methods

A small spraying trial was done in an orchard of Cox apple trees at Horticulture Research International – East Malling, to provide an initial assessment of ranked set sampling. Trees were sprayed in October 1997 with the fluorescent, water soluble, tracer Tinopal CBS-X, at 2% concentration in water. The spraying mixture also contained a wetting agent. Two nine-tree plots, in separate rows of the orchard, were chosen for spraying. One plot was sprayed at high volume, using coarse nozzles on the sprayer to give a large average droplet size. The other plot was sprayed at low volume, using fine nozzles to give a small average droplet size. Both sides of the tree rows were sprayed.

Twenty sets of five leaves were sampled from the central five trees of each plot. Leaves were sampled haphazardly, *i.e.* without any intentional positional bias, but equally not according to a formal randomisation scheme. Leaves of similar size were selected from throughout the tree canopy, to encompass the full range of variability in spray deposit. Leaves were transported back to the laboratory where a single observer ranked the leaves within each set based on the visual appearance of the deposits on the upper leaf surfaces when viewed under ultraviolet light. The deposits on the lower surfaces of the leaves were ranked separately. Once ranking was complete, an image analysis system (Optimax V) was used to estimate the percentage area of the leaf surface that was covered with deposit (here termed %COVER). The upper and lower leaf surfaces of all five leaves were measured.

The spray deposit on the upper surface of each leaf was washed into a test tube with a directed jet of water (5 ml). The relative concentration of tracer (here termed DEPOSIT) in each extract was measured using a Perkin Elmer 3000 fluorescence spectrometer at an excitation wavelength of ???nm and emission wavelength of ???nm,. As preliminary experiments had demonstrated that the recovery of the tracer was essentially quantitative from the upper leaf surface, but was less than 50% from the lower surface, only the deposit on the upper surface was measured by this method.

Results and Discussion

For brevity, we discuss only the data for the upper leaf surfaces. Table 3 shows mean values of DEPOSIT and %COVER. For statistical analysis, these variables were transformed, to make the within-treatment variances more equal. DEPOSIT values were transformed using the $\log_e()$ transformation, and for %COVER the logit transformation

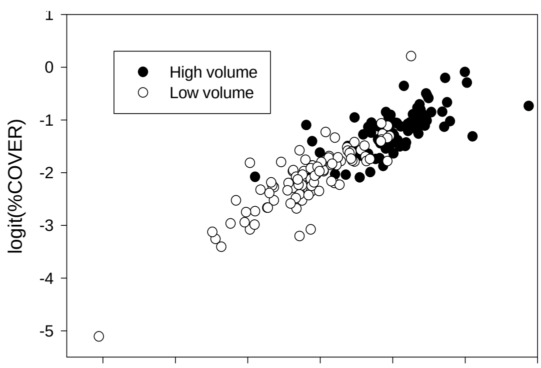
$$\text{logit}(\%\text{COVER}) = \log_e\left(\frac{\%\text{COVER}}{100 - \%\text{COVER}}\right)$$

was used.

Table 3. *Mean values of DEPOSIT and %COVER for low and high volume sprays. Main entries refer to transformed data, with means on the original scale given in brackets.*

Variable	Low volume	High volume	SED (198 df)
$\log_e$(DEPOSIT)	5.88	7.00	0.083
(DEPOSIT)	(426)	(1268)	
logit(%COVER)	-2.08	-1.22	0.075
(%COVER)	(12.4)	(23.6)	

Both variables had significantly higher mean values at high spray volume than at low spray volume (P<0.001). The variables were correlated (r=0.86 for high volume data, r=0.69 for low



volume data) as shown in Fig. 1.

Fig. 1. Relationship between transformed values of DEPOSIT and %COVER

Table 4 shows estimates of *RP* and *RE* for estimating the mean values of DEPOSIT and %COVER. Estimates were calculated on the transformed scale and on the original scale. Generally, a slightly higher *RP* value was obtained for the transformed data. Estimates of *RP* at low and high spray volumes were similar for %COVER, but differed substantially for DEPOSIT.

Table 4. *Estimates of relative precision* (RP) *and relative efficiency* (RE) *for estimating the mean values of DEPOSIT and %COVER at low and high spray volumes.*

	RP		*RE*	
Variable	Low volume	High volume	Low volume	High volume
$\log_e$(DEPOSIT)	2.17	1.67	1.74	1.34
(DEPOSIT)	2.18	1.39	1.74	1.11
logit(%COVER)	1.85	1.97	0.93	0.99
(%COVER)	1.75	1.89	0.88	0.95

The theoretical upper limit for the value of *RP* here is (5+1)/2 = 3. Failure to achieve this value is partly because the distribution of these variables in the population is not uniform, but is also a consequence of ranking errors. For illustration, Table 5 compares the true ranking with the observer's ranking. Only 49/100 samples were assigned the correct rank. However, although *RP* is considerably lower than would have been achieved with perfect ranking, *RP* was still considerably greater than one. This emphasises that ranked set sampling can still yield worthwhile improvements in precision, even when errors occur in the ranking process.

Measuring %COVER and DEPOSIT for a single sample took approximately 0.5 min and 2.0 min respectively. Collection of additional samples and ranking added an overhead of approximately 0.5 min for each measured sample. These values were used to calculate the *RE* values in Table 4. For %COVER, the overhead of ranked set sampling is large, and the method is less efficient than simple random sampling. In contrast, for DEPOSIT the overhead associated with ranked set sampling is only a quarter of the basic measurement time, and ranked set sampling is more efficient than simple random sampling.

Table 5. *Comparison of the observer's ranking and the true ranking of 20 sets of 5 leaves sampled from the low volume treatment. The true ranking is based on %COVER.*

	Observer's ranking				
True ranking	1	2	3	4	5
1	13	3	3	0	1
2	5	6	6	2	1
3	2	9	5	3	1
4	0	1	5	11	3
5	0	1	1	4	14

In more recent work, we have used a different image analysis system that allows a much wider range of summary measures to be obtained from a digitally stored image of a sprayed leaf. Moreover, the images may be stored indefinitely. However, this additional flexibility is obtained at the cost of speed; image capture and processing now takes about 2.5 min per leaf and ranked set sampling is therefore likely to be beneficial. A separate and more detailed study has been carried out to assess this, and to look at additional factors such as the variation in ranking performance of different observers. The results of this study will be reported elsewhere.

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