Moving Frames and Noether’s Conservation Laws – the General Case

Tânia M. N. Gonçalves
Universidade Federal de São Carlos (Brazil)

Elizabeth L. Mansfield
University of Kent (United Kingdom)

tmng@kentforlife.net

Abstract

In recent works, the authors have considered Lagrangians invariant under a Lie group action, in the case where the Lagrangian may be parametrized so that the independent variables are each invariant under the action. We were able to calculate the Euler-Lagrange equations for the invariants in terms of the standard Euler operator and a ‘syzygy’ operator specific to the action and which is readily calculated. Further, we were able to obtain the linear space of conservation laws in terms of vectors of invariants and the adjoint representation of a moving frame for the Lie group action. This allowed us to simplify the calculation for the extremals in the original variables, once the Euler-Lagrange equations for the invariants were solved, for all three SL(2) and the standard SE(3) actions.

In this talk, we show how our ideas may be extended to cases where reparametrization of the independent variables is difficult, impossible, or undesired. We take for our main expository example the standard linear action of SL(2) on the two independent variables. This choice is motivated by applications to variational fluid problems which conserve potential vorticity. We note that Kogan and Olver previously handled the one-dimensional case using a variational tricomplex.

Keywords
Variational problems, Invariant calculus of variations, Noether’s conservation laws, Moving frames