Abstract

The talk is about general properties of the algebra of polynomial integro-differential operators
\[ I_n := K\langle x_1, \ldots, x_n, \frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n}, \int_1, \ldots, \int_n \rangle. \]

We show that the algebra \( I_n \) is a prime, central, catenary, self-dual, non-Noetherian algebra
of classical Krull dimension \( n \) and of Gelfand-Kirillov dimension \( 2n \). Its weak dimension is
\( n \), and \( n \leq \text{gl.dim}(I_n) \leq 2n \). All the ideals of \( I_n \) are found explicitly, there are only finitely
many of them (\( \leq 2^n \)), they commute (\( ab = ba \)) and are idempotent ideals (\( a^2 = a \)). An
analogue of the Hilbert’s Syzygy Theorem is proved for \( I_n \). The group of units of the algebra
\( I_n \) is described (it is a huge group). A canonical form is found for each integro-differential
operators (by proving that the algebra \( I_n \) is a generalized Weyl algebra). All the mentioned
results hold for the Jacobian algebra \( A_n \) (but \( \text{GK}(A_n) = 3n \), note that \( I_n \subset A_n \)). It is proved
that the algebras \( I_n \) and \( A_n \) are ideal equivalent.

The group \( G_n \) of automorphisms of the algebra \( I_n \) is found:
\[
G_n = S_n \ltimes \mathbb{T}^n \ltimes \text{Inn}(I_n) \supseteq S_n \ltimes \mathbb{T}^n \ltimes \mathbb{GL}_\infty(K) \ltimes \cdots \ltimes \mathbb{GL}_\infty(K),
\]
\[
G_1 \simeq \mathbb{T}^1 \ltimes \mathbb{GL}_\infty(K),
\]
where \( S_n \) is the symmetric group, \( \mathbb{T}^n \) is the \( n \)-dimensional torus, \( \text{Inn}(I_n) \) is the group of inner
automorphisms of \( I_n \) (which is huge). It is proved that each automorphism \( \sigma \in G_n \) is uniquely
determined by the elements \( \sigma(x_i) \)'s or \( \sigma(\frac{\partial}{\partial x_i}) \)'s or \( \sigma(\int_i) \)'s. The stabilizers in \( G_n \) of all the
ideals of \( I_n \) are found, they are subgroups of finite index in \( G_n \). It is shown that the group \( G_n \)
has trivial centre. For each automorphism \( \sigma \in G_n \), an explicit inversion formula is given via
the elements \( \sigma(\frac{\partial}{\partial x_i}) \) and \( \sigma(\int_i) \).

Keywords

The algebras of polynomial integro-differential operators, the group of automorphisms, the
canonical form, the Jacobian algebras.