Lecture courses for LMS-EPSRC Short Course on Classical and Quantum Integrable Models

1. Classical and quantum integrable systems (Sklyanin)

   - Liouville integrability
   - Lax pairs
   - Separation of variables
   - Bethe ansatz
   - Bäcklund transformation and Q-operators
   - Examples: Toda lattice, Calogero-Moser model, Gaudin model

   Reading list:

   (a) E.K. Sklyanin, Separation of variables. New trends. Progr. Theor. Phys. Suppl. No. 118 (1995) 35–60. http://arxiv.org/abs/solv-int/9504001

   (b) E.K. Sklyanin, Canonicity of Bäcklund transformation: r-Matrix Approach. I, L.D. Faddeev's Sem. Math. Phys. 277-282, Amer. Math. Soc. Transl.Ser 2, 201.

   (c) Amer. Math. Soc., Providence, RI, (2000); Canonicity of Bäcklund tranformations: r-Matrix Approach.II, Proc. Steklov Inst. Math. 226, 121–126, (1999).

2. Quantum integrable systems (Doikou)

   - Notation, and brief review of $sl(2)$ and its representations. XXX and XXZ Hamiltonians.
   - Yang-Baxter equation and braid groups
   - The quantum Lax operator and quantum algebras. $U_q(sl(2))$ representation theory.
   - Quantum integrability: transfer matrix, Hamiltonian. Algebraic Bethe ansatz
   - Reflection equation and B-type Hecke algebras

Reading list: A. Doikou, S. Evangelisti, G. Feverati and N. Karaiskos, Introduction to Quantum Integrability.
http://arxiv.org/PS cache/arxiv/pdf/0912/0912.3350v2.pdf

Exercise classes: Based on the following exercises from the above review paper:

(a) Sections 3.1. ex. 1-6

(b) Sections 4.1, 4.2. ex. 1-7

(c) Section 5.1. ex. 1-4

(d) Section 6.1. ex. 1

(e) Sections 8.1., 8.3. ex. 1-3

3. Hamiltonian approach to integrable discretization (Suris)

- Lie-Poisson bracket, (linear) r-matrix structure, examples: Toda lattice, Euler top
- AKS factorization scheme, explicit solutions and Bäcklund transformations through factorization, application to the Toda lattice
- R-matrix recipe for discretization, application to integrable lattice systems, localizing changes of variables
- Discrete time Lagrangian mechanics, discrete time Lagrangian reduction on Lie groups
- Integrable discrete time Lagrangian systems of classical mechanics, including Lagrange top.

Reading list: Yu.B. Suris. The Problem of Integrable Discretization: Hamiltonian Approach. Progress in Mathematics, Vol. 219. Basel: Birkhäuser, 2003.

Exercise classes: These will cover the following topics:

(a) Tensor formalism of Lie-Poisson and r-matrix brackets

(b) Discrete time Toda lattice: nonlocal version and localizing change of variables

(c) Discrete time Lagrange top, including details of the Lagrangian derivation, Lax pair, and integrals of motion.