Autumn Term 2014	University of Kent
Exercises on Stochastic Processes	L. Breuer / F. Leisen

Assessment 2

NOTE: Students of MA836, please return your solutions to the SMSAS general office before noon on Tuesday, the 2nd of December.

Exercise 1: 10 Marks

Let $\mathcal{Y} = (Y_t : t \geq 0)$ denote a Markov process with state space $E = \{1, 2, 3\}$ and generator matrix

$$G = \begin{pmatrix} -2 & 2/3 & 4/3 \\ 1 & -2 & 1 \\ 1 & 0 & -1 \end{pmatrix}$$

Draw the transition graph for $\mathcal{Y}$ and write down the balance equations.

Exercise 2: 10 Marks

An electrical switch has two states, 0 (for off) and 1 (for on). Assume that the holding time in each state is exponentially distributed with parameters $\lambda_0, \lambda_1 > 0$. Describe the time–dependent behaviour of the switch in terms of a Markov process $\mathcal{Y} = (Y_t : t \geq 0)$ by giving the generator matrix of $\mathcal{Y}$. Determine the stationary distribution of $\mathcal{Y}$.

Exercise 3: 20 Marks

Consider the following queue with retrials. Customers arrive at a single server according to a Poisson process with parameter $\lambda > 0$. If a customer finds the server idle, her service will commence immediately, otherwise she will join the queue. The service time distribution is exponential with parameter $\mu > 0$. After a service is finished, the customer just served will commence a new service with probability $q < 1$. All service times are independent with identical distribution. The arrival process is independent of the service times.

(a) Provide a model of this queueing system in terms of a Markov process. Determine the holding time parameters and the transition matrix of the embedded Markov chain. From these, determine the generator matrix.

(b) What condition do λ, μ and q need to satisfy in order to guarantee existence of a stationary distribution? Given this condition, determine the stationary distribution.