On the Relevance of Capital Income Taxation

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1 Introduction

In a standard model of the intertemporal labour supply/consumption/saving decision, with weak separability between consumption and leisure in each period; identical preferences across households; and a Mirrlees optimal nonlinear tax on labour earnings; the Atkinson-Stiglitz Theorem (AS) implies that there is no case for taxing future consumption and therefore the return to saving. An extension of AS by Konishi (1995), Laroque (2005) and Kaplow (2006), (KLK), replaces the assumption of an optimal nonlinear tax system with that of the planner being able to choose any smooth function from gross to net income, and shows that an allocation with both direct and indirect taxation can always be Pareto-dominated by one with direct taxation alone. This in the intertemporal context again implies no taxation of the return to saving. Thus taxation of capital income is purported to be irrelevant, superfluous or non-optimal. This paper is intended to contribute to a recent body of literature that contests this view.

The assumptions of weak separability, identical preferences and optimal or unconstrained nonlinear taxation are of course strong, and a number of studies have shown that the result, which in policy-related discussions is often expressed as "capital income ought to be untaxed", is not robust to their relaxation. This paper contributes to this literature by analysing the implications for the form of the optimal taxes on labour earnings and capital income first of introducing household production as a form of time use, along with market work and leisure; secondly, of imposing the constraint that taxation must take the

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1 For a recent review see Diamond and Saez (2013).
2 See for example Mankiw et al (2009).
3 See Boadway (2012) for a concise but comprehensive survey, and Banks and Diamond (2010), Diamond and Spinnewijn (2013), and Saez (2002) for specific models.
piecewise linear form; and finally of recognising that households typically consist of two earners. We continue to assume identical preferences, since this finesse the problem of making interpersonal comparisons when preferences differ.\footnote{In contrast to Diamond and Spinnewijn (2013) and Saez (2002) who assume differences in preferences across households.} We also show that the assumption of weak separability of leisure and consumption, which though dubious empirically seems to be almost universally adopted in life cycle models, becomes neither necessary nor sufficient for the conclusion that capital income should not be taxed, when we allow household and market consumption goods to be Hicksian substitutes or complements.

There are strong arguments on empirical grounds for making these extensions. One has only to consider the main types of household production - child care, meal preparation, domestic accounting and financial management, laundry, house maintenance and cleaning - to see that they all have close but usually imperfect substitutes and complements that can be bought on markets.\footnote{Sandmo (1990), Kleven et al (2000), Kleven (2004) and Alesina et al (2011) analyse the issues of direct and/or indirect taxation in the presence of household production in an atemporal context.} Virtually all real world tax systems are piecewise linear.\footnote{For the analysis of optimal piecewise linear taxation in an atemporal context see Sheshinski (1989), Slemrod et al. (1994) and Apps, Long and Rees (2013).} All and tax systems have to deal with the issue of how to tax two-earner households.

The paper is structured as follows. In the next section we set out the basic two-period model of the individual consumer/worker who saves in a perfect capital market, consumes a household good, a market good and leisure in each period, and divides her time endowment among market work, leisure and household production in the first period and, in the second, between household work and leisure. In Section 3 we assume there are two wage types and examine the determinants of optimal taxation of the return to saving when there is also optimal nonlinear taxation of labour income given non-verifiability of wage types. In Section 4 we assume a continuum of wage types and analyse the joint determination of optimal labour and capital income taxation when tax systems are constrained to be piecewise linear.\footnote{This corresponds to the so-called "dual system" of taxation that has been introduced by a number of European countries. See Sørenson (1994), (2005) for further discussion.} Section 5 considers the comparison of income and consumption taxation when households consist of two earners, a key aspect of the comparison being that individual consumptions, as opposed to labour earnings, cannot be observed.\footnote{The problem is not just that standard data sets provide very little information on individual consumptions of household members, but also that it would be extremely difficult to measure who's consumption is reduced by how much to finance a given amount of household saving.} Section 6 concludes.

## 2 The Individual Model

A consumer $i$’s utility is given by a strictly concave, increasing function

$$u_i = u(x_{i0}, y_{i0}, l_{i0}) + \rho u(x_{i1}, y_{i1}, l_{i1})$$

(1)
where $x$ is a market consumption good, $y$ is a good produced within the household, and $l$ is leisure. We place for the moment no a priori restrictions on the structure of within-period utilities, while adopting the standard assumption that the across-period utilities are additively separable, with $\rho \in [0, 1]$ a felicity discount factor. In the first period $i$ supplies market labour $L_i$, at a wage rate $w_i$ and in both periods supplies $h_{it}$, $t = 0, 1$ to household production. She retires in the second period. The time constraints therefore are

$$L_i + h_{i0} + l_{i0} = 1$$

$$h_{i1} + l_{i1} = 1$$

Household production is described simply by

$$h_{it} = a_{it}y_{it}, \ t = 0, 1$$

so that the time requirement (inverse productivity) coefficients $a_{it}$ can vary across individuals as well as over time. This variation is essentially what distinguishes time spent in household production from leisure. In the standard formulation an hour of leisure produces the same utility for all individuals at all times, but here one hour spent in producing the good $y$ can yield varying utilities, even though all utility functions are identical. Thus leisure is defined as a household good in production of which all households always have identical productivity, so that the implicit productivity coefficients can all be normalised to 1. The important point from the standpoint of an optimal tax model is that the $a_{it}$ represent differences in productivities, of the same nature as the differences in wage rates $w_i$, rather than differences in preferences, and so present no difficulties in making interpersonal comparisons. If $a_{it} < a_{jt}$ then $i$ is unambiguously better off than $j$ at any given time allocation, other things, especially wage rates, being equal.

Defining $z_i \equiv w_i L_i$ and assuming that each $i$ saves a positive amount at the same per period interest rate $r$ in period 0 allows us to define the wealth constraint, absent taxation, as

$$\sum_{t=0}^{1} \delta^t x_{it} \leq z_i$$

where $\delta = (1 + r)^{-1}$, and the consumer chooses optimal life time profiles of consumptions, time allocations and saving by maximising (1) subject to (2)-(5). In what follows however we simplify by using the household production and time constraints to write the utilities as

$$u^i = u(x_{i0}, y_{i0}, 1 - a_{i0}y_{i0} - \frac{z_i}{w_i}) + \rho u(x_{i1}, y_{i1}, 1 - a_{i1}y_{i1})$$

Then only the wealth constraint remains.
3 Optimal Linear Consumption Tax with Non-linear Income Taxation

The planner can observe only labour incomes \(z_i\) and consumptions \(x_{it}\) in each period, \(t = 0, 1\). There are just two wage types \(i = 1, 2\) and we assume \(w_2 > w_1\). Household production and leisure are unobservable and so untaxable. Producer prices are normalised at 1 in each period and without loss of generality first period consumption is chosen as the numeraire, and so indirect (= capital income) taxation takes the form of a consumer price \(p = 1 + \tau\) for second period consumption. The planner’s optimal tax problem can be modelled as choice of \(z_i, p\) (or \(\tau\)) and \(c_i\), where this last is defined by

\[
x_{i0} + \delta px_{i1} = c_i \quad i = 1, 2
\]  

(7)

Thus the planner chooses labour supply in the first period and the present value of consumption, which is equivalent to setting labour income taxation, while setting \(p\) is equivalent to taxing the return to first period saving. The consumer maximises the utility function in (6) with \(z_i\) given, subject to the wealth constraint in (7) with \(p\) and \(c_i\) also given. This yields consumption demands \(x_{it}(c_i, z_i, p), y_{it}(c_i, z_i, p)\) and indirect utility functions \(V^i(c_i, z_i, p)\), with derivatives

\[
V^i_c = \lambda_i; V^i_z = -u'^i/w_i; V^i_p = -\delta \lambda_i x_{i1}(c_i, z_i, p)
\]  

(8)

where \(\lambda_i\) is \(i\)'s marginal utility of wealth. Note also that we can maximise \(u^2\) with \((c_1, z_1, p)\) as parameters, i.e. with individual 2 "mimicking" 1, in which case we denote

\[
\hat{x}_{21} = x_{21}(c_1, z_1, p); \hat{V}^2 = V^2(c_1, z_1, p); \hat{V}^2_c = \hat{\lambda}_2
\]  

(9)

The utilitarian planner solves the problem

\[
\max_{c_i, z_i, p} \sum_i \phi_i V^i(c_i, z_i, p)
\]  

(10)

where \(\phi_i\) is the proportion of consumers of type \(i\), subject to the government’s per capita revenue constraint

\[
\sum_i \phi_i [z_i - c_i + \tau \delta x_{i1}(c_i, z_i, p)] \geq G
\]  

(11)

and the incentive compatibility constraint\(^9\)

\[
V^2(c_2, z_2, p) \geq \hat{V}^2(c_1, z_1, p)
\]  

(12)

First consider this problem with the constraint \(p = 1\) \((\tau = 0)\), as if indirect taxation is ruled out, so that we just have the standard Mirrlees optimal tax problem. The Lagrange function for the planner’s problem is then

\[
L = \sum_i \phi_i V^i + \lambda \left( \sum_i \phi_i [z_i - c_i + (p - 1) \delta x_{i1}] - G \right) + \mu [V^2 - \hat{V}^2]
\]  

(13)

\(^9\)We assume that the single crossing condition \(\partial[-V^i/V^p]/\partial w < 0\) holds.
We then have:

**Result 1**: Denoting the value function of this problem by \( W(p) \), we have at \( p = 1 \)

\[
W'(p) = \mu \lambda_2 \delta(\hat{x}_{21} - x_{11}) \leq 0
\]  

**Proof**: From the first order conditions for this problem we obtain

\[
\lambda \phi_1 = \lambda_1 \phi_1 - \mu \lambda_2
\]  
\[
\lambda \phi_2 = \lambda_2 \phi_2 + \mu \lambda_2
\]

Then applying the Envelope Theorem and using (15),(16) gives the result.

This result tells us that capital income should be untaxed \((p = 1)\) if and only if \((\hat{x}_{21} - x_{11}) = 0\), which is the AS case, while future consumption should be taxed relatively more (less) heavily than current consumption if \((\hat{x}_{21} - x_{11}) > (>) 0\). Thus the case for capital income taxation in this model rests on whether the higher wage consumer’s second period consumption when she mimics the lower wage consumer differs from that of the latter.

In the standard model in which time is divided between market work and leisure, this is ruled out when the utility function exhibits weak separability between consumption and leisure, as in AS, but in the present model this condition is neither necessary nor sufficient for this result, because of the existence of the third time use, household production. In that case it is sufficient (though not necessary) to assume weak separability between consumption of the market good and both the household good and leisure,\(^{11}\) that is, the utility function takes the form

\[
U^i = U(x_{i0}, u(y_{i0}, 1 - a_{i0}y_{i0} - \frac{z_i}{w_i})) + \rho U(x_{i1}, u(y_{i1}, 1 - a_{i1}y_{i1}))
\]

Whatever one may think about the common assumption of separability of consumption and leisure, to extend this to household goods is quite counterfactual, as the examples we gave in the Introduction are meant to suggest.

Some intuition is suggested by the presence of the shadow price of the incentive compatibility constraint \( \mu > 0 \) in (14). When \((\hat{x}_{21} - x_{11}) > 0\), an increase in \( p \), or equivalently \( \tau \), relaxes the incentive constraint by making the utility gained by the higher wage type, 2, when she chooses \( c_1, z_1 \), lower relative to that she obtains when she chooses \( c_2, z_2 \), and this increases social welfare at the optimal type-contingent labour earnings tax levels \((z_i - c_i)\). A similar argument applies in the converse case in which \((\hat{x}_{21} - x_{11}) < 0\) and \( p \), or equivalently \( \tau \), should be reduced. Further intuition will be given when we consider the relationship between this key difference \((\hat{x}_{21} - x_{11})\) and the relationship of substitution or complementarity between the market and household goods.

First however, by reinstating \( p \) as an unconstrained instrument in solving the planner’s optimisation problem we obtain:\(^{12}\)

\(^{10}\)A similar result for the atemporal case is given in Boadway (2012) p69.

\(^{11}\)This, though intuitively obvious, is shown more formally below.

\(^{12}\)Again see Boadway (2012) p.69 for a similar condition in the atemporal case.
Result 2: The optimal capital income tax rate $\tau^*$ is given by

$$\tau^* = -\frac{\mu \hat{\lambda}_2 (\hat{x}_{21} - x_{11})}{\lambda \sum_i \phi_i s_i}$$  \hspace{1cm} (18)$$

where $s_i < 0$ are the compensated demand derivatives of the $x_{i1}$ with respect to $p$.

Proof: From the first order conditions for the problem in (10)-(12) we now obtain:

$$\lambda \phi_1 = \lambda_1 \phi_1 - \mu \hat{\lambda}_2 + \lambda \phi_1 \tau^* \frac{\partial x_{11}}{\partial c_1}$$  \hspace{1cm} (19)$$

$$\lambda \phi_2 = \lambda_2 \phi_2 + \mu \lambda_2 + \lambda \phi_2 \tau^* \frac{\partial x_{21}}{\partial c_2}$$  \hspace{1cm} (20)$$

$$- \sum \lambda_i \phi_i + \lambda \delta \sum \phi_i (x_{i1} + \tau^* \frac{\partial x_{i1}}{\partial p}) + \mu (V^2_p - \hat{V}^2_p) = 0$$  \hspace{1cm} (21)$$

Substituting from (19) and (20) into (21) and using the Slutsky equations $\partial x_{i1}/\partial p = s_i - \delta x_{i1} \partial x_{i1}/\partial c_i$ gives the result.

Thus the optimal capital income tax rate is given by a trade off between the gain in welfare it gives by relaxing the incentive constraint and the deadweight loss resulting from the distortion in optimal consumption choices, represented by the sum of compensated demand derivatives in the denominator of (18).

Note there is some similarity between this condition and that for the optimal marginal tax rate in the standard optimal linear taxation model. There, the denominator is the same while the numerator is the covariance between the marginal social utility of income across consumers and the amount of the income being taxed. The difference results from the fact that in the linear tax model, the tax rate is determined by the trade off between equity and efficiency. Here on the other hand, this role is being played by the optimal nonlinear taxes, implicitly defined by the optimal $z_i$ and $c_i$ values. The role of the indirect tax is to reduce the distortion away from the first lump sum labour income taxes created by the need to ensure incentive compatibility.

At this point it is natural to enquire into the relationship between this key difference $(\hat{x}_{21} - x_{11})$ and the substitute/complement relationships between the goods in the individuals' utility functions. From the work that has been done on the atemporal case, where the consumer's time is divided between only work and leisure,\(^{13}\) we might guess that this will be positive when $x$ and $y$ are Hick- sian complements and negative when they are substitutes. This result in turn bears what Boadway with justice calls\(^{14}\) an "uncanny resemblance" to the famous Corlett-Hague theorem\(^{15}\) on the determinants of the relative tax rate on a consumption good in the case of two taxable consumption goods and non-taxed leisure - a relatively higher tax is imposed on the good that is more

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\(^{13}\)See again for example Boadway (2012), pp.61-65 and the work cited there.

\(^{14}\)Ibid. p61.

\(^{15}\)See Corlett and Hague (1953). Note that Kleven et al. (2000) show that this result must be modified in the presence of household production because of the complement/substitute relationships between the household and market goods.
complementary to leisure. However, the structure of the present model is such that this uncanny resemblance may be precisely reversed. In fact we have:

**Result 3:** If leisure is weakly separable from the two consumption goods $x$ and $y$, and $a_{1t} = a_{2t}, t = 0, 1$, then the tax rate on $x_1$ is relatively higher when $x$ and $y$ are Hicksian substitutes, while when $a_{1t} \neq a_{2t}$, all cases are possible.

**Proof:** See the Appendix.

The analysis proceeds as follows. Given $c_1, z_1, p, a_{1t}$ and $w_1$, the type 1 consumer chooses $x_{11}$. If $a_{1t} = a_{2t}$, the type 2 consumer would choose $x_{21} \neq x_{11}$ when also faced with $c_1, z_1, p$, only because $w_2 > w_1$. Thus to derive the relationship between $x_{21}$ and $x_{11}$ we just have to carry out the standard comparative statics analysis of the effect of an increase in 1’s wage on her equilibrium choices. In the Appendix it is shown that

$$\frac{\partial x_{11}}{\partial w} = \beta u_{120} \tag{22}$$

where $\beta < 0$ is a complicated term involving partials of the utility function, which can however be signed under the above assumptions, and $u_{120} \equiv \frac{\partial^2 u}{\partial y_{10} \partial x_{10}}$. Thus if $x$ and $y$ are substitutes $u_{120} < 0$ and $\partial x_{11}/\partial w > 0$ and therefore $x_{21} > x_{11}$. In that case $\rho > 1$. The intuition is as follows. Given $c_1, z_1$, an increase in $w$ reduces the time spent working and so type 2 has both more leisure and time spent in household production relative to 1. If the household good is a substitute for the market good consumption $x_{20}$ will be lower and therefore, given the wealth constraint, $x_{21}$ will be higher.\(^{16}\) Imposing a tax on $x_1$ then reduces the gain from 2’s higher wage when she mimics 1, and so the incentive constraint is relaxed. The weak separability of leisure implies that the change in leisure consumption would have no effect, but this does not matter in this case as long as $u_{120} \neq 0$. It is then easy to see that the case of complements, $u_{120} > 0$, gives the converse result.

However, for this nice determinacy we do require that household productivities of the two wage types are identical. Allowing them to vary with the wage gives the ceteris paribus results

$$\frac{\partial x_{11}}{\partial a_0} = \alpha_0 u_{120} \quad \alpha_0 > 0 \tag{23}$$

$$\frac{\partial x_{11}}{\partial a_1} = \alpha_1 u_{121} \quad \alpha_1 < 0 \tag{24}$$

The effect of an increase in $a_0$ is the opposite of that of the wage rate because increasing $\alpha_0$ increases the time required to produce $y_{10}$ and therefore reduces its consumption. In the case of substitutes this increases consumption $x_{20}$ and reduces that of $x_{21}$. On the other hand an increase in the time requirement in the second period will increase $x_{21}$ in the case of substitutes. Thus assuming, say, a positive correlation between market and household productivity (inverse of $a_{it}$) in both periods would result in overall indeterminacy because of the

\(^{16}\)The intertemporal structure of the model, with labour supply in the first period and taxed consumption in the second, is what causes the difference to the Corlett-Hague results.
opposing signs of $\alpha_0$ and $\alpha_1$. Thus even with weak separability between leisure and consumption it is possible to have a case in which $(\hat{x}_{21} - x_{11}) = 0$.

4 Piecewise Linear Dual Income Taxation

Traditionally the issue of taxation of capital income was discussed\(^\text{17}\) in the context of the problem of choosing the tax base, which was framed as a choice between some comprehensive measure of income on the one hand and consumption expenditure on the other. One way of calculating the latter tax base is to exclude from the measure of taxable income the return to saving.\(^\text{18}\) In the case of income taxation it was generally assumed that all forms of income would be subjected to the same tax schedule, and the arguments of the relative merits of the two tax bases was carried on in these terms. However, if we accept that the arguments for excluding capital income from taxation are not robust, the question of how exactly to tax it arises. In particular, we can ask the question: Is it necessarily optimal to tax all forms of income at the same rate?

"Dual income taxation" (DIT) is the system whereby labour earnings and capital income are taxed according to different schedules, and, pioneered in the Nordic countries, has recently been introduced in a number of other countries, including Germany. The central idea is that alongside a standard piecewise linear labour income tax schedule,\(^\text{19}\) there is a flat rate tax on capital income, where the rate is typically at or below the "standard rate" on labour earnings. Since there is also a tax-free capital income threshold before the flat rate is applied, the DIT effectively involves a two-bracket piecewise linear capital income tax with a zero rate in the lower bracket. In this section, we take an economy in which labour earnings and capital income are subject to separate, but interrelated, two-bracket piecewise linear taxes and derive the optimal parameters of these, showing how they are related. We also consider the question of the conditions under which either or both of the marginal tax rates on capital income should be zero.

On the one hand, we generalise the model in the previous section by assuming a continuum of wage types defined by an interval $\Omega = [w_0, w_1] \subset \mathbb{R}_+$, distributed on $\Omega$ according to the cdf $\Phi(w)$. On the other hand, we simplify it by dropping leisure, which has no substantive implications for the analysis, and by assuming that there is a perfect correlation between productivity in household production, the (inverse of the) $\alpha_{ht}$ parameter in the previous model, and the individual wage. This ensures that under the tax system we consider, in equilibrium under the given tax system individual utility is strictly increasing with the wage.

We further assume for simplicity additive separability in within period util-

\(^{17}\)See Meade (1978) as a classic example

\(^{18}\)See the next section for details.

\(^{19}\)Though, perhaps uniquely in the world, Germany does not have a piecewise linear system, since within the two middle income brackets the marginal tax rate is a linear function of income.
ity\textsuperscript{20} and write the utility function now as
\[ u(w) = u(x_0(w)) + v(y_0(w)) + \rho[u(x_1(w)) + v(y_1(w))] \] (25)

Given the time constraints
\[ L(w) + h_0(w) = 1 \] (26)
we can use the household production relationships \( h_0(w) = a(w)y_0(w) \) to define
\[ \psi(z(w)) \equiv -v((1 - z/w)/a) = -v(y_0(w)) \] (27)
and thus finally write the utility function as
\[ u(w) = u(x_0(w)) - \psi(z(w)) + \rho u(x_1(w)) \] (28)
where we drop the utility of \( y_1 \) since this is now a constant.\textsuperscript{21}

Now \( z \) and \( x_1 \) are to be taxed according to a piecewise linear structure\textsuperscript{22} where \( t_1, t_2 \) are the marginal tax rates for taxation of labour income and \( \hat{z} \) is the bracket limit, and \( \tau_1, \tau_2 \) are the marginal tax rates on capital income taxation and \( \hat{x} \) is the bracket limit. More formally we can describe the tax systems by the functions \( T_z(z(w)), T_x(x_1(w)) \), where:
\[ T_z(z(w)) = t_1 z(w) \quad z(w) \leq \hat{z} \] (29)
\[ T_z(z(w)) = t_2[z(w) - \hat{z}] + t_1 \hat{z} \quad z(w) > \hat{z} \] (30)
\[ T_x(x_1(w)) = \tau_1 x_1(w) \quad x_1(w) \leq \hat{x} \] (31)
\[ T_x(x_1(w)) = \tau_2[x_1(w) - \hat{x}] + \tau_1 \hat{x} \quad x_1(w) > \hat{x} \] (32)
and so the consumer’s budget constraint takes the general form
\[ x_0(w) + \delta x_1(w) \leq g + z(w) - T_z(z(w)) - T_x(x_1(w)) \] (33)
where \( g > 0 \) is a lump sum payment per capita.

The presentation of the results is greatly simplified if we assume:

A. The optimal tax structure has \( t_2 > t_1, \tau_2 > \tau_1 \), so that the case of nonconvex budget sets for consumers is excluded.\textsuperscript{23}

B. The unique set of wage types for which in equilibrium under the optimal tax system \( z(w) = \hat{z} \) is identical to the unique set of wage types at which \( x_1(w) = \hat{x} \).

\textsuperscript{20}Because we are restricting possible tax functions to be piecewise linear we are not in the domain of the Atkinson/Stiglitz Theorem in this model.

\textsuperscript{21}Note that strict concavity of \( v(\cdot) \) implies strict convexity of \( \psi(\cdot) \).

\textsuperscript{22}Somewhat surprisingly perhaps, given its prevalence in practice, the literature on optimal piecewise linear taxation is quite small. See Apps, Long and Rees (2013) for a recent contribution and discussion of the literature.

\textsuperscript{23}For analysis of this case see Apps, Long and Rees (2013) and the evidence given there that this is an empirically reasonable assumption.
for each of according to where they fall on the budget constraint defined at the tax optimum complexity. As a result of it we can partition the set of wage types into 3 subsets based but is not difficult to relax, at the cost however of a big jump in notational complexity. We write their indirect utility function as

\[ L = u(w) + \lambda(w)[g + (1 - t_1)z - x_0 - \delta(1 + \tau_1)x_1] \] (34)

We model this subset’s problem as being to maximise utility subject to the same budget constraint as that in (34), but also subject to the constraints

\[ 1. \quad \Omega_1 = \{w \in \Omega \mid x(w) < \hat{x}, \ z(w) < \hat{z}\} \]

These individuals are in equilibrium in the lower of both tax brackets. Their Lagrange function is

\[ L = u(w) + \lambda(w)[g + (1 - t_1)z - x_0 - \delta(1 + \tau_1)x_1] + \mu(w)(\hat{x} - x) + \nu(w)(\hat{z} - z) \] (35)

Their indirect utility function is

\[ V_g = \lambda(w); V_{t_1} = -\lambda(w)z; V_{\tau_1} = -\lambda(w)\delta x_1 \] (36)

We drop the \( w \) index where no confusion is likely to result.

\[ 2. \quad \Omega_2 = \{w \in \Omega \mid x(w) = \hat{x}, \ z(w) = \hat{z}\} \]

We model this subset’s problem as being to maximise utility subject to the same budget constraint as that in (34), but also subject to the constraints

\[ x(w) \leq \hat{x}, \ z(w) \leq \hat{z}, \] which are binding at the optimum. Thus their Lagrange function is

\[ L = u(w) + \lambda(w)[g + (1 - t_1)z - x_0 - \delta(1 + \tau_1)x_1] + \mu(w)(\hat{x} - x) + \nu(w)(\hat{z} - z) \] (37)

Their indirect utility function is

\[ V_g = \lambda(w); V_{t_1} = -\lambda(w)\hat{z}; V_{\tau_1} = -\lambda(w)\delta \hat{x} \] (38)

\[ V_{\hat{x}} = \mu(w) = \rho u'(x_1) - \lambda(w)\delta(1 + \tau_1) > 0 \] (39)

\[ 3. \quad \Omega_3 = \{w \in \Omega \mid x(w) > \hat{x}, \ z(w) > \hat{z}\} \]

These individuals are in both upper tax brackets. They have the Lagrange function

\[ L = u + \lambda(w)[g + (1 - t_2)z + (t_2 - t_1)\hat{z} + \delta(\tau_2 - \tau_1)\hat{x} - x_0 - \delta(1 + \tau_2)x_1] \] (40)

and indirect utility function

\[ V_g = \lambda(w); V_{t_1} = -\lambda(w)\hat{z}; V_{\tau_1} = -\lambda(w)\delta \hat{x} \] (41)

\[ V_{t_2} = -\lambda(w)(z - \hat{z}); V_{\tau_2} = -\lambda(w)\delta(x - \hat{x}) \] (42)

\[ V_{\hat{x}} = \nu(w)\delta(\tau_2 - \tau_1) > 0; V_{\hat{z}} = \lambda(w)(t_2 - t_1) > 0 \] (43)

Assumption B is of course a special case and has no particular empirical basis but is not difficult to relax, at the cost however of a big jump in notational complexity. As a result of it we can partition the set of wage types into 3 subsets according to where they fall on the budget constraint defined at the tax optimum for each of \( z \) and \( x_1 \). In what follows we define each subset, give the appropriate Lagrange function for that household type’s optimisation problem, and specify the indirect utility functions and their derivatives, which are what we need for the optimal tax analysis. The purpose of this analysis is to characterise the optimal values of the tax parameters \( a, t_1, t_2, \tau_1, \tau_2, \hat{x}, \hat{z} \).

1. \( \Omega_1 = \{w \in \Omega \mid x(w) < \hat{x}, \ z(w) < \hat{z}\} \)

2. \( \Omega_2 = \{w \in \Omega \mid x(w) = \hat{x}, \ z(w) = \hat{z}\} \)

3. \( \Omega_3 = \{w \in \Omega \mid x(w) > \hat{x}, \ z(w) > \hat{z}\} \)

These individuals are in both upper tax brackets. They have the Lagrange function

\[ L = u(w) + \lambda(w)[g + (1 - t_1)z - x_0 - \delta(1 + \tau_1)x_1] \] (34)

We write their indirect utility function as

\[ V_g = \lambda(w); V_{t_1} = -\lambda(w)z; V_{\tau_1} = -\lambda(w)\delta x_1 \] (35)

2. \( \Omega_2 = \{w \in \Omega \mid x(w) = \hat{x}, \ z(w) = \hat{z}\} \)

We model this subset’s problem as being to maximise utility subject to the same budget constraint as that in (34), but also subject to the constraints

\[ x(w) \leq \hat{x}, \ z(w) \leq \hat{z}, \] which are binding at the optimum. Thus their Lagrange function is

\[ L = u(w) + \lambda(w)[g + (1 - t_1)z - x_0 - \delta(1 + \tau_1)x_1] + \mu(w)(\hat{x} - x) + \nu(w)(\hat{z} - z) \] (36)

Their indirect utility function is

\[ V_g = \lambda(w); V_{t_1} = -\lambda(w)\hat{z}; V_{\tau_1} = -\lambda(w)\delta \hat{x} \] (37)

\[ V_{\hat{x}} = \mu(w) = \rho u'(x_1) - \lambda(w)\delta(1 + \tau_1) > 0 \] (38)

\[ V_{\hat{z}} = \nu(w) = \lambda(w)(1 - t_1) - \psi'(z_1) > 0 \] (39)

3. \( \Omega_3 = \{w \in \Omega \mid x(w) > \hat{x}, \ z(w) > \hat{z}\} \)

These individuals are in both upper tax brackets. They have the Lagrange function

\[ L = u + \lambda(w)[g + (1 - t_2)z + (t_2 - t_1)\hat{z} + \delta(\tau_2 - \tau_1)\hat{x} - x_0 - \delta(1 + \tau_2)x_1] \] (40)

and indirect utility function

\[ V_g = \lambda(w); V_{t_1} = -\lambda(w)\hat{z}; V_{\tau_1} = -\lambda(w)\delta \hat{x} \] (41)

\[ V_{t_2} = -\lambda(w)(z - \hat{z}); V_{\tau_2} = -\lambda(w)\delta(x - \hat{x}) \] (42)

\[ V_{\hat{x}} = \nu(w)\delta(\tau_2 - \tau_1) > 0; V_{\hat{z}} = \lambda(w)(t_2 - t_1) > 0 \] (43)

We drop the \( w \) index where no confusion is likely to result.
Note also that $t_1, \tau_1, \hat{x}, \hat{z}$ have income effects on the demands for $x_1$ and $z$ for consumers in $\Omega_3$.

We are now in a position to solve the optimal tax problem. The planner wants to maximise the utilitarian social welfare function
\[
\int_{\Omega_1} V(g, t_1, \tau_1) d\Phi + \int_{\Omega_2} V(g, t_1, \tau_1, \hat{x}, \hat{z}) d\Phi + \int_{\Omega_3} V(g, t_1, t_2, \tau_2, \hat{x}, \hat{z}) d\Phi
\] (44)
subject to the budget constraint
\[
\int_{\Omega_1} (t_1 z + \delta \tau_1 x_1) d\Phi + (t_1 \hat{z} + \delta \tau_1 \hat{x}) = \int_{\Omega_2} (t_2(z - \hat{z}) + \delta(\tau_2(x_1 - \hat{x}) + \tau_1 \hat{x})) d\Phi \geq g + G
\] (45)

We then have:

Result 4: The condition corresponding to $g$ takes the form
\[
\int_{\Omega} (m(w) - 1) d\Phi = 0
\] (46)
where $m(w)$ is the marginal social utility of income, defined to be net of effects on tax revenue arising from income effects of the tax parameters, to individuals of type $w$.

Therefore the optimal $g$ equates the average value of the marginal social utility of income across the population to its marginal social cost of 1, a result familiar from standard optimal linear tax theory.

Result 5:
The conditions characterising the four marginal tax rates can be written as:
\[
t_1 \hat{s}_{zt} + \tau_1 \hat{s}_{xt} = \int_{\Omega_1} (m(w) - 1) z d\Phi + \hat{z} \int_{\Omega_2 \cup \Omega_3} (m(w) - 1) d\Phi
\] (47)
\[
t_1 \hat{s}_{zt} + \tau_1 \hat{s}_{xt} = \int_{\Omega_1} (m(w) - 1) x_1 d\Phi + \hat{x} \int_{\Omega_2 \cup \Omega_3} (m(w) - 1) d\Phi
\] (48)
\[
t_2 \hat{s}_{zt} + \tau_2 \hat{s}_{xt} = \int_{\Omega_3} (m(w) - 1)(z - \hat{z}) d\Phi
\] (49)
\[
t_2 \hat{s}_{zt} + \tau_2 \hat{s}_{xt} = \delta \int_{\Omega_3} (m(w) - 1)(x_1 - \hat{x}) d\Phi
\] (50)
where $\hat{s}_{xt}, \hat{s}_{zt}$ are the compensated Hicksian derivatives in brackets 1 and 2 respectively, averaged over the relevant subsets of wage types, $\Omega_1$ for (47), (48) and $\Omega_3$ for (49), (50). Those characterising $x$ and $\hat{z}$ respectively are
\[
\int_{\Omega_2} (\frac{\mu(w)}{\lambda} + \delta \tau_1) d\Phi = -\delta(\tau_2 - \tau_1) \int_{\Omega_3} (m(w) - 1) d\Phi
\] (51)
\[
\int_{\Omega_2} (\frac{\gamma(w)}{\lambda} + t_1) d\Phi = -(t_2 - t_1) \int_{\Omega_3} (m(w) - 1) d\Phi
\] (52)
where $\lambda$ is the shadow price of the government’s budget constraint.

As an aid to interpreting these conditions, first assume the cross-price derivatives $\bar{s}_{zt}^1, \bar{s}_{zt}^2 = 0$. We can then write the first four conditions as

$$t_1 = \frac{\int_{\Omega_1} (m(w) - 1) zd\Phi + \hat{z} \int_{\Omega_2 \cup \Omega_3} (m(w) - 1) d\Phi}{\bar{s}_{zt}^1}$$  (53)

$$\tau_1 = \frac{\int_{\Omega_1} (m(w) - 1)x_1 d\Phi + \hat{x} \int_{\Omega_2 \cup \Omega_3} (m(w) - 1) d\Phi}{\bar{s}_{zt}^1}$$  (54)

$$t_2 = \frac{\int_{\Omega_2} (m(w) - 1)(z - \hat{z}) d\Phi}{\bar{s}_{zt}^2}$$  (55)

$$\tau_2 = \frac{\int_{\Omega_3} (m(w) - 1)(x_1 - \hat{x}) d\Phi}{\bar{s}_{zt}^2}$$  (56)

Each of these defines the optimal tax rate as determined by a trade off between distributional equity, represented by the numerator term, and allocative efficiency, the denominator term. The latter is the average over the relevant subset of the compensated derivative of respectively earnings (labour supply) or first period consumption (saving) with respect to the corresponding tax rate, as a measure of marginal deadweight loss.

We give the interpretation of the numerator terms for the case of earnings taxation, those for capital income taxation follow similarly. In (53), the first term in the numerator is a measure of the closeness of the association between the marginal social utility of income $m(w)$ and labour earnings over the individuals in the lower tax bracket. Unlike the case of straightforward linear taxation, this is not strictly a covariance, since $1$ is the population average of $m(w)$.

We expect $m(w)$ to be decreasing with wage type, in which case this term could conceivably be positive which, since the denominator is negative, could suggest a negative tax rate. However, the second term is certainly negative and it can be shown that if $m'(w) < 0, \forall w \in \Omega$, the numerator overall must be negative. This second term reflects the fact that varying $t_1$ yields an intramarginal income gain to the individuals in the subset $\Omega_2$ and in the higher tax bracket $\Omega_3$, that is proportional to $\hat{x}$, and this represents a social cost since the average of $m(w)$ across these subsets must be less than the population average. In the case of the higher bracket tax rate the second type of term is of course absent because there is no higher bracket.

The two conditions (51) and (52) characterise the choice of bracket limits $\hat{x}$ and $\hat{z}$ respectively. The left hand sides of these conditions represent the marginal social benefit of a small relaxation of the corresponding bracket limit. This consists of the gain in utility to the wage types (in subset $\Omega_2$) who are effectively constrained at that bracket limit and are able now to increase their saving or consumption and achieve a net utility increase, together with the gain in tax revenue resulting from the corresponding increases in saving and labour supply. The right hand sides of these conditions represent the marginal social costs of relaxing the bracket limits, which take the form of the lump
sum income gains to those individuals in the highest tax bracket, for whom the average difference from the population average marginal social utility of income

\[ \int_{w_0}^{w_1} (m(w) - 1) d\Phi \]

is negative.

The full set of conditions in (47)-(52) reflect the fact that the tax rate on labour earnings in a given tax bracket will affect at the margin the saving of individuals in that bracket, and the tax on capital income will likewise affect labour supply, while at the same time lower bracket tax rates have only income effects on saving and labour supply decisions in the higher brackets. They also serve to emphasise the fact that under dual taxation all tax rates, on earnings from capital as well as labour, should be jointly determined, and in general there is no reason to expect at the optimum either that \( \tau_1 = 0 \), or that \( \tau_2 = t_1 \).

5 Choosing the tax base: income vs consumption

The previous section showed that when the tax system is restricted to be piecewise linear there is no plausible case for exempting capital income, that is, for treating capital income as fundamentally different to labour earnings. In this section we turn to a more explicit discussion of the choice of the tax base.

It is fair to say that at least since the Meade Report (1978) there has been a strong body of opinion among economists in favour of moving from income to consumption taxation. The most recent expression of this is in the Mirrlees Commission Review\(^{25}\) which presents three main ways in which this can be achieved:

1. A cash flow expenditure tax, or **EET**: Withdrawals from income for saving are tax exempt (E), income from saving is tax exempt (E), and then the proceeds are taxed when spent on consumption (T).

2. A labour earnings tax with exemption for income from saving, or **TEE**: Saving is made out of taxed income with no exemption (T), but returns (E) and final consumption of proceeds (E) are tax-exempt.

3. An income tax with an "allowed rate of return", or **TtE**: Saving is made out of taxed income (T), returns to saving at a rate below a certain level, the "normal rate of return", are tax exempt, while returns above this are taxed at the same rate as labour earnings (t), and withdrawal of savings for consumption are tax-exempt (E).

In rationalising these proposals, the review adopts as a "guiding principle" the neutrality of capital income taxation as the basis for determining tax policy. This principle asserts that there should be no distortion of the time pattern of consumption chosen by households and no distortion of their allocation of saving among assets. Now the central principle of second best theory is that in general, given an unavoidable distortion in one sector of the economy, for example that created in the labour market by the taxation of labour earnings, it will in general be (second best) optimal to create distortions in related sectors,

\(^{25}\)See Mirrlees et al. (2011).
for example in the capital market by taxing the income from saving. Then, we are necessarily concerned with the optimal levels of two instruments, taxes on labour and on capital income, and the relationship between them. This has been the approach in the previous two sections of this paper. Although the Review discusses arguments in support of applying this principle in the case of capital income taxation at some length, in the end it concludes that "it would be better to make neutrality the central goal of savings tax policy".26

The core of the review’s approach is captured by the following quotation:27

"...in an ideal world, we would like to tax people according to their life time earning capacity - broadly equivalent to their potential consumption[....]. It might appear that taxing savings is an effective way to redistribute [....]. But someone with savings is not necessarily better off over their life time than someone without savings. The two might earn and spend similar amounts over their lifetimes, but at different times: one earns his money when young and saves it to spend when he is old, while for the other the timings of earning and spending are close together. We can tax people on their total resources by taxing their money income at its source (taxing earnings) or when it is finally used for consumption (taxing expenditure). We can tax better-off people more heavily by making the rate scale applied to earnings or expenditure more progressive. If [...] people’s saving decisions tell us nothing about their underlying earning capacity, just about their taste for consuming tomorrow rather than today, then taxing saving cannot help us to target high ability people more accurately than taxing earnings or expenditure."

To paraphrase: we should not tax savings that simply result from differences either in time preferences for consumption or in the timing of endowed incomes relative to the desired time stream of consumption; there is an equivalence in a life cycle sense between taxing consumption and taxing labour earnings; and time preferences for consumption, if heterogeneous, are uncorrelated with an individual’s productivity type.28

However, the assumption underlying the model on which this view rests is that the unit of taxation is a single individual dividing his time between work and leisure and using a perfect capital market to allocate consumption over his life time in accordance with his preferences for consumption of goods vs. leisure at various points in time. In that context the view is plausible. Such households might conceivably exist. The more typical case however is a household with two actual or potential earners in which household production, particularly child care, is an important form of time use. This observation has

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26Interestingly, this conclusion is in contradiction to the comment made in a working paper contributed to the Review by Banks and Diamond (2008), that the models from which the arguments supporting this conclusion flow (primarily Atkinson and Stiglitz (1976), Chamley (1986) and Judd (1985)) are based on "considerations of economic behaviour and the nature of economic environments that are too restrictive when viewed in the context of both theoretical findings in richer models and the available econometric evidence". See also Banks and Diamond (2011) and Auerbach (2009) for fuller discussion of the more recent literature that underpins this view.

27Op cit., p 293.

28It is this assumption that is relaxed in Saez (2002).
we believe far-reaching consequences for the type of proposal, as well as the underlying arguments, presented in the Review:

- Because of the importance of household production, household income, whether measured in a single period or over a lifetime, is not a reliable measure of the capacity of the household to generate wellbeing for its members, which is the type of capacity that we really want to tax. The same applies to consumption expenditure on market goods.\(^\text{29}\) This point has particular force when there is significant heterogeneity across households in the second earner’s market labour supply and therefore production of household goods.

- When there are two earners, choice of the tax base, whether individual or joint income, becomes of central importance to the analysis of both the efficiency and equity effects of taxation.\(^\text{30}\)

One important consideration in the comparison of consumption taxation with the taxation of labour income is that it is not possible, without making arbitrary assumptions, to tax consumption \emph{per se} individually, since, as already pointed out, the individual consumptions of household members are not observable, and so consumption taxation is necessarily joint. Earnings can of course be individually observed, assigned and taxed. An important advantage of labour income taxation is then that the tax rates can be varied on individual labour earnings across households with the same total joint income but differing relative contributions of the two earners. This allows the choice of tax parameters that have the effect of taxing indirectly part of the untaxed additional production in households where this is relatively high. Given the high degree of heterogeneity in second earner labour supply across households, this will in general achieve greater efficiency and equity than a tax on household consumption.\(^\text{31}\) When individual incomes provide a better tax base on both equity and efficiency grounds, moving from taxation of earnings to taxation of consumption risks a worsening of the tax system in both these respects. We now go on to provide a somewhat more formal discussion of these points.

The idea underlying the quotation from the Mirrlees Review can be illustrated very simply. Take the model in which an individual of type \(w\) solves the problem

\[
\max_{x_0, x_1, z} u(w) = u(x_0(w)) - \psi(z(w)) + \rho u(x_1(w))
\]  

\(^{29}\) Virtually all economists would agree that the values of neither imports nor exports would be a good measure of the standard of living of a country or economy, but some seem to find it difficult to see that the same is true for market labour income ("exports") and expenditure on market goods ("imports") of a family household. In both cases non-traded goods produced for domestic consumption are a significant component of total household income and consumption.


\(^{31}\) This is discussed at much greater length in Apps and Rees (2009), Chs 6-9. See also Apps and Rees (2012).
subject to
\[ x_0(w) + \delta x_1(w) \leq z(w) \] (58)

Her consumption/saving choice is undistorted if the condition
\[
\frac{u'(x_0)}{u(x_1)} = \frac{\rho}{\delta}
\] (59)

holds. This will be satisfied if taxation implies that her budget constraint takes one of the following three equivalent forms:\textsuperscript{32}

**Consumption taxation:**
\[ (1 + \tau)[x_0(w) + \delta x_1(w)] \leq z(w) \] (60)

**TEE:**
\[ x_0(w) + \delta x_1(w) \leq (1 - t)z(w) \] (61)

which yields the same consumption path as (60) if \( t = \tau/(1 + \tau) \).

**EET:**
\[
(1 + \tau)x_1(w) = (1 + r)s
\] (63)

which again yields the same consumption path as (60) if \( t = \tau/(1 + \tau) \).

In each case consumption choices are undistorted while of course labour supply decisions are distorted, with
\[
\frac{u'(x_0)}{\psi'(z(w))} \neq 1
\] (64)

Consider now however a model of a two-earner household in which household production is a significant form of time use and, moreover, there is heterogeneity in the second earner’s labour supply across each subset of households with the same wage pair \((w_1, w_2)\), which now defines a household’s type, due to variations in her productivity in household production.\textsuperscript{33} It can be shown\textsuperscript{34} that the household’s problem now becomes

\[
\max_{x_0, x_1, z_1} u(w_1, w_2) = u(x_0(w_1, w_2)) - \psi_1(z_1(w_1, w_2)) - \psi_2(z_2(w_1, w_2)) + \rho u(x_1(w_1, w_2))
\] (65)

subject to
\[ x_0(w_1, w_2) + \delta x_1(w_1, w_2) \leq z_1 + z_2 \] (66)

\textsuperscript{32}The EtT form cannot be assessed in this simple framework because all saving earns only the "normal rate of return" \( r \). Full discussion of this case really requires a model with super-normal returns to saving and/or risk.

\textsuperscript{33}This model, and its implications for optimal piecewise linear taxation systems, are more fully analysed in Apps and Rees (2012).

\textsuperscript{34}See Appendix.
Restricting taxation to linear or piecewise linear systems, how would we interpret the Review’s proposals in this case? Clearly full equivalence of consumption taxation and EET requires the latter to involve joint taxation. Put another way, a move to consumption taxation from an individual-based earnings taxation system without capital income taxation, effectively a TEE system but with differentiated marginal tax rates between primary and second earners for many households, involves a move from individual to joint taxation. In such a move, account would have to be taken of the degree of progressivity in earnings tax rates under the TEE system in defining the marginal rate structure of a piecewise linear consumption tax system. It is very hard to see that such a move would be a costless transition between equivalent tax systems.

Under the EET system, the question would arise of how the exemption for saving would be assigned to the individual households. It is impossible, as already pointed out, to observe the reductions in consumption of each member of the household that have contributed to the household’s overall saving. If left to the households themselves, clearly the incentive is to assign it to the individual with the higher marginal tax rate, where these differ. This is likely to lead to, effectively, a significant decline in the progressivity of the tax system. Again therefore it is hard to believe that the simple equivalence suggested by the model underlying the recommendations of the Mirrlees Review would hold in practice.

Furthermore, consider the case where two households, A and B, have the same market consumption expenditure, and so would be taxed at the same rate and pay the same amount of tax under a consumption tax system. Household A consists of two low-wage individuals working full time and therefore buying in market goods that are close substitutes for household goods they would otherwise produce themselves. Household B consists of two high wage individuals with only the primary earner working in the market and the second earner working at home to produce a significant amount of untaxed household goods. This household could have a higher household income than A, and so would be saving more. A move from individual-based earnings taxation to joint consumption taxation is then likely to reduce both equity and efficiency. The regressive effects are obvious, but also the move from individual to joint taxation that is implied reduces the incentive for the second earner in household A to work in the market and induces her to substitute domestic for market production.

6 Conclusions

To be completed

Appendix (to be completed)
Bibliography (to be completed)
References


